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Influence of Steel Hardness in Face-Milling

By J. B. ARMITAGE¹ AND A. O. SCHMIDT,² MILWAUKEE, WIS.

Face-milling tests are reported on seven different kinds of steels for power consumption and tool life at hardnesses of approximately 200, 300, and 400 Bhn. At any given hardness power requirements varied but little, but differences in tool life were considerably greater. Machining the workpiece at an elevated temperature was found to decrease the power required and permit good cutter life. When tool-life or production tests are not available, the hardness of steel can be taken as a guide in choosing preliminary values of cutting speed and feed.

SEVEN different kinds of steels were tested for power consumption and tool life at hardnesses of approximately 200, 300, and 400 Bhn. Differences in power required at any given hardness were comparatively small but the differences in tool life were more noticeable. Tool life was practically zero when the 400-Bhn test pieces were milled at 428 fpm cutting speed, the same speed as used on 200-Bhn test bars. However, a decrease in the cutting speed to 130 fpm and in the feed to 0.0045 ipt caused an increase in the tool life to a good performance when milling test pieces of 400 Bhn.

After machining a steel workpiece of 200 Bhn for some time, the power increased because of tool wear and was frequently higher than that used by a new tool cutting a much harder workpiece. However, a cutter operating on a softer material generally held up better and did not fail as readily as one machining a hard workpiece. One explanation of this phenomenon is that continuous machining must have worn the cutter in such a way that its shape was stronger, or that originally there were no weak spots or fine cracks in the blade which might have caused early failure.

The machining of hard alloy steels is still a major problem. It requires a powerful rigid machine, such as is available only in the latest models, and also a cutter able to withstand high cutting forces, a large number of heavy impacts per blade, and designed in such a way as to eliminate chip interference. Machining the workpiece at an elevated temperature was found to decrease the power required and permit good cutter life.

DETAILS OF MILLING TESTS

When other machinability data are not readily available, the hardness of steel is frequently taken as an index by shopmen in arriving at preliminary feed and speed estimates. The milling tests reported herein constitute an effort to show the relationship of hardness to power requirements and tool wear or tool life. A grade of sintered tungsten titanium carbide, which had been proved by laboratory and shop experience to be suitable for most carbide-steel milling operations, was used as the tool material in all tests.

For tool-life determinations, test bars $1\frac{1}{2} \times 6 \times 12$ in. of seven different steels were heat-treated to approximate Brinell hardness values of 200, 300, and 400. The steels tested included

NE 8744, NE 9445, SAE 1055, SAE 3140, SAE 4150, SAE 4150 sulphurized, and SAE 4335.

Comparative horsepower requirements were determined with the aid of a calorimeter (1).³ In the latter tests, bars of 1 in. diam of the same composition and heat-treatment as the tool-life test bars were used.

The test cutters were 3-in.-diam face mills with two solid-carbide inserted blades. The blades were ground to 12 deg negative radial rake, 0 deg axial rake, and 15 deg corner angle. All tests were carried out on a Kearney & Trecker 2K vertical milling machine.

In comparative power tests, the results for different materials were similar for each range of hardness. Fig. 1 illustrates the horsepower requirements for all the steels tested. As in previous tests these cutters were considered sharp, since they did not remove more than 0.3 cu in. of material. There was thus no change in the power requirements due to tool wear (2). At 200 Bhn there is little scattering of individual data points. The same test bars at 300 Bhn require more power than at 200 Bhn, and the performances are also more erratic, as indicated by the increased scattering of values for this hardness range. A further increase in power requirement is accompanied by a further increase in scattering at 400 Bhn.

In the tool-life tests the test bar ($1\frac{1}{2} \times 6 \times 12$ in.) was held in a vise and face-milled at a cutting speed of 430 fpm, a feed of 0.0115 ipt, and a depth of 0.125 in. for the 200 and 300-Bhn specimens. It was found necessary to reduce the cutting speed to 130 fpm and the feed to 0.0045 ipt when milling the 400-Bhn samples, since the tool life was practically nothing (only a few chips), and the type of failure was very undesirable at the former cutting speed and feed. The tool-life tests at 200 and 300 Bhn were made with both blades cutting. However, when tests were made for the purpose of taking photographs of tool wear after every pass or every other pass, only one blade was cutting, the other blade having been moved in far enough so that it could not possibly machine any of the test bar. This same procedure was adopted in milling the 400-Bhn bars in order to obtain more uniform results. A graphical presentation of tool life for the 200 and 300-Bhn steel bars is given in Fig. 2, and the tool-life data for 400-Bhn steels are given in Fig. 3, since these data were obtained at a lower speed and feed and are not directly comparable to those in Fig. 2.

These test results have been confirmed in many shop operations. A hard steel must be milled at lower speeds and feeds, usually between 150 and 300 fpm with a feed per tooth between 0.003 and 0.010 in. General recommendations (3) indicate a cutting speed of 350 to 800 fpm and a feed per tooth of 0.010 to 0.020 in. for mild steel, which is usually considered to mean steel not harder than 200 Bhn. Medium-hard steels, which would include steels around 300 Bhn, should be milled in the cutting-speed range of 250 to 400 fpm with feeds per tooth ranging from 0.007 to 0.015 in. Although these recommendations cover considerable latitude, they nevertheless permit a start or proper approach to the ultimate selection of optimum speed and feed. Speeds in the upper part of a recommended range and feeds in the lower part of a recommended range should generally be chosen if the cut is to be a finishing operation. Just the opposite procedure should be used in the case of roughing operations.

³ Numbers in parentheses refer to the Bibliography at the end of the paper.

¹ Vice-President, Kearney & Trecker Corporation. Fellow ASME.

² Research Engineer, Kearney & Trecker Corporation. Mem. ASME.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society. Paper No. 48–A-30.

TABLE 1 HEAT-TREATMENT OF STEEL TEST BARS

Steel	Bhn	Heat to deg F	Heat treatment ^a Quench	Draw deg F
NE 8744	217	1550	Oil	1300
NE 8744	293	1550	Oil	1050
NE 8744	418	1550	Oil	750
NE 9445	212	1550	Oil	1250
NE 9445	302	1550	Oil	950
NE 9445	418	1550	Oil	600
SAE 1055	196	1550	Oil	1200
SAE 1055	302	1550	Brine	900
SAE 1055	387	1550	Brine	850
SAE 3140	223	1550	Oil	1200
SAE 3140	311	1550	Oil	1000
SAE 3140	387	1550	Oil	600
SAE 4150	217	1550	Oil	1200
SAE 4150	321	1550	Oil	1000
SAE 4150	418	1550	Oil	750
SAE 4150 ^b	196	1550	Oil	1350
SAE 4150 ^b	311	1550	Oil	1150
SAE 4150 ^b	418	1550	Oil	850
SAE 4335	202	1550	Oil	1350
SAE 4335	302	1550	Oil	1200
SAE 4335	403	1550	Oil	900

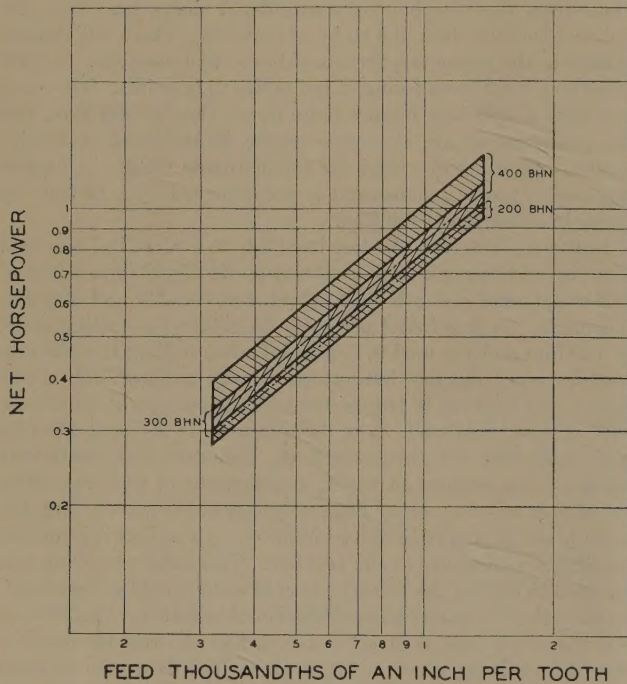
^a Cool in furnace for all steels.^b Sulphurized.

FIG. 1 HORSEPOWER REQUIREMENTS FOR STEELS OF DIFFERENT HARDNESS

(This graph is based upon the materials listed in Table 1.)

As can be seen in Figs. 1, 2, and 3, hardness is only an approximate indicator of machinability. A power increase of only about 10 per cent is indicated in Fig. 1, between the hardnesses of 200 and 300 Bhn for the same steel, while the tool life decreased more than 100 per cent, as can be seen from the number of passes plotted in Fig. 2. The power increased only about 25 per cent when the hardness of the steel was increased from 200 to 400 Bhn, but had the tool life at 400 Bhn also been shown in Fig. 2, it would have been close to zero in all cases.

WEAR OF TUNGSTEN-TITANIUM BLADES

When milling under the most favorable conditions, breakage of carbide blades will seldom occur. Should the setup and machine be such that vibration, deflection, and overload are absent, or at a minimum, a number of other factors will still influence the performance of the carbide blade. Even when feeds and speeds

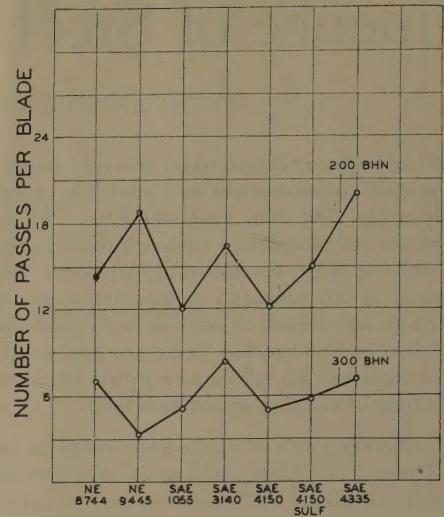


FIG. 2 AVERAGE NUMBER OF PASSES PER BLADE WHEN MILLING STEEL OF 200 AND 300 BHN

(Cutting speed in these tests was 428 fpm and feed 0.0115 ipt.)

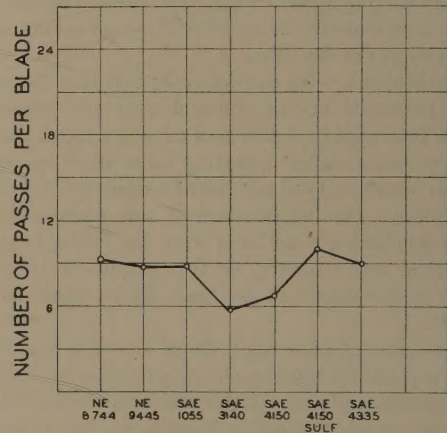
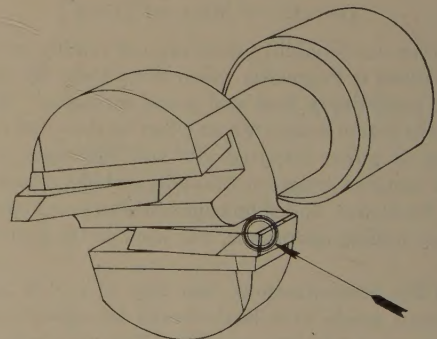
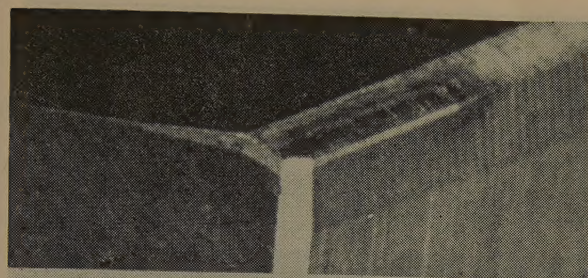


FIG. 3 AVERAGE NUMBER OF PASSES PER BLADE WHEN MILLING STEEL OF 400 BHN

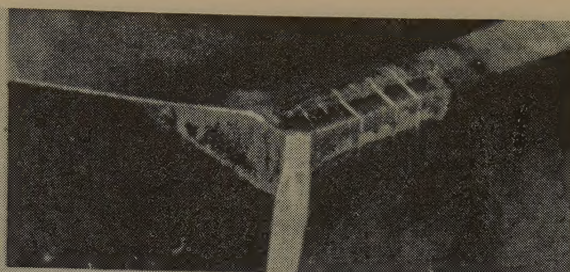
(Cutting speed, 130 fpm; feed 0.0045 ipt.)

FIG. 4 CUTTER USED IN WEAR TESTS
(Part shown in Figs. 5, 6, 7, is encircled.)

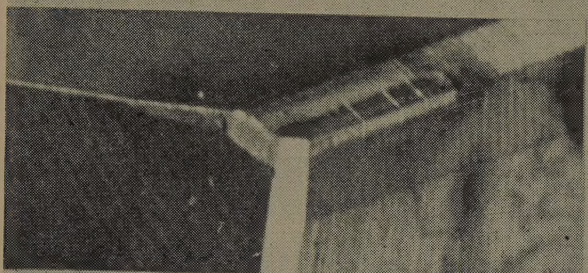
are chosen properly and all conditions over which an operator has control are kept constant, various types of tool wear and failure will result. In analyzing performance data of a set of cutters run for several months on the same kind of workpiece, eight different types of wear (4) were established, four of which were classed as objectionable because blades in which these types of



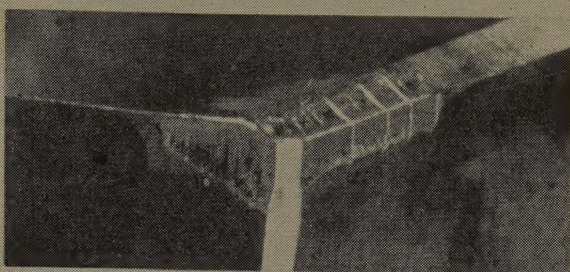
2 PASSES



12 PASSES



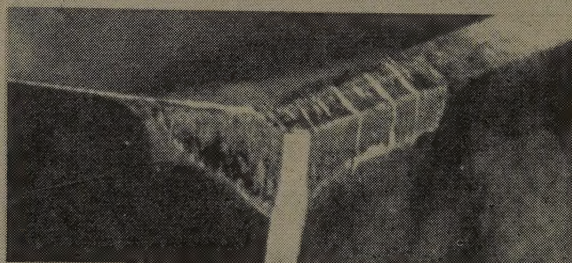
4 PASSES



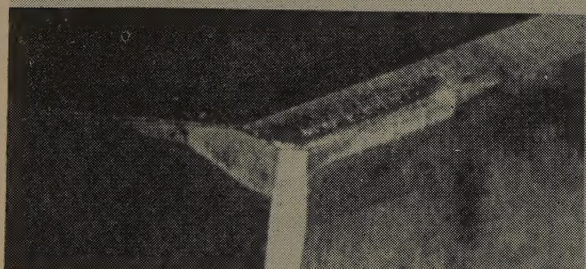
14 PASSES; $\frac{1}{32}$ -IN. PERIPHERAL WEAR



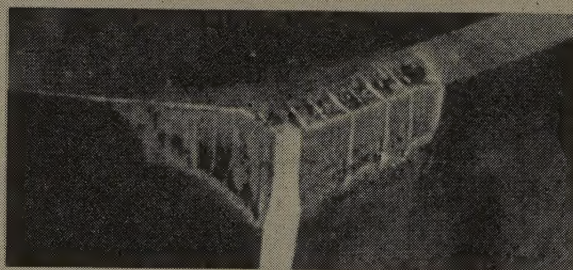
6 PASSES



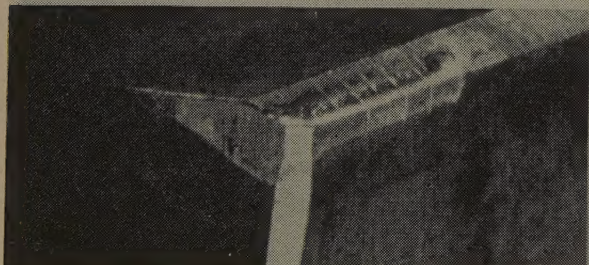
16 PASSES



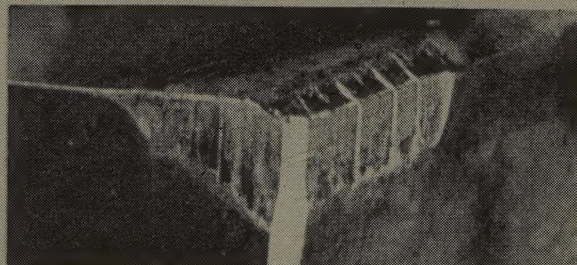
8 PASSES



18 PASSES



10 PASSES



20 PASSES; $\frac{1}{16}$ -IN. PERIPHERAL WEAR

FIG 5 PROGRESSIVE WEAR ON A TUNGSTEN-TITANIUM CARBIDE MILLING TOOTH WHEN MILLING STEEL WORKPIECES OF 200 BHN (Cutting speed, 428 fpm; feed, 0.0115 ipt. For section and position of blade see Fig. 4.)

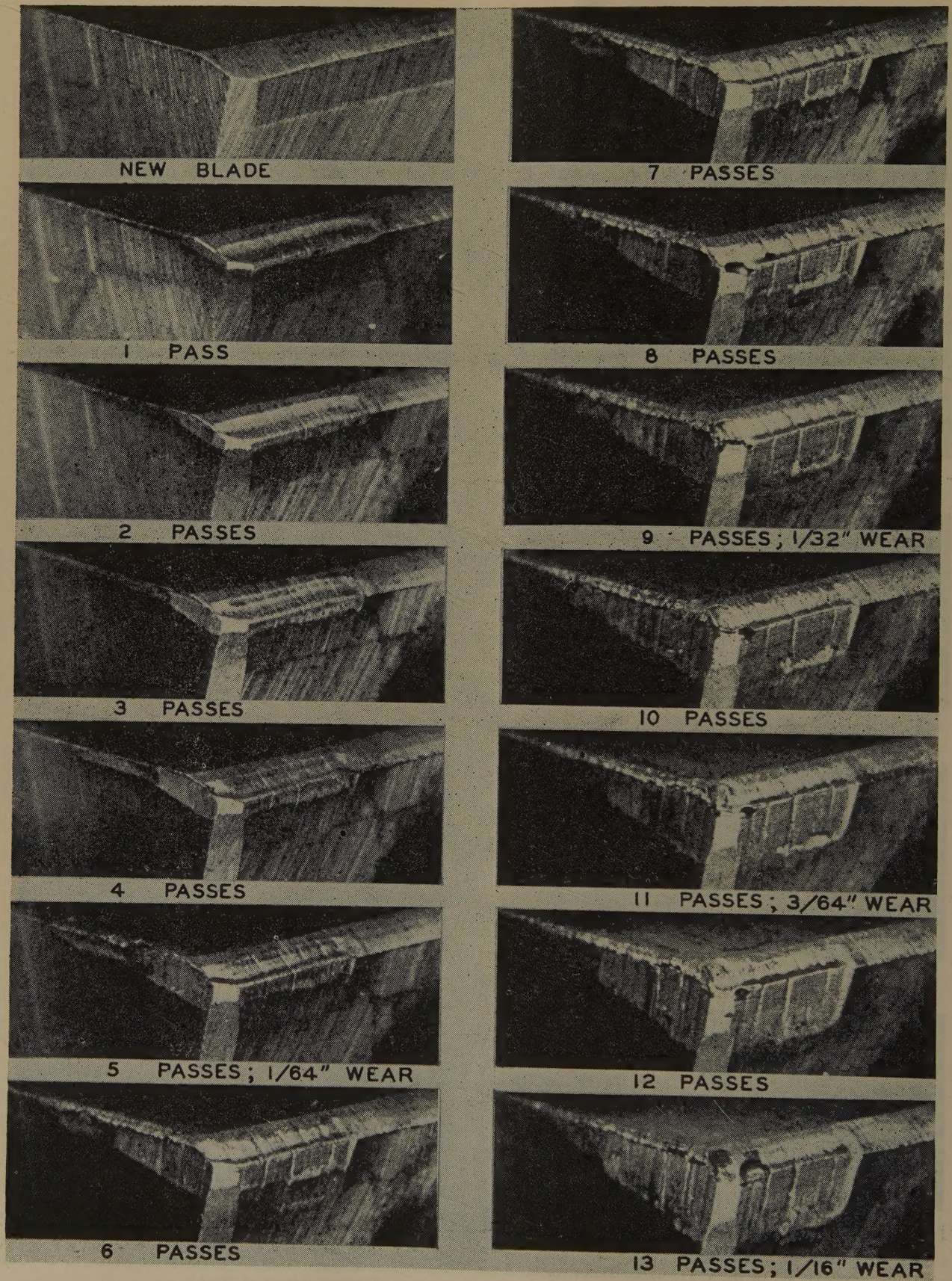


FIG. 6 PROGRESSIVE WEAR ON A TUNGSTEN-TITANIUM CARBIDE MILLING TOOTH WHEN MILLING STEEL WORKPIECES OF 300 BHN
(Cutting speed, 428 fpm; feed, 0.0115 ipt. For section and position of blade see Fig. 4.)

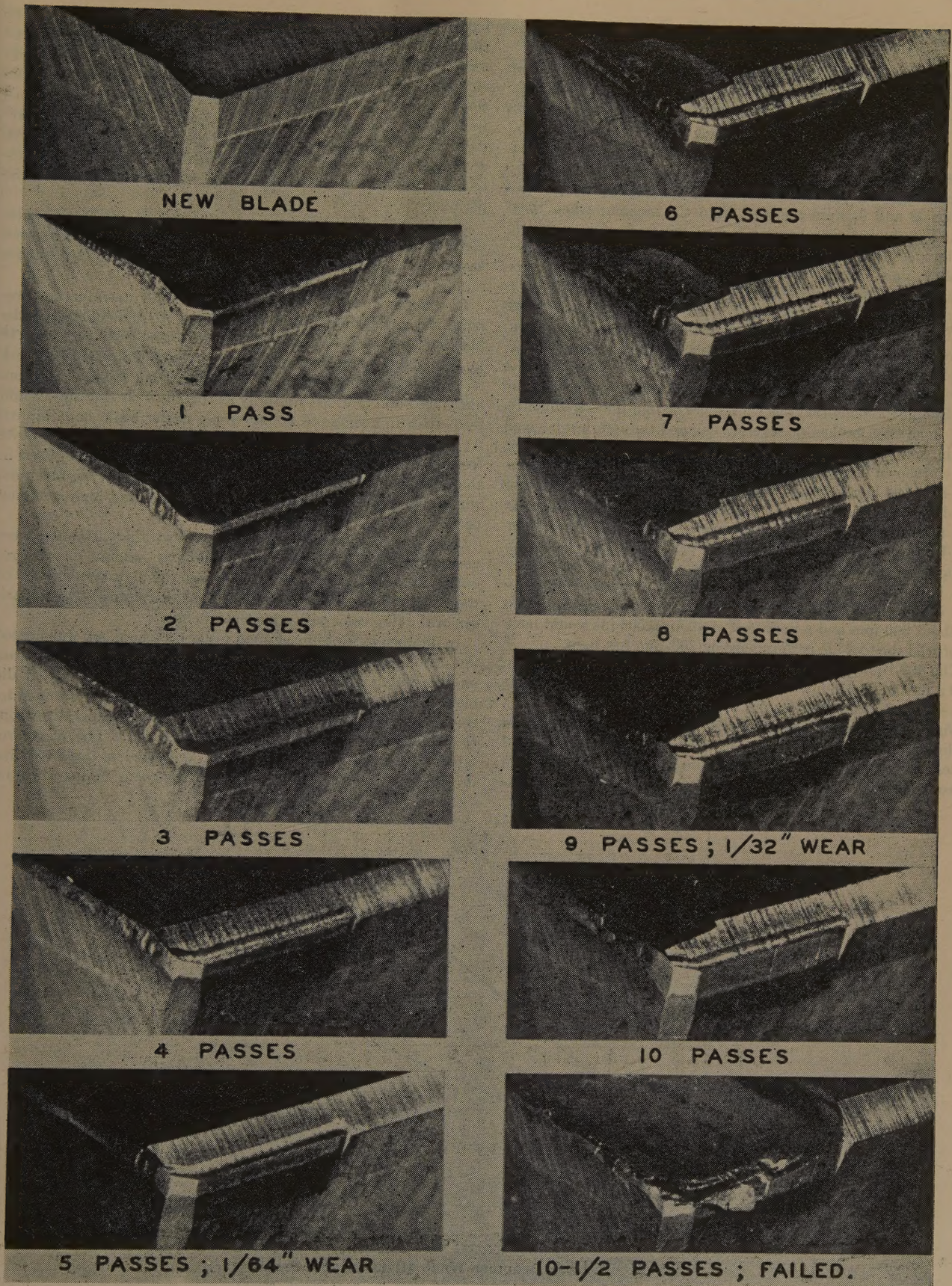


FIG. 7 PROGRESSIVE WEAR ON A TUNGSTEN-TITANIUM CARBIDE MILLING TOOTH WHEN MILLING STEEL WORKPIECES OF 400 BHN
(Cutting speed, 130 fpm; feed, 0.0045 ipt. For section and position of blade see Fig. 4.)

wear occurred could not be reconditioned easily by simple re-grinding.

Progressive wear on a tungsten-titanium carbide blade is illustrated in Fig. 5. A freshly ground carbide milling blade was photographed after each pass on a 200-Bhn steel test bar $1\frac{1}{2}$ in. wide and 12 in. long when face-milling at a depth of 0.125 in. with a feed of 0.011 ipt at a cutting speed of 428 fpm. After 14 passes, which was an average performance in these tests, the peripheral wear was $\frac{1}{32}$ in. Cutters whose performances are plotted in Figs. 2 and 3, were removed from the machine when they showed $\frac{1}{32}$ -in. peripheral wear. It is at this stage of wear, which can be measured easily, that cutters should be removed for re-grinding, since the rate of wear will increase rapidly beyond this point and the likelihood of spalling, chipping, and breakage of the blade will become greater. When tool wear has progressed this far, the power required generally will have increased between 30 and 40 per cent. Re-grinding a blade at this point will be simple and economical. The few extra passes that may be obtained beyond this point do not justify the increase in cost of re-conditioning a blade that has worn more or that quite frequently may have been broken or severely chipped through use beyond $\frac{1}{32}$ -in. peripheral wear.

As is shown in Fig. 6, the tool will wear more rapidly when a 300-Bhn test bar is milled under the same conditions as was the 200-Bhn test bar. After the ninth pass $\frac{1}{32}$ -in. peripheral wear had been reached. This performance is better than average for 300-Bhn test bars plotted in the lower part of Fig. 3. Generally, chipping of the blade, which can be seen after the eighth pass in Fig. 6, occurred much earlier and the cutter frequently failed before it had worn $\frac{1}{32}$ in. on the periphery. A slower cutting speed of about 300 fpm and a finer feed of about 0.006 ipt would have improved the tool-life performance on 300-Bhn test bars.

The 400-Bhn specimens were machined at a speed of 130 fpm and a feed of 0.0045 ipt, since the tool life was practically zero at the speed and feed used for the 200 and 300-Bhn samples.

Fig. 7 illustrates progressive wear of the carbide blade when milling a 400-Bhn steel test bar of the same dimensions as used in tests on softer material. These photographs cannot be used in direct comparison with those in Figs. 5 and 6 since the conditions

have been changed. The wear on the carbide tip is uniform for the first five passes and after the sixth pass flaking can be observed, but this flaking is on a part of the blade which does no cutting. About midway through the eleventh pass a large portion of the carbide blade face had flaked off, and, with a little continued cutting, a crater had begun to form on the new rough surface. It is this flaking and chipping of the carbide tip which can be observed when milling harder workpieces. Cutters will fail frequently before they have reached the $\frac{1}{32}$ -in. wear mark on the periphery.

Even when milling the same workpieces of mild steel under identical conditions, with the exception that cutting speeds were varied from 130 to 3300 fpm, the same carbide tips wore more rapidly at the higher cutting speeds (3).

Low cutting speeds and feeds and a rigid machine are prerequisites for milling materials difficult to machine, e.g., die blocks used in drop-forging. A number of die blocks $24 \times 6 \times 18$ in., which had to be reworked, were face-milled at 0.250 in. depth of cut with a 10 in. C.S.M. cutter at a cutting speed of 100 fpm and a feed of 2 ipm. It was only at these low cutting speeds and feeds that an average of 12 passes per cutter could be obtained. At higher cutting speeds and feeds the cutter would fail quite frequently after only one or two passes. The machine used in these tests was a Kearney & Trecker C.S.M. bed-type machine, which due to its rigid, bed-type construction and high accuracy permitted this type of milling.

The power requirements, as measured with a wattmeter, were very high. Another indication of the large amount of work required in this particular milling job was the high temperature of the chips, which would reach 1400 F, as determined by comparison of the chip color with a heat-color chart.

In another series of machining tests, this die block was heated in a furnace to 1800 F and then mounted in an insulated fixture on the milling-machine table. The milling of this heated die block started when the block was about 1500 F, as determined by comparison with a heat-color chart, and the temperature continued to drop slowly as the tests proceeded. The power required, as measured by a wattmeter, is plotted as the lower line in Fig. 8. After the block had cooled to room temperature its

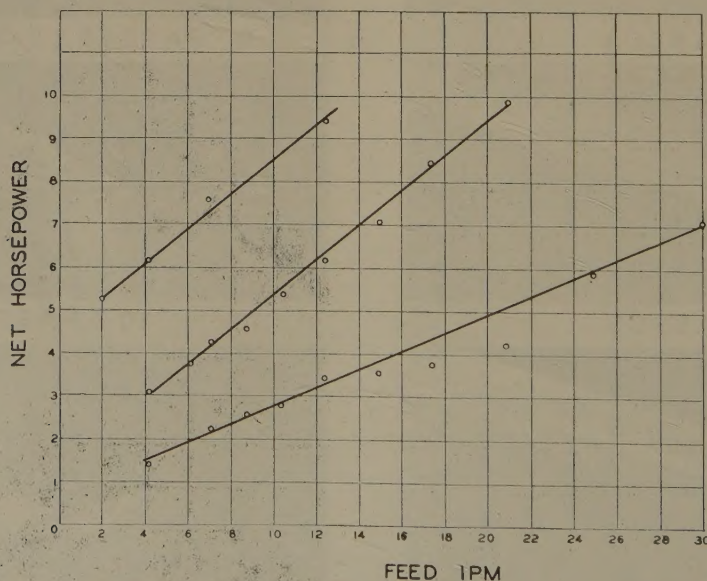


FIG. 8 HORSEPOWER REQUIRED WHEN MILLING A DIE BLOCK
(Upper line indicates power required to mill block in "as-received" condition. Lower line shows power required when block was at a temperature of about 1500 F, and center line indicates power after block had cooled again to room temperature.)

hardness had dropped from an average of 400 Bhn to 350 Bhn, which also reduced the power needed to that indicated by the middle line in Fig. 8. When the block was at room temperature in the state as received, the power consumption was as shown by the upper line. Due to the interrupted type of cut in milling, tool life is comparatively good when machining a heated workpiece, because the tooth has a chance to cool in the air between cuts. Changes of microstructure and accuracy in the workpiece are unavoidable under these conditions, but a difficult milling job can be made easier this way. Care should be taken to provide proper protection against the heat flow from the workpiece into the machine, either by insulating with layers of asbestos or by circulating a coolant in the table or fixture.

CONCLUSIONS

When production or tool-life tests are unavailable, the hardness of steel can be taken as a guide in choosing preliminary values of cutting speed and feed.

In general, the harder a steel is, the greater will be the work required to machine it and the shorter will be the tool life if setup conditions, cutting speed, and feed are the same as those used for identical, but softer, steels.

A reduction in the cutting speed and use of finer feed will increase the tool life when milling harder steels. Heating a work-

piece to about 1500 F will decrease the power requirements appreciably, and, combined with the intermittent cutting characteristic in milling, will also result in good tool life. Machine rigidity and optimum cutter design become increasingly more important with the machining of the harder alloy steels.

ACKNOWLEDGMENT

Acknowledgment is made to J. R. Roubik and Harold Francke for the help in the tests reported herein and the photographic work, respectively.

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Mechanics of Formation of the Discontinuous Chip in Metal Cutting

By MICHAEL FIELD¹ AND M. E. MERCHANT,² CINCINNATI, OHIO

The geometry, mechanics of formation, and plasticity conditions for the "continuous" type of chip have been analyzed in detail as a result of previous research. The present paper formulates a similar analysis of the "discontinuous" type of chip. In this type of chip formation, segments are formed by rupture which occurs intermittently on the shear plane as the cutting tool progresses. As the tool advances into the cut immediately following rupture of a segment, the metal is first deformed plastically and the angle of shear is large. Rupture is prevented during this stage by the fact that the compressive stress on the shear plane is high enough to permit the metal to undergo considerable shearing strain without fracture. As the tool progresses, the shear angle falls until the compressive stress becomes sufficiently low (or the shearing strain becomes sufficiently high) to permit fracture. The cycle then repeats.

NOMENCLATURE

The following nomenclature is used in this paper:

A = instantaneous cross-sectional area of "chip" before removal from workpiece = tw_1
 A_0 = cross-sectional area of chip before removal from workpiece = t_1w_1
 C = machining constant = $\text{arc cot } k$
 d = distance traveled by tool measured from instant of chip rupture
 F_c = cutting force; force component acting in direction of motion of tool relative to workpiece
 F_n = force component acting in direction normal to shear plane
 F_s = shearing force; force component acting in direction of shear of metal, in shear plane
 F_t = thrust force; force component acting in direction perpendicular to F_c , and to surface generated
 k = slope of linear curve relating shear stress to compressive stress
 K = slope of linear curve relating shearing strain of metal to compressive stress under rupture conditions
 N = normal force; force component acting perpendicular to tool face
 R, R' = resultant forces; forces acting between chip and tool and between chip and workpiece
 r_c = cutting ratio
 S_0 = shear strength of metal at atmospheric pressure

S_n = compressive stress acting perpendicular to shear plane
 S_s = mean shear stress on shear plane
 S_s' = mean shear strength of metal being cut
 t = instantaneous thickness of chip in discontinuous cutting
 t_1 = thickness of chip before removal from workpiece = maximum depth of cut for discontinuous chip
 t_2 = thickness of chip after removal from workpiece, as measured in direction perpendicular to face of cutting tool
 V_c = cutting velocity, velocity of tool relative to workpiece
 V_f = chip flow velocity; velocity of chip relative to cutting tool
 V_s = velocity of shear; velocity of chip relative to workpiece
 w = width of chip before removal from workpiece
 W_c = work done in cutting, per unit volume of metal removed
 W_f = work done in overcoming friction between chip and tool, per unit volume of metal removed
 W_s = work done in shearing of metal, per unit volume of metal removed
 α = oblique rake angle of tool measured in plane perpendicular to its cutting edge = true rake angle for case of orthogonal cutting
 ϵ = shearing strain undergone by chip under process of removal
 ϵ_0 = shearing strain of metal at rupture under atmospheric pressure
 ϵ_r = shearing strain required to produce rupture
 μ = coefficient of friction acting between sliding chip and tool face = F/N
 τ = friction angle = $\text{arc tan } \mu$
 ϕ = shear angle; angle between shear plane and surface being generated measured in plane perpendicular to cutting edge of tool = shear angle to horizontal surface
 ϕ_1 = shear angle at rupture
 ϕ_2 = shear angle to inclined surface

INTRODUCTION

The process of metal cutting is a vital link in all stages of manufacturing. It would, indeed, be difficult to think of any product that did not directly or indirectly involve some phase of the metal-cutting process, namely, turning, drilling, milling, shaping, planing, hobbing, or boring. For this reason, the advancement of our present-day technology is intimately bound up with the advancement of our knowledge of the machining process. The importance of assuring that this knowledge be of an engineering rather than an empirical nature is evident.

Anyone who is familiar with machining operations is aware of the fact that in some cases the process of metal removal results in chips which are broken up into segments, while in other cases chips are formed as continuous ribbons of varied length. Ernst (1)³ has pointed out that there are actually three basic types of chips, namely, Type 1, the discontinuous chip; Type 2, the continuous chip; and Type 3, the continuous chip with built-

¹ Research Engineer, The Cincinnati Milling Machine Company. At present, Director of Applied Metal Cutting, Metcut Research Associates, Cincinnati, Ohio. Jun. ASME.

² Senior Research Physicist, The Cincinnati Milling Machine Company. Mem. ASME.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society. Paper No. 48—A-136.

³ Numbers in parentheses refer to the Bibliography at the end of the paper.

up edge. Merchant (2) has made a fairly complete mathematical analysis of the geometry, mechanics, and plasticity conditions governing the formation of the Type 2 chip. The present paper formulates a similar analysis of the Type 1 chip. The importance of engineering knowledge of such a nature is evident when one stops to realize that, almost without exception, a discontinuous chip is formed in all machining operations carried out on such important materials as cast iron, brass, and bronze.

J. Thime (3), in 1878, was the first to study and report on the basic difference between the continuous and discontinuous types of chips. Among the subsequent investigators who recognized the distinct nature of a discontinuous chip and devoted some study to it were G. Sellergren (4) (1896), Rosenhain and Sturney (5) (1925), M. Okoshi (6) (1930), O. W. Boston (7) (1930), F. Schwerd (8) (1932), and Ernst and Martellotti (9) (1935). In 1938, H. Ernst (1) set forth his clear exposition of the three basic chip types enumerated in the foregoing.

MECHANICS OF THE CONTINUOUS CHIP

The mechanism of formation of the continuous chip without built-up edge (Type 2) is basic to the remaining types of chip formation. An understanding of the mechanics associated with the Type 2 chip is therefore a necessary approach to the present analysis.

The geometry of formation of the Type 2 chip, in the case of orthogonal cutting, is illustrated schematically in Fig. 1, which also shows the force system acting on this type of chip. In this simple case, the chip is formed by a process of shear which is con-

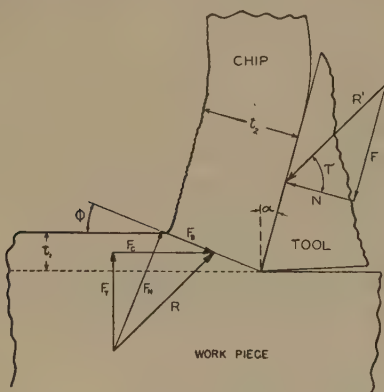


FIG. 1 SCHEMATIC REPRESENTATION OF GEOMETRY AND FORCE SYSTEM IN CASE OF ORTHOGONAL CUTTING AND A TYPE 2 CHIP

fined to a narrow zone extending from the cutting edge of the tool to the surface of the workpiece. This zone of shear may be approximated by a single plane known as the shear plane, shown making an angle ϕ with the work surface in Fig. 1. As the cutting tool advances, the metal lying ahead of it passes through this plane of shear where it is deformed to produce the chip, which then slides up the face of the tool. In the case of the Type 2 chip, this shear occurs without fracture of the metal and so a continuous ribbon is formed.

Important geometrical quantities in this process of chip formation, in addition to the shear angle ϕ , are the ratio of the chip thickness before removal to that after removal (t_1/t_2) which is known as the cutting ratio r_c , the rake angle of the tool α , the shearing strain undergone by the chip metal during plastic deformation ϵ , the cutting speed V_c , the shearing speed of the metal during deformation V_s , and the speed of sliding of the chip over the cutting tool V_f .

The mathematical relationships existing between these quanti-

ties in the case of orthogonal cutting and a Type 2 chip are as follows

$$\tan \phi = \frac{r_c \cos \alpha}{1 - r_c \sin \alpha} \dots \dots \dots [1]$$

$$\epsilon = \cot \phi + \tan (\phi - \alpha) \dots \dots \dots [2]$$

$$V_s = V_c \frac{\cos \alpha}{\cos (\phi - \alpha)} \dots \dots \dots [3]$$

$$V_f = V_c \frac{\sin \phi}{\cos (\phi - \alpha)} \dots \dots \dots [4]$$

The force system shown in Fig. 1, acting during chip formation, is derived from the principle that the chip must be held in stable mechanical equilibrium by the forces acting on it. Quantities important to this force system are the cutting force in the direction of tool travel F_c , the thrust force perpendicular to the surface generated F_t , the coefficient of friction between chip and tool μ , the friction angle τ , the friction force between chip and tool F_f , the shearing force responsible for plastic deformation of the metal F_s , the shearing stress on the plane of shear S_s , the compressive stress on the plane of shear S_n , the work consumed in overcoming friction between chip and tool W_f , the work consumed in shearing the metal at the shear plane W_s , and the total work done in cutting W_c . The mathematical relationships existing between these quantities during the formation of the Type 2 chip in the case of orthogonal cutting, are as follows

$$\mu = \tan \tau = \frac{F_t + F_c \tan \alpha}{F_c - F_t \tan \alpha} \dots \dots \dots [5]$$

$$F = F_t \cos \alpha + F_c \sin \alpha \dots \dots \dots [6]$$

$$F_s = F_c \cos \phi - F_t \sin \phi \dots \dots \dots [7]$$

$$S_s = \frac{F_c \sin \phi \cos \phi - F_t \sin^2 \phi}{A_0} \dots \dots \dots [8]$$

$$S_n = S_s \tan (\phi + \tau - \alpha) \dots \dots \dots [9]$$

$$W_f = \frac{F \sin \phi}{A_0 \cos (\phi - \alpha)} \dots \dots \dots [10]$$

$$W_s = S_s \epsilon \dots \dots \dots [11]$$

$$W_c = W_f + W_s \dots \dots \dots [12]$$

$$F_c = \frac{S_s A_0 \cos (\tau - \alpha)}{\sin \phi \cos (\phi + \tau - \alpha)} \dots \dots \dots [13]$$

The equilibrium conditions which control the size of the shear angle, i.e., the direction of plastic deformation ahead of the tool, are determined by applying the principle of minimum energy to the process of chip formation. The resulting plasticity condition (for the case of orthogonal cutting and a Type 2 chip) is given by the equation

$$\phi = \frac{C - \tau + \alpha}{2} \dots \dots \dots [14]$$

Here α is the rake angle of the cutting tool and τ is the angle whose tangent is the coefficient of friction between chip and tool (see Fig. 1). The quantity C is known as the machining constant. It is related in theory to the plastic properties of the material being cut by the equation

$$C = \text{arc cot } k \dots \dots \dots [15]$$

The quantity k is the slope of the straight line relating shear

strength to compressive stress. Equations [14] and [15] are first approximations, but have been found to agree well with experiment in the case of the continuous type of chip (Type 2).

FORMATION OF DISCONTINUOUS CHIP

The discontinuous chip is produced when cutting inherently "brittle" materials such as cast iron or when machining ductile materials at low cutting speeds in the absence of cutting fluids. In the latter case the discontinuous chip is associated with the high friction that is known to exist between the tool and chip. It was the object of this investigation to formulate an analysis consistent with the foregoing facts.

The basic difference between the mechanism of formation of the discontinuous chip and that of the continuous chip is that instead of shear occurring ahead of the cutting tool continuously without fracture, rupture intermittently occurs on the shear plane. This introduces new factors into the geometry of chip formation, which must be treated in the present analysis. In addition, the factors governing the mechanics of the intermittent fracture on the shear plane must be subjected to analysis.

The cutting action which produces a discontinuous chip was studied by taking moving pictures, through a microscope, of bronze being cut by a planer-type tool under orthogonal cutting conditions. As the tool advanced with respect to the work, chip segments were produced at approximately a uniform rate. Selected frames of such a movie showing the formation of a single chip segment, are shown in Fig. 2. Sketches corresponding to these photographs are shown in Fig. 3. In frame 1 the tool is just beginning to advance into the inclined work surface that was produced by the previously ruptured chip segment. In frame 16 the metal distortion indicates that the shear angle is high and that the shear plane extends to the inclined work surface. As the tool advances, the shear angle ϕ rapidly decreases. When the tool is at the position indicated in frame 20, the shear plane extends to the horizontal surface, as indicated by the start of curvature of this horizontal work surface. As the tool advances further the shear angle continues to decrease and eventually rupture occurs along the shear plane, producing a chip segment, as in frame 39, after which the cycle repeats itself.

In order to account for the fact that shear occurs to the in-

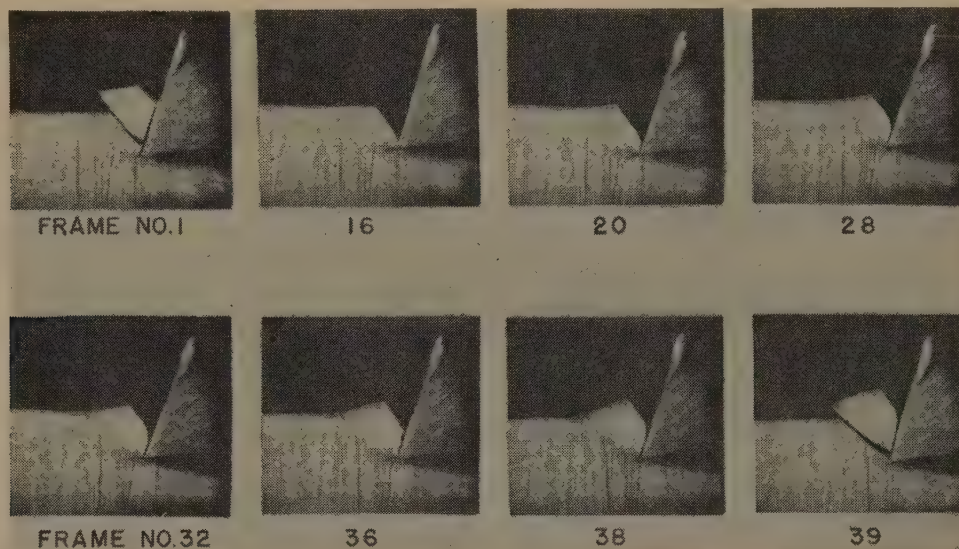


FIG. 2 SUCCESSIVE FRAMES FROM MOTION PICTURE SHOWING STAGES IN COMPLETE CYCLE OF DISCONTINUOUS-CHIP-SEGMENT FORMATION

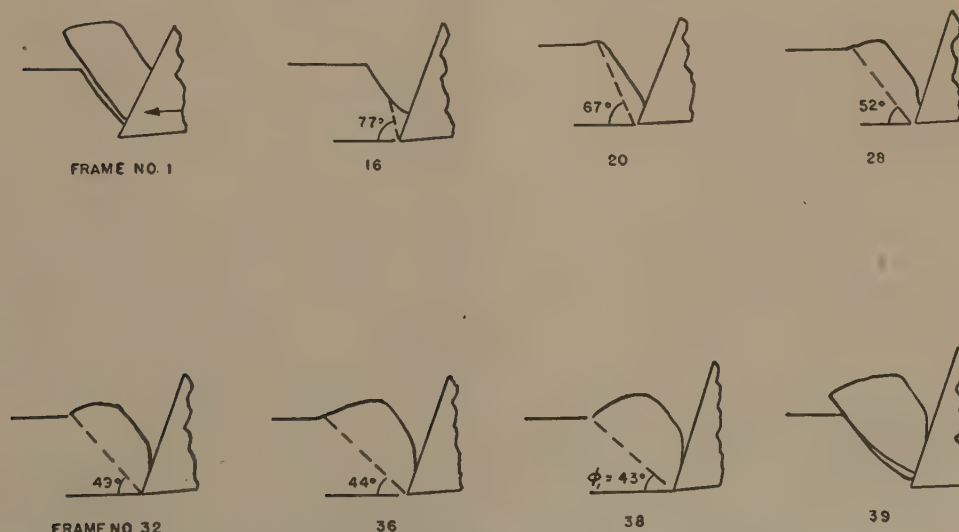


FIG. 3 SKETCHES TRACED FROM PHOTOGRAPHS OF FIG. 2

elined surface at the start of the cycle and then changes over to the horizontal surface during the latter part of the chip-segment formation, the geometry, mechanics, and plasticity conditions during these two phases of chip-segment formation were investigated for the case of orthogonal cutting.

PLASTICITY CONDITIONS

Let Condition A represent the case where shear is occurring from the cutting edge to the inclined surface and Condition B the case where shear is occurring to the horizontal surface. Condition A is the situation that exists during the start of the chip-segment formation, while Condition B exists during the latter stage of the segment formation.

Condition A: Shear Occurring to the Inclined Surface. Fig. 4 shows the tool after it has advanced a short distance d into the

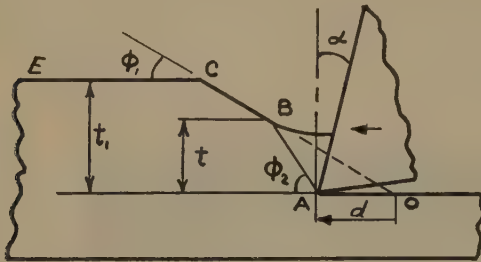


FIG. 4 GEOMETRY OF INITIAL STAGE OF CYCLE OF CHIP-SEGMENT FORMATION

workpiece which originally had the shape OCE .

Let

t_1 = total depth of cut

t = instantaneous depth, i.e., vertical distance from machined surface to intersection of shear plane AB with inclined surface OC

ϕ_1 = shear angle at rupture

ϕ_2 = shear angle while shear plane extends to inclined surface

ϕ = shear angle while shear plane extends to horizontal surface

d = distance traveled from point O

w = width of workpiece

Depth t can be expressed in terms of ϕ_1 , ϕ_2 , and d as follows

$$\begin{aligned} \frac{\sin(\phi_2 - \phi_1)}{d} &= \frac{\sin \phi_1}{AB} \\ t &= AB \sin \phi_2 \\ &= \frac{d \sin \phi_1 \sin \phi_2}{\sin(\phi_2 - \phi_1)} \quad \dots \dots \dots [16] \end{aligned}$$

The instantaneous cross-sectional area A is

$$A = tw = \frac{wd \sin \phi_1 \sin \phi_2}{\sin(\phi_2 - \phi_1)} \quad \dots \dots \dots [17]$$

The instantaneous cutting force F_c becomes, on substituting A from Equation [17] for A_0 in Equation [13]

$$F_c = S_s wd \frac{\sin \phi_1 \cos(\tau - \alpha)}{\sin(\phi_2 - \phi_1) \cos(\phi_2 + \tau - \alpha)} \quad \dots \dots [18]$$

where ϕ_2 indicates that shear is occurring to the inclined surface rather than the horizontal surface. The value of the shear angle ϕ_2 will be determined by the minimum-energy principle, i.e., since the force F_c is responsible for the total work, ϕ_2 must assume such a value as to make F_c a minimum.

Assume that the shear strength S_s' of the material being cut is

a linear function of the compressive stress acting on the shear plane or

$$S_s' = S_0 + k S_n \quad \dots \dots \dots [19]$$

then, from Equation [9]

$$S_s' = \frac{S_0}{1 - k \tan(\phi_2 + \tau - \alpha)} \quad \dots \dots \dots [20]$$

where ϕ_2 has replaced ϕ of Equation [9]. Substituting Equation [20] into [18]

$$F_c = \frac{S_0 wd \sin \phi_1 \cos(\tau - \alpha)}{\sin(\phi_2 - \phi_1) \cos(\phi_2 + \tau - \alpha) - k \sin(\phi_2 - \phi_1) \sin(\phi_2 + \tau - \alpha)} \quad \dots \dots \dots [21]$$

Apply the minimum-energy principle to Equation [21], i.e., differentiate F_c with respect to ϕ_2 and equate to 0. The result is

$$\cot(2\phi_2 + \tau - \alpha - \phi_1) = k \quad \dots \dots \dots [22]$$

or

$$\phi_2 = \frac{C - \tau + \alpha + \phi_1}{2} \quad \dots \dots \dots [23]$$

where, as in the case of the continuous chip

$$C = \text{arc cot } k \quad \dots \dots \dots [15]$$

Condition B: Shear Occurring to the Horizontal Surface. As the tool advances into the workpiece, eventually the shear plane is observed to extend to the horizontal surface instead of to the inclined surface. Under this condition the depth of cut becomes t_1 and the shear angle is called ϕ instead of ϕ_2 .

The continuous-chip theory is assumed to hold for this condition up to the time of rupture. The shear angle ϕ is thus expressed by the equation

$$\phi = \frac{C - \tau + \alpha}{2} \quad \dots \dots \dots [14]$$

for the general case, where the shear strength of the material is assumed to be a linear function of the compressive stress acting on the shear plane. As in the case of the angle ϕ_2 in Equation [23], the shear angle ϕ in Equation [14] continually decreases as the tool advances, reaching a minimum value at the instant of rupture. The reason for this steady decrease in shear angle is that the friction angle τ in Equations [23] and [14] is also a variable and increases as the tool advances, due to the cleansing action of the chip as it slides over the tool face. Angle τ reaches its maximum value at the instant of rupture.

The experimental study which is described in detail later in the paper reveals that Equations [23] and [14] are poor approximations to the conditions existing during the formation of the chip segment prior to rupture. Evidently the simple assumptions made concerning the conditions governing plastic deformation do not apply well to this portion of the cycle of chip-segment formation. Therefore the use of these equations in analyzing this portion of the cycle should be limited to "qualitative" interpretations only. However, Equation [14] was found to hold quite well at the instant of rupture, i.e., when $\phi = \phi_1$.

GEOMETRICAL AND FORCE RELATIONSHIPS

It is evident from the foregoing discussion of the process of formation of the discontinuous chip that the same basic Equations [1] through [13], which describe the geometry and force system in the case of the continuous chip (Type 2), can be applied to

the discontinuous chip, provided that the proper values of instantaneous shear angle and instantaneous chip thickness or cross-sectional area are used in them. There is no need to repeat these equations here. However, the plasticity conditions represented by Equations [14] and [15] apply only "qualitatively," except at the instant of fracture.

If ϕ is replaced by ϕ_1 in Equations [1] through [14], these equations then can be used to analyze the conditions existing at the instant of rupture in the process of discontinuous-chip formation. Values of the rupture angle ϕ_1 and of the instantaneous tool forces must of course be observed to do this.

At the instant of rupture the shear angle is well defined and can be determined quite accurately either by observation from a photograph (see Fig. 2), by visual observation of the cutting process through the microscope, or by direct measurements on the chip segments produced in cutting.

In general, the chip is observed to have the trapezoidal shape $OPQR$, Fig. 5. The long leg of the trapezoid OP is an accurate indication of the length of the shear plane at rupture. Hence

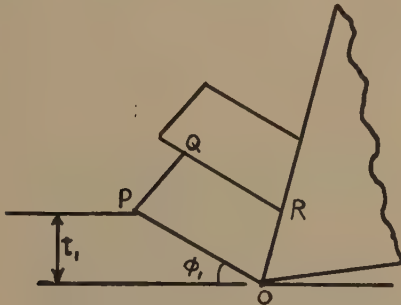


FIG. 5 APPROXIMATE CHIP GEOMETRY AT FRACTURE

the shear angle at rupture can be determined from the relation

$$\sin \phi_1 = \frac{t_1}{OP} \dots \dots \dots [24]$$

The length OP can be measured fairly accurately on the chip by using a Brinell microscope. A better method, however, is first to mount the chip in plastic and then polish the surface to give better definition of the cross-sectional area.

EXPERIMENTAL STUDY OF DISCONTINUOUS-CHIP FORMATION

The question of what variables and relationships control the fracture process that is responsible for segmental chip formation is not answered by the foregoing theory and analysis. Experimental data are needed to solve this problem. Such an investigation requires the simultaneous determination of the instantaneous forces acting between tool and work as well as the instantaneous conditions of shear within the workpiece.

The arrangement shown in Fig. 6 was used for such a study. The workpiece was clamped in a dynamometer which deflected under load. The deflection was amplified by an integral mechanical lever arrangement and the amplified deflection was measured by a $1/10,000$ -in. dial indicator. The tool was carried in another dynamometer attached to the overarm of a milling machine. This dynamometer measured the horizontal deflection of the tool by means of another $1/10,000$ -in. dial indicator. By suitable calibration, the deflections of the tool and work dynamometers gave the cutting force F_c and the thrust force F_t , respectively.

The two orthogonal force readings F_c and F_t were sufficient to determine the entire force system of Fig. 1, and, together with

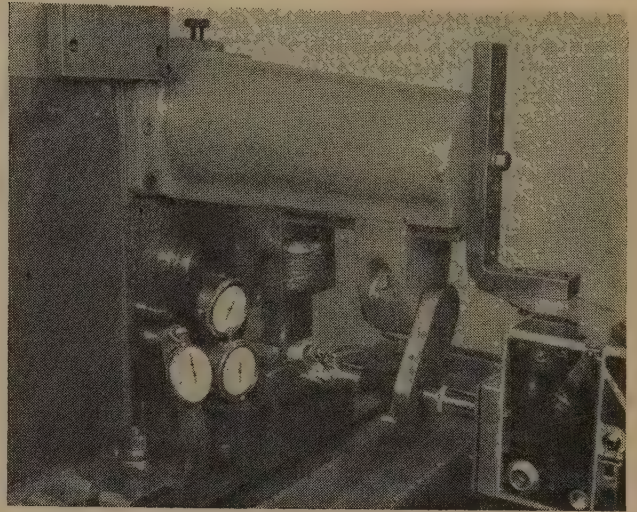


FIG. 6 EXPERIMENTAL SETUP FOR STUDY OF DISCONTINUOUS-CHIP FORMATION THROUGH MICROSCOPE

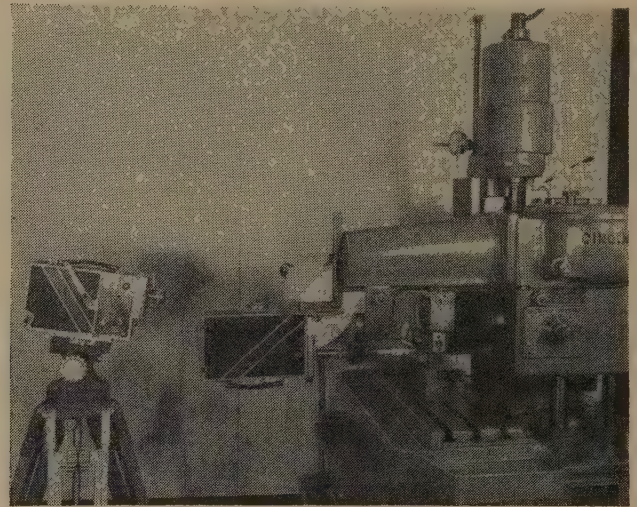


FIG. 7 ARRANGEMENT OF MOTION-PICTURE CAMERAS FOR SIMULTANEOUS RECORDING OF TOOL FORCES AND CHIP GEOMETRY

the knowledge of the depth of cut, width of cut, and cutting velocity, provided all the data necessary for the determination of the shearing strain, stresses on the shear plane, coefficient of friction, etc.

The deformation of the workpiece while cutting was observed through a low-power microscope which was attached to the overarm, Fig. 6.

In order to record simultaneously the cutting action seen through the microscope and the readings of the tool and work indicators, two "Ciné Kodak Special" movie cameras, Fig. 7, were used. The microscope camera was attached to the overarm by a bracket so that the axis of the microscope was in line with that of the camera shutter. The lens was removed from this movie camera.

The second movie camera was set up on a tripod and was focused on the two dial indicators as well as on a stop watch which was attached to the milling-machine table. The illumination for this latter camera came from a No. 2 photo flood lamp placed 30 in. from the workpiece.

The synchronization of the force readings with their correspond-

TABLE 2 MOVIE SCENE G. RUPTURE CONDITIONS, DEPTH OF CUT = 0.015 IN.

Seg- ment	Observed			Calculated						
	F_c , lb	F_t , lb	ϕ_1 , deg	$\tau - \alpha$ deg	τ deg	μ	ϵ	S'_s , psi	S_n , psi	C^* deg
1	84.7	2.27	42	1.5	15.5	0.277	1.64	49600	47000	85.5
2	84.7	5.38	30	3.6	17.6	0.317	2.02	42700	28300	63.6
3	84.7	8.22	42	5.6	19.6	0.356	1.64	46500	46800	89.6
4	95.2	2.83	41	1.7	15.7	0.281	1.66	55500	51200	83.7
5	95.2	5.67	48	3.4	17.4	0.313	1.58	53300	66800	99.4
6	100.5	2.27	52	1.3	15.3	0.273	1.56	57200	76700	105.3
7	100.5	4.53	43	2.6	16.6	0.298	1.63	58200	59200	88.6
8	105.7	8.78	39	4.7	18.7	0.339	1.70	58200	55500	82.7
9	105.7	3.12	35	1.7	15.7	0.281	1.81	58800	37700	71.7
10	105.7	3.12	45	1.7	15.7	0.281	1.60	61700	65500	91.7

* $C = 2\phi_1 + \tau - \alpha$.

interest was aroused by the wide variation that was obtained for ϕ_1 , the shear angle at rupture. The variation in ϕ_1 in this series of segments is seen to be between 30 deg and 52 deg. Similar wide variations were observed when taking the other cuts. The reason for this wide variation seems evident. Polycrystalline metal is far from homogeneous. Thus the values of those properties of the metal which control fracture will vary from point to point along the path of the tool and so the individual values of shear angle at fracture, ϕ_1 , may vary considerably from segment to segment. Only average values of shear angle at fracture, taken from a number of successive segments, have significance in relation to the average or bulk properties of the work material.

RUPTURE CONDITIONS IN CUTTING

With the aid of the data obtained in the experimental study, the conditions controlling the rupture process in discontinuous-chip formation were studied in detail. It is known from studies on metals, such as those reported by Bridgman (10), that the amount of plastic shearing strain necessary to produce fracture increases as compressive stress is applied to the planes of shear. The greater the compressive stress, the greater the ductility. The relationship between shearing strain required for rupture and compressive stress is roughly of the form

$$\epsilon_r = \epsilon_0 + K S_n \dots \dots \dots [25]$$

where
 ϵ_r = shearing strain required to produce rupture
 ϵ_0 = shearing strain required to produce rupture at zero compressive stress
 K = slope of shearing strain - compressive stress curve.
Thus in metal cutting, fracture will occur on the shear plane (and therefore a discontinuous chip will be formed) whenever the shearing strain ϵ of Equation [2] becomes equal to or greater than ϵ_r . This rupture condition can be expressed mathematically by combining Equations [2] and [25], bearing in mind that at rupture the shear angle is called ϕ_1 . Thus

$$\cot \phi_1 + \tan (\phi_1 - \alpha) \geq \epsilon_0 + K S_n \dots \dots \dots [26]$$

To a first approximation, Equation [26] establishes the value of the minimum shear angle ϕ_1 at which fracture will occur when cutting a given metal. As a tool enters a cut and advances into the metal, the shear angle will at first fall, due to the increasing friction between chip and tool, as shown by the experimental study already described. If, during this process, the shearing strain rises to a high enough value, or the compressive stress drops to a low enough value (or both), to satisfy Equation [26], then a discontinuous chip will result. If, however, the equilibrium value of the shear angle (controlled by Equation [14]) is arrived at before the shearing strain rises high enough or the compressive stress falls low enough to satisfy Equation [26], then a continuous chip will result.

In the case of the experimental study on Ampco bronze re-

ported herein, it was found that the shearing strain varied little with shear angle over the range of angles which occurred. This is illustrated, for example, by the data in Table 1. The reason for this is found in the fact that shearing strain, as calculated from Equation [2], is nearly independent of shear angle over the range from 30 deg to 70 deg, in the case of a 14-deg-rake-angle tool, and this happens to be just about the range of the shear angle ϕ occurring when the bronze was cut. This is illustrated in Fig. 11. Therefore, in the case of this material, the occurrence of fracture must be attributed mainly to the large decrease in compressive stress which takes place as the shear angle falls during the advance of the tool. This rapid decrease of compressive stress is illustrated, for example, by the data in Table 1.

The mechanism by which Equation [26] becomes satisfied during the course of chip-segment formation is illustrated in Fig. 12. Data covering the formation of three different chip segments, taken from the motion-picture films, is plotted there. The

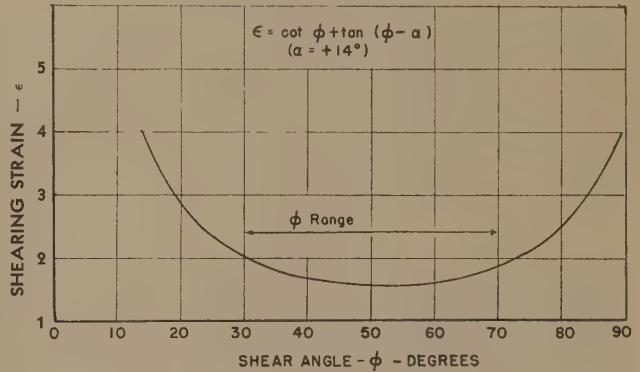


FIG. 11 GEOMETRICAL RELATION OF SHEARING STRAIN TO SHEAR ANGLE, SHOWING RANGE OF SHEAR ANGLE ϕ OBSERVED IN EXPERIMENTAL STUDY

broken line, marked "Assumed relation at rupture," represents a plot of Equation [25] for the values of ϵ_0 and K assumed to hold for this metal. The exact values of these two constants are not too important; independent tests made on this material indicated the approximate values to be used. It can be seen from Fig. 12 that, at the beginning of chip-segment formation in each case, the combination of shearing strain and compressive stress lies well away from the required values for fracture. However, as the tool advances, although the shearing strain changes little, the compressive stress decreases rapidly until finally the shearing strain-compressive stress curve intersects the line representing the fracture condition, thus satisfying Equation [26], whereupon fracture occurs and the cycle is ready to repeat.

CONCLUSION

The authors realize that the present study is not complete, and leaves a number of questions unanswered. However, it is be-

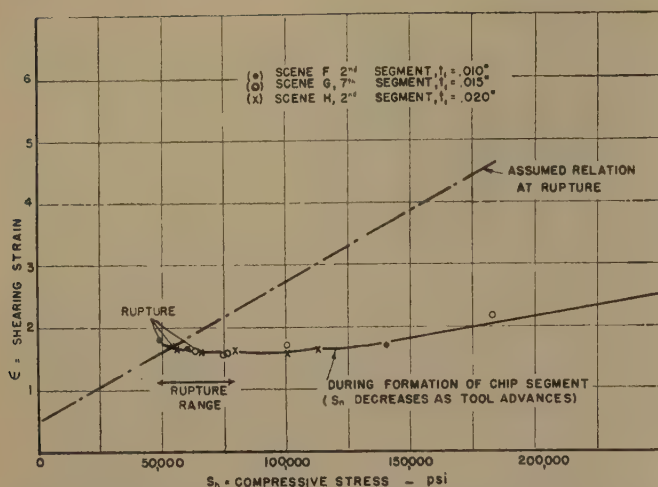


FIG. 12 TYPICAL PROGRESS OF INSTANTANEOUS SHEARING STRAIN AND COMPRESSIVE-STRESS VALUES THROUGHOUT CHIP-SEGMENT FORMATION, IN RELATION TO RUPTURE CONDITION

lieved that the findings presented in this paper do represent the essentials of the geometry and mechanics of discontinuous-chip formation. It is hoped that this paper will stimulate further research by others on this important subject.

ACKNOWLEDGMENTS

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Discussion

O. W. BOSTON.⁴ The writer has enjoyed the presentation of the authors' papers on a study of continuous and discontinuous types of chips as presented in various papers during the last several years. Their company is to be commended in permitting them to devote their time to these studies, which have made many people conscious of the value of an understanding of the fundamentals of metal cutting.

The present paper, analyzing the formation of the discontinuous type of chip and setting up mathematical formulas for its treatment, is another valuable contribution in this field. There are still numerous phases of this subject as treated which leave the writer doubtful of the premise upon which the present mathematics is based. Certain metal-cutting conditions are necessary to secure the discontinuous chip. One of these is low cutting speed. The authors have used an extremely low speed of 0.515 fpm and a width of cut on a land of 0.050 in. The depth of cuts range from 0.005 to 0.020 in. The writer has found that when the width of such a land is narrow, the unit force in a direction of cut is reduced, all other factors remaining constant. For steel, this critical width was found to be around 0.3 in. It is less for the less ductile metals.⁵

It is believed that for the narrow width of 0.050 in., the formation of the chip is definitely influenced by the edge effect, which is similar to the effect of boundary lubrication in a journal bearing, and that the true state of the deformed metal would be well within the interior of the chip. Certainly the surface as observed by a microscope would not represent the conditions occurring on the interior.

It is well known that cutting speed definitely affects cutting forces and the formation of the chip. This has been brought out in a paper⁶ by Messrs. Boulger, Shaw, and Johnson of the Battelle Memorial Institute. Their curves, Fig. 7, show that the feed in hundred thousandths of an inch of the carriage, as a turning tool is fed along the work under a constant force, is increased as the speed is reduced. There is a definite and marked change in the direction of the curves with lower values of feed for each of five low-carbon steels, including two Bessemer screw-stock steels, when the cutting speed is below 80 to 50 fpm. The writer has in mind a value slightly lower than this, 20 to 30 fpm, as being the critical speed at which this change in force occurs, based upon his own work. Therefore it seems certain that the conditions arising from the extreme low speeds used by the authors vary a great deal from what might be called normal.

The writer also has observed through a microscope at 80 magnification the formation of the discontinuous type of chip when cutting at very low speeds, and his own analysis of the formation does not subscribe to the authors' shear plane in its true sense. It is believed that the authors are using the shear plane as a term indicating a plane of appreciable thickness, but it seems that the value of the shear angle could be measured with a considerable plus and minus tolerance. Certainly this work is of a pioneering nature, and it is hoped, as the authors have expressed, that more interest may be centered on this problem and more help obtained by those interested in obtaining data which will lead to a complete understanding of the subject.

⁴ Chairman, Department of Metal Processing, College of Engineering, University of Michigan, Ann Arbor, Mich. Fellow ASME.

⁵ "Research in the Elements of Metal Cutting," by O. W. Boston, Trans. ASME, vol. 48, 1926, p. 793.

⁶ "A Constant-Pressure Lathe Test for Measuring the Machinability for Free Cutting Steels," by F. W. Boulger, H. L. Shaw, and H. E. Johnson, presented at the Annual Meeting, New York, N. Y., November 28-December 3, 1948, of The American Society of Mechanical Engineers.

F. P. BUNDY.⁷ The analyses given in this paper for the formation of No. 2 and No. 1 types of chips are clear, logical, and appear to be mathematically correct. The authors are to be congratulated for having done a nice job on this difficult problem. However, as they point out themselves, the analyses do not explain in detail all the observed facts, and hence a more complete theory which takes into account second-order effects will have to be worked out.

For example, the simple theory does not predict the curvature, or curl, of the No. 2 type of chip. Obviously, this must be due to an uneven distribution of stress across the thickness of the chip as it is formed. Likewise, in the photomicrographs shown in Fig. 2 of the paper the contour of the chip in process of formation does not have the exact appearance called for by the theory. It appears that the concentration of stresses at the very tip of the tool has a pronounced influence on the phenomenon.

The paper gives no theoretical or experimental values for the slope constant k in Equation [19], $S'_s = S_0 + k S_n$. The general understanding is that for most materials k is small and positive. Hence the "machining constant," $C = \text{arc cot } k$, should always be less than (but very nearly equal to) 90 deg, and the shear angle ϕ should be influenced only by a few degrees. Actually, the data presented in Table 3 herewith (data taken from authors' Table 1) show k to have a negative value (-0.20), which calls for C being greater than 90 deg (actually 101 deg), and hence ϕ should be greater than 50 deg in this case. Table 2 of the paper shows the observed values of ϕ_1 , the shear angle at fracture, to range from 30 to 52 deg. The 30-deg value of ϕ , calls for a k value of $+0.50$, and the 52-deg value calls for $k = -0.27$. Thus it appears that in practice the value of k varies widely in a given specimen and does not influence the chip behavior very much. The criterion of whether a No. 1 or a No. 2 type of chip will be formed apparently depends more on the shear-strain versus compressive-stress relationship.

TABLE 3 MOVIE SCENE G, 7TH SEGMENT

Frame	S'_s , psi	S_n , psi	ϕ , deg
1			
16	41300	182000	77
20	44000	101000	67
28	65500	76600	52
32	65400	76500	49
38	65500	63200	43

A discrepancy in notation exists between Figs. 9 and 10 of the paper. Apparently, from other data given in the paper, the depth of cut for Scene G was 0.015 in. and that for Scene H was 0.020 in.

R. S. HAHN.⁸ The authors mention the classification of chips as Type 1, the discontinuous chip; and Type 2, the continuous chip. In addition, continuous chips often occur which are serrated, as shown in Fig. 13 of this discussion. It appears that, in these chips, plastic flow at first takes place in a zone around the "shear plane" and, as deformation proceeds, this general flow rapidly localizes to a sudden slip (or a shearing rupture which is self-healing). This phenomenon has been observed in ductile metals such as copper, bronze, aluminum, mild steel and semi-hardened steel, and appears to be independent of the vibrational characteristics of the tool and work-holding systems, at least of the lower frequency modes of vibration.

For example Fig. 13 shows a mild-steel chip whose serrations correspond to a frequency of over 1,000,000 cycles per sec, the

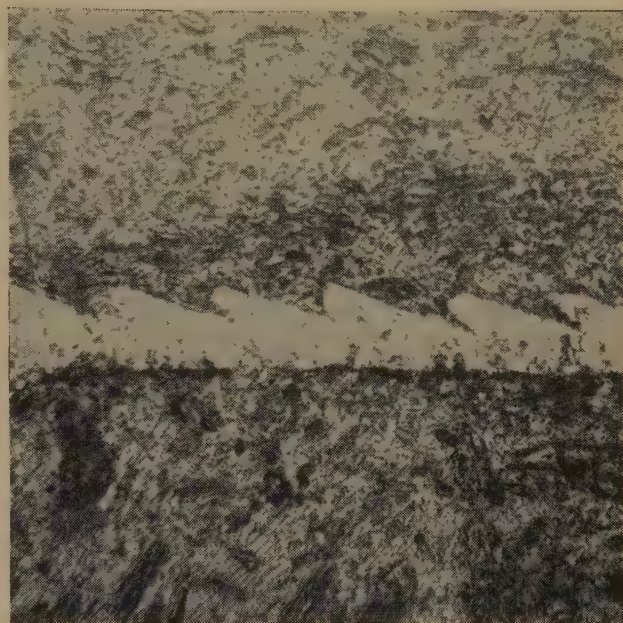
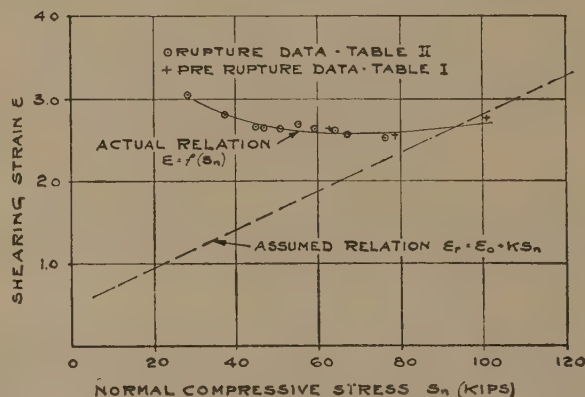
FIG. 13 PHOTOMICROGRAPH OF SERRATED CHIP (Mild steel; cutting speed 7900 fpm; $\times 400$.)

FIG. 14 RELATION BETWEEN SHEARING STRAIN AND COMPRESSIVE STRESS

spacing being about 0.0015 in. It seems reasonable to suppose that the phenomenon studied by the authors is related to the foregoing, and to the well-known jumplike character of plastic deformation described by M. Classen-Nekludowa⁹ and others.

In view of this and the fact that the shearing strain at rupture is practically independent of normal stress, as shown in Fig. 14 herewith, a possible explanation of the phenomenon might be as follows: As the tool proceeds into the workpiece, the deformation, at first being homogeneous, suddenly changes to localized slip, with perhaps instantaneous temperature flashes on the localized shear plane (similar to Bowden's temperature flashes between sliding blocks) aiding in this instability, since the strength of a metal is a strong function of the temperature. At this moment the frictional force on the tool face is relieved, and, due to the compressed material just below the shear plane, the chip is accelerated upward, thereby generating a tensile force which is a necessity if rupture is to occur.

Fragmentation of crystals along the shear plane probably also

⁷ Research Laboratory, General Electric Company, Schenectady, N. Y.

⁸ Research Engineer, The Heald Machine Company, Worcester, Mass. Jun. ASME.

⁹ "Über die sprungartige Deformation," by M. Classen-Nekludowa, *Zeitschrift für Physik*, vol. 55, 1929, pp. 555-568.

generates tensile stress over microscopic domains. In this way the moment of rupture would be more a function of the shear strength of the metal on the shear plane, i.e., when localized slip occurred than it would be of the shearing strain as the authors have assumed. The large variation of the conditions at rupture, shown in Table 2 of the paper, would seem then to be associated with variations in shear strength and, as pointed out in the paper, is quite reasonable for a polycrystalline material.

As mentioned previously, if the authors' data of Table 2 are plotted as shown in Fig. 14 of this discussion, it will be observed that the strain at rupture is nearly independent of the normal stress, quite contrary to the authors' assumed relation. If the assumed relation were valid, the data should at least scatter around an upward sloping line. On the contrary it follows a remarkably smooth curve sloping downward if anything and connecting smoothly with the strain prior to rupture (after correcting ϵ for segment 9 Table 2 for a numerical error).

AUTHORS' CLOSURE

The authors wish to thank Messrs. Boston, Bundy, and Hahn for their comments and suggestions regarding the research results reported in this paper. These comments represent considerable thought and study on the part of the discussers, and the suggestions made are on the whole very constructive.

Professor Boston draws attention to the fact that the cutting conditions used by the authors do not correspond to those employed in machine shops. This is true. An experimental determination of the process of formation of the discontinuous chip is complicated by the necessity for the simultaneous observation of (1) the force system, and (2) the plastic behavior and rupture of the work material. Furthermore complexity is increased by the rapidity with which a chip segment is formed, and by the fact that no steady-state condition is attained, as in the case of the continuous chip. In this research, as simple an experimental approach as possible was devised to aid in checking the theoretical development of the physical process of chip-segment formation. The aim was to establish basic principles, not to duplicate exact conditions found in normal practice.

The experimental setup undoubtedly could be improved in future investigations so as to provide more accurate instantaneous force measurements. An exact observation of the plastic behavior of the metal during segment formation will remain an extremely difficult problem.

Mr. Bundy points out that during the formation of a chip segment the machining constant C does not remain constant. As stated by the authors, this fact is indicative that the actual physical behavior of the material is somewhat more complex than that depicted by the mathematical development. However, at the instant of rupture, the constancy of C is found to be good, and any variation can be accounted for by local differences from point to point within the polycrystalline metal. Thus the quantity C (or k) does have an important influence on chip

behavior, even in the case of the discontinuous chip. Its role in the case of the continuous chip has been well established previously; in that case the value of the equilibrium shear angle depends directly upon the value of C . With some metals, C is considerably less than 90 deg; with others, it may even be slightly greater than 90 deg.

The occurrence of serrated continuous chips is referred to and illustrated by Dr. Hahn. Such chips are of course obtained under chatter conditions in cutting, but Dr. Hahn is of the opinion that the examples he cites are not due to tool vibration. However, there is another fairly obvious cause of this type of chip formation, and one which actually has been observed in our metal-cutting studies; namely, a rapid oscillation in the size of the shear angle. Under certain conditions, the shear angle is unable to attain a constant equilibrium value, due to interactions between the variables which control that angle, and so it oscillates rapidly in size, giving chip formation of the type discussed and illustrated by Dr. Hahn.

Dr. Hahn is of the opinion that the rupture data, which he plots in Fig. 14 of the discussion, is not in agreement with the rupture condition proposed by the authors. He suggests that the experimental points which he plots should all scatter at random along the assumed rupture relation if the authors' proposal is correct. This contention overlooks two important facts: (1) The progress of the shearing-strain and compressive-stress values, during the formation of any one chip segment, follows a perfectly smooth curve which eventually is terminated by rupture, as indicated in Fig. 12 of the paper. (2) The exact point along that curve where rupture will occur will vary somewhat from one segment to another, in successive cycles of chip-segment formation, since the material being cut is bound to have local inhomogeneities at successive points along the path of the cutting tool. Thus the curve which Dr. Hahn plots in his Fig. 14, is merely the tail end of the smooth curve plotted in Fig. 12, and each of the experimental points on his curve is merely a marker indicating where, along that curve, the particular segment in question finally ruptured.

The actual point along the curve at which any given segment will rupture must lie somewhere in the vicinity of the assumed relation at rupture (given by the broken line), but may lie either somewhat to the right or somewhat to the left of this line, depending upon the local properties of the metal from point to point along the path of tool travel. Thus the experimental points at rupture should scatter, but they should all scatter along the smooth curve and lie in the vicinity of the broken line, as indeed they do.

Unfortunately, Dr. Hahn has made an error in plotting the data in Fig. 14, having plotted all the strain values one unit too high (see Table 2 of the paper). On the other hand, the assumed relation at rupture is correctly plotted. It will be noted that if the strain values be properly plotted, the experimental points do indeed scatter about equally on both sides of the broken line, along the smooth curve.

Constant-Pressure Lathe Test for Measuring the Machinability of Free-Cutting Steels

By F. W. BOULGER,¹ H. L. SHAW,² AND H. E. JOHNSON³

This paper describes a machinability testing method developed during a co-operative research program at Battelle Memorial Institute sponsored by The Carnegie-Illinois Steel Corporation. The test evaluates materials on the basis of the feed resulting from a fixed horizontal tool pressure. Its advantage is that it requires only a short testing time. A description of the lathe and auxiliary equipment is given, and typical testing results are presented. The data indicate that the method has a high order of sensitivity and adequate reproducibility of test ratings. Constant-tool-pressure machinability-test ratings were in close agreement with tool-life test values when the evidence indicated reliable sampling, and in adequate agreement with large-scale commercial machining operations. The test seems to offer a means of rating the cutting quality of free-cutting steels rapidly and reliably with the use of small samples.

INTRODUCTION

FROM the metallurgical viewpoint, machinability is probably best defined as a complex property of a material which controls the facility with which it may be cut to the size, shape, and surface finish required commercially. Attempts to measure machinability are usually based upon determinations of tool wear (1),⁴ chip behavior (2, 3), energy consumption (4), or rate of metal removal (5).

The problems in measuring machinability are manifold. Because of the numerous variables involved, machinability ratings must be based upon shop operations under comparatively uncontrolled conditions, or on laboratory tests with conditions simplified by various assumptions. The latter method seems preferable.

In a general way, low strength and low ductility during cutting seem to be desirable for good machinability. Since these properties are mutually contradictory for most metals, the desired balance between them would be expected to vary for different machining operations. Consequently, it seems likely that steels which are good for certain types of cutting may be unsatisfactory for other machining operations. Caution should be exercised in using machinability ratings based upon one type of cutting for predicting the behavior in other types of machining. For the same reason, discretion should be used in accepting machinability ratings based upon physical or mechanical properties rather than upon performance in simulative tests.

There are many types of laboratory machinability tests, because inductive reasoning leads people to make different assumptions and assign different degrees of importance to various factors. Laboratory machinability tests usually involve machining operations, because investigators generally realize that our knowledge of the nature of machining does not yet permit substituting another test in place of machining. Ordinarily, the machining operation, cutting fluids (if any), tool shapes, and tool materials are selected arbitrarily and held constant. The materials under study are then evaluated by measuring some other important variable believed to bear a close relationship to cutting quality. The quantity of metal removed per unit of time for a constant feed load (6) or tool life (5), or the pressures (7), wear, or life (8) of the tools under known conditions, are often used as criteria for machinability. Measurements of tool and work temperatures, or the amount of heat generated during cutting operations (9), have also been used for machinability comparisons. Merchant's methods provide a more fundamental approach to the study of metal cutting. His analysis of chip geometry and force systems for specific machining operations permits determinations of the shear strength and the effect of compressive stress normal to the shear plane on the shear strength of the metal, and the coefficient of friction between chip and tool during cutting.

CONSTANT-PRESSURE MACHINABILITY TEST

This investigation on the machinability of free-cutting steels deals with materials which are usually machined in automatic screw machines. It seemed desirable therefore to base machinability ratings on lathe tests. Since one objective of the investigation was to determine the cause of variations in cutting quality between steels of the same grade, the testing method had to be sensitive to small differences in machinability. Furthermore, a satisfactory method must give reliable and reproducible ratings, and it is desirable that the testing procedure require only a minimum of time, material, and labor. It is also desirable that specimens of different sizes or of widely different composition be compared on the same scale.

The laboratory lathe test, which is described, is based on the finding that steels of different machining quality cut at different rates, for constant surface speed, when a fixed lateral pressure is applied to the tool. The machinability ratings are based on the assumption that a steel with superior machinability will cut with a heavier feed than one of poorer machinability when a fixed pressure is used to produce tool travel. The validity of the assumption that variations in feeds obtained on different steels under fixed pressures and controlled conditions in laboratory tests are closely related to variations in machinability under commercial conditions has been established by a large number of comparisons.

The equipment used for the constant-pressure machinability test is comparatively simple. It consists basically of a lathe with a tool carriage disconnected from the fixed-feed mechanism and mounted on ball bearings; a means of applying a predetermined lateral tool pressure; and a device for recording the number of spindle revolutions occurring during a certain distance of tool travel. Fig. 1 shows the lathe and auxiliary parts.

Test Lathe. The lathe used in the machinability investigation

¹ Supervising Metallurgist, Battelle Memorial Institute, Columbus, Ohio.

² Research Engineer, Battelle Memorial Institute.

³ Formerly Research Associate, Carnegie-Illinois Steel Corporation, Pittsburgh, Pa.

⁴ Numbers in parentheses refer to the Bibliography at the end of the paper.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society. Paper No. 48-A-86.

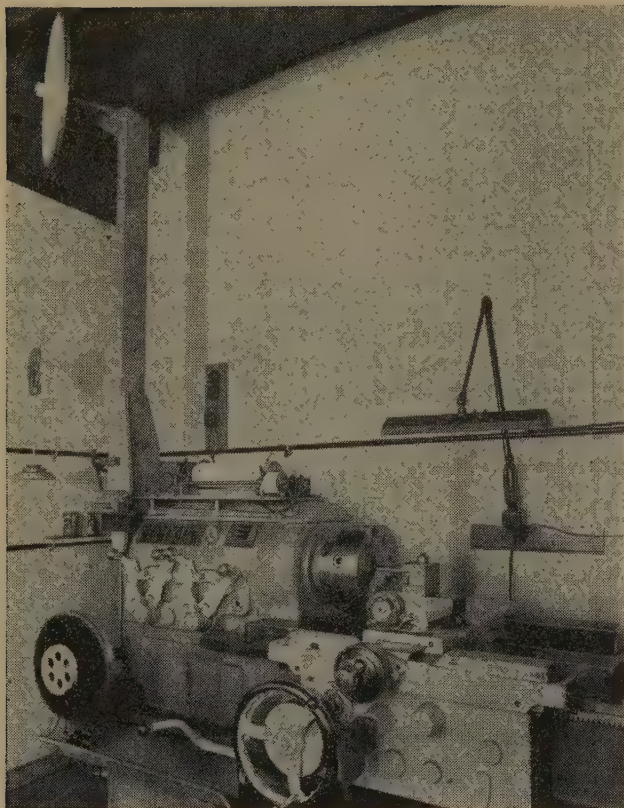


FIG. 1 MACHINABILITY TESTING EQUIPMENT

has a $5\frac{1}{2}$ -hp motor and 16 spindle speeds, ranging from 24 to 1000 rpm. It has a $16\frac{1}{2}$ -in. swing and 30 in. between centers. The spindle is bored out to take bars up to 1.5 in. diam. Most of the fittings and gears of a standard lathe are missing from the equipment, as shown in Fig. 1, by the blank bosses on the apron of the tool carriage. The tool carriage is equipped with metal shields to prevent chips from falling on the ways and getting under the bearings.

The feed load is connected to the handwheel of the lathe by a cable and pulley, and transmitted to the carriage by a gear and a pinion moving along the rack of the lathe. Since the gear is accurate and mounted in ball bearings, and the pinion mates with the rack very well, this drive mechanism performs satisfactorily. A counterweight was attached to the handwheel opposite the handle.

The peripheral edge of the 7-in.-diam drum, mounted behind the handwheel, is knurled, and a light pawl is attached to the carriage. The drum concerned is the one carrying the cable attached to the test weight. The thin aluminum pawl rides the knurled edge of the drum freely when the carriage is moving toward the chuck but prevents rotation of the drum in the opposite direction. Backward movement of the carriage is prevented because the drum is attached to the wheel driving the carriage. This device acts as a ratchet and pawl to prevent carriage chatter which is encountered occasionally in testing certain steels at certain ranges of spindle speeds. The cable attached to the weight, shown at the left in Fig. 1, moves the carriage by producing a torque on the drum mounted behind the handwheel. The rate of carriage travel of course is controlled by the amount of weight used, the resistance of the steel to cutting, and the friction of the chip moving over the tool surface. The small pulley at the lower left is mounted on ball bearings and changes the direction of the cable.

Measuring Tool Travel. The black wheel behind the pulley at the lower left in Fig. 1 is connected to the tool carriage by means of a shaft, sprocket, and roller chain. The periphery of the wheel moves 1 in. for each 0.2 in. tool travel. The cogs on the circumference of the bakelite disk mounted in front of the wheel are spaced uniformly and operate the switch mounted below the device, thereby indicating tool travel. The switch closes an electrical circuit, after each 0.2-in. increment of tool travel, causing a signal bell to ring. The mechanical counter, on the column at the left of the lathe, which records the number of spindle revolutions is driven by roller chain and sprockets connected to the lathe spindle. The counter is geared to indicate every second revolution of the spindle.

Toolholder. The special toolholder designed for the test, illustrated in Fig. 2, consists of a solid block of steel rigidly fastened to the compound of the tool carriage. It holds the tool so the cutting edge forms a right angle with the axis of the test piece and the direction of tool travel. The tool is held by means of machine bolts pressing the tool against a wedge resting on the tapered bottom of the groove. The taper and wedge permit the tool to be adjusted quickly so that the cutting edge is at the same height as the center line of the test bar. A dial-gage fixture is used to measure the distance from the top of the tool post to the cutting edge of the tool, and this tool position is constant for all tests.

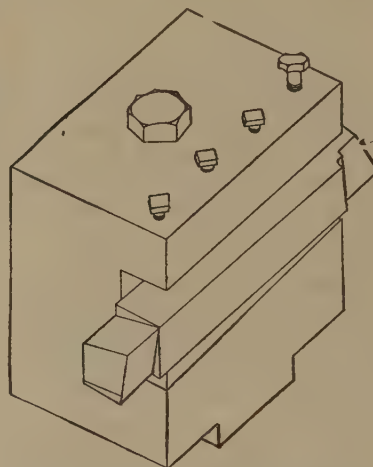


FIG. 2 SKETCH SHOWING DESIGN OF TOOLHOLDER USED WITH MACHINABILITY TESTING LATHE

Tool Preparation. The shape of the single-point tools ordinarily employed in the laboratory lathe tests is illustrated in Fig. 3. Regular high-speed-steel $\frac{3}{4}$ -in.-square blanks are ground to the shape indicated. Tools are reground after each test series with a 100-mesh alundum wheel on a grinder operating dry at 3450 rpm. A jig is used to hold the tool in order that the side-relief and side-rake angles can be maintained at 12 deg. A limited number of experiments were made with high-speed-steel tools, ground with negative top rake angles and various positive top rakes up to 24 deg. They gave about the same ratings as the shape selected as standard, which was chosen because it was easy to reproduce and gave a satisfactory useful life.

Considerable care must be exercised in preparing tools in order to get reproducible results, and they are reground before appreciable wear occurs. After each test series, usually 4 ft of steel, the tools are lightly reground, with precautions taken to avoid overheating. The tools are honed after grinding because this makes them more uniform, as illustrated in Fig. 4. After grinding and honing, the fresh tool is used for a cut about 6 in. long on a B1111 steel before starting the tests, because experience has shown first few cuts on a tool sometimes give erratic readings. In other words, the tool is "broken in" before being used in a test series.

Testing Procedures. The two types of machining operations ordinarily employed in the laboratory machinability tests are differentiated by the terms "facing" and "turning" tests. In facing-cut tests, the depth of cut equals one half the diameter

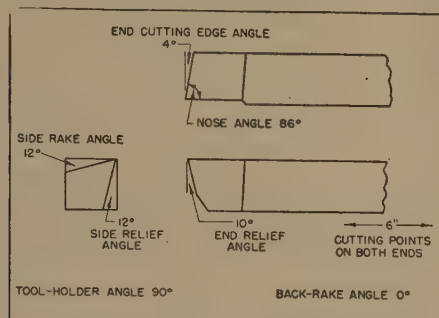


FIG. 3 SKETCH SHOWING SHAPE OF RIGHT-CUT HIGH-SPEED-STEEL TOOLS USED IN CONSTANT-PRESSURE MACHINABILITY TESTS

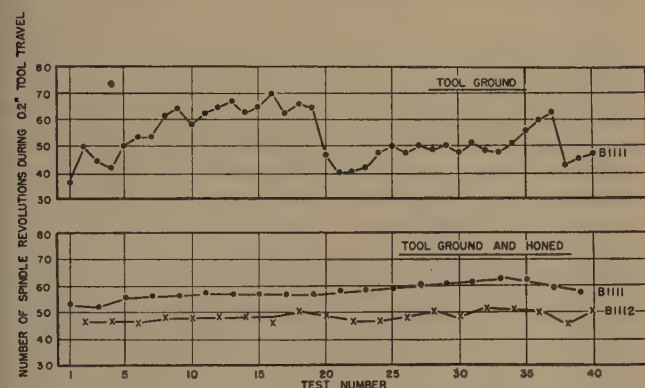


FIG. 4 EFFECT OF TOOL PREPARATION AND TOOL WEAR ON REPRODUCIBILITY OF TEST VALUES IN CONSECUTIVE TURNING TESTS; EACH 2 IN. LONG

of the specimen; in turning-cut tests, it is fixed at a smaller depth such as $\frac{1}{8}$ in. In order to facilitate comparisons between samples tested at different times and to reduce the effect of slight differences in tool conditions, a bar from a special lot of B1111 steel is tested several times in each series and used as a reference standard. A supply of this material, rolled to $1\frac{1}{16}$ -in. rounds from a particular blow, was available for use as an arbitrary standard. This standard material is considered to have a machinability rating of 100 in all tests. Steels which cut faster (heavier feeds) than the standard steel in the laboratory test under otherwise fixed conditions are given higher numerical ratings. The ratings are calculated by either of the following methods.

$$\text{Machinability index} = 100 \times \frac{\text{Average feed on unknown steel}}{\text{average feed on standard B1111 steel}}$$

$$= 100 \times \frac{\text{Average number of spindle revolutions during test on standard B1111 steel for 2-in. cut}}{\text{Average number of spindle revolutions during test on unknown steel for 2-in. cut}}$$

The two methods are equivalent because the feed is defined as the distance of tool advance per spindle revolution. The machinability indexes are usually calculated by the second method,

because the number of spindle revolutions is available from the primary test records.

Standard conditions for testing have been used whenever possible, because this practice seems safer for making comparisons between materials. All tests for comparing different steels have been made without using cutting fluids. The large majority of tests consisted of $\frac{1}{8}$ -in. turning tests on $\frac{7}{8}$ -in. rounds, and facing tests on the $\frac{5}{8}$ -in. rounds remaining from the first tests. A spindle speed of 377 rpm is usually used, and the actual feed load applied on the tool has been about 81 lb for turning tests, and 110 lb in facing tests. The lever action of the gears and pulleys multiplies the load applied at the hanger by a factor of about 13.2. Because some samples were received as larger or smaller bars, a number of tests were run under conditions other than those mentioned. Faster speeds and heavier loads were considered undesirable because they caused larger changes in chip and work temperatures during testing. The usual or "standard" conditions were selected because they were convenient for testing the bulk of the samples received and gave satisfactory sensitivity, reproducibility, and tool life.

Ordinarily, a group of samples is tested with one tool in a series which includes three tests on the standard B1111. The order of testing each material is varied systematically in each series in order to compensate for tool wear. Each material is usually included in six series of tests in order to increase the reliability of the machinability ratings and to minimize the effect of different tools. The machinability index of any material is based on a comparison with the average feed of the standard steel in the same series. The machinability index used in reporting results on a steel, is the average obtained in all tests, usually six.

STATISTICAL APPRAISAL OF REPRODUCIBILITY OF DATA AND SIGNIFICANCE OF RATINGS

The first questions in considering any new testing method concern the limits within which data can be reproduced in tests on the same material, and the significance to be attached to differences in ratings between two materials. Any test value is an estimate of the properties of a particular material reflected by the test. The error of the estimate, or the amount by which it differs from the true value, is expected to be decreased by averaging the observations made from a number of tests under similar conditions. Statistical analysis provides mathematical methods for applying this principle to evaluating the significance and reproducibility of test data. Such appraisals were made on several groups of data during this investigation and some of the results will be presented.

Using conventional statistical methods, the variation in results obtained on the standard B1111 steel in constant-pressure lathe tests was studied first.

Under standardized conditions, the average feed obtained in turning tests on the standard steel was 0.00274 in., and the standard deviation was 4.14 per cent of the test reading, for 285 tests, covering a three-month period. The use of control-chart methods as the criterion for checking statistical control indicated the data were obtained under reproducible conditions and that there were no significant differences in results during the period. The average readings obtained in different series varied of course but were within ± 5.8 per cent of the average in two thirds of the cases. Statistical methods predict that the feed value of the standard B1111 steel should fall within 0.00260 and 0.00291 in. when calculated from six test series. Checks of the average reading on the standard in six successive tests showed this was true for the three-month period covered. This is important, because it means that samples tested at different times can be compared as reliably as those tested in the same series.

The next point to consider in estimating the reproducibility of the test is the variation in indexes to be expected in samples compared to the standard. This question is one of estimating how closely machinability indexes, based on the performances of the standard and unknown samples, can be duplicated in different tests. The results of separate tests are bound to differ because of inhomogeneity of the materials or small unavoidable differences in cutting tools and other testing conditions.

This problem was studied by applying statistical methods to two groups of data. It was first assumed that other free-cutting steels would show the same reproducibility as the standard B1111 steel; conclusions based on this comparison were checked by comparing the machinability indexes, of a particular steel, obtained over a long period of time. The information necessary for the latter study was available for a certain B1112 steel tested 180 times during the same three-month period. Based on all of these tests, this B1112 steel had an average machinability index of 124.3 and the index based on six tests had a standard deviation of 4.09 per cent of this average. Statistical calculations based on the B1111 standard alone and on the machinability indexes of the higher-sulphur steel, both suggested that variations in indexes of 5 per cent probably indicate real differences in two cases out of three; 90 per cent of the time differences of 10 per cent in ratings should be significant.

The general idea of the reproducibility of machinability indexes obtained by this test method can be obtained from Table 1. This table gives the machinability indexes for individual tests on 24 samples, the average index for 5 or 6 tests, and statistical data. The statistics indicate a probability of nine chances in ten that the true indexes of the samples fall in the limits indicated in the last column. The limits set for each sample on the basis of the tests on that material are all within 9 per cent, the value predicted previously on the basis of the results on the other B1112 steel discussed previously.

The calculations indicate that the differences between the last sample and the others in each group are highly significant, that is, it is extremely improbable that such differences in ratings could be obtained without a real cause. Actually, the poor performance of samples 1-10, 2-10, 3-10, 4-10, and 6-5 through 9-5, was associated with differences in microstructure and composition. When variations can be correlated with metallurgical factors, the reliability of the data is greater than indicated by mathematical analyses because the results on one sample are supported by ratings on similar materials.

The statistical appraisal and observations on several hundred samples of free-cutting steel suggested that machinability indexes should be based on the average performance in six tests. This requires samples approximately 18 in. long and gives sufficient sensitivity to detect variations in cutting quality between samples of the same grade and even between specimens from the same heat or blow. Materials can be rated with only slightly less precision, however, on the basis of four tests with some saving in time and effort.

Hardness and Machinability. It is almost axiomatic that the hardness or strength of a steel should affect the ease with which it can be machined. Although a steel that is considerably harder than another is usually more difficult to machine, this is not necessarily true when the hardness differences are small. Ordinarily the hardness variations encountered in a particular grade of steel are less important than differences in microstructure or chemical composition.

On first thought, the constant-pressure lathe test for machinability might be suspected of being unduly sensitive to variations in hardness of different samples. Therefore the data in Fig. 5 are presented. These observations, on samples from mill production, show that the machinability indexes were found to bear no relation to the corresponding Brinell-hardness values. These 36 samples met specifications for one grade of free-cutting steel, but the spread in composition was sufficient to outweigh any effects of hardness variations on machinability test results.

Effect of Pressure on Feed and Test Rating. The effect of tool pressure on the feed obtained in this machinability test was investigated by a large number of tests on eleven different steels. Two of the materials were open-hearth steels, representing C1050 and C1144 grades. The tests on Bessemer steels included B1111, B1112, B1113, low-sulphur, and cold-drawn samples. In these tests, the load applied to the tool during testing was determined by means of an electrical strain gage measuring the deflection of the tool. Since all studies were made while taking a $1/8$ -in. turning cut on $7/8$ -in.-diam bars, the actual tool loads are directly proportional to the tool pressures. The loads applied at the hanger varied from 3 to 9 lb. and the loads at the tools ranged from 40 to 119 lb.

Fig. 6 is a typical chart presenting the data obtained on five steels tested to determine the relationship between feed and tool load in the machinability tests. Each point plotted on the chart is the average feed based on at least six observations. The results show that plotting tool load against the resultant feed,

TABLE 1 MACHINABILITY RATINGS AND STATISTICS FOR TESTS ON SAMPLES FROM BLOW "A"

Sample no.	Individual indexes						Average index	Standard deviation of average index		Limits for true index (chances 9 in 10), per cent
	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6		Units	Per cent	
1-1	203	181	189	210	201		197	5.2	2.6	± 4.3
1-3	189	173	179	208	196		189	6.1	3.2	5.3
1-5	184	190	181	190	177	187	185	2.3	1.2	2.0
1-10	145	126	141	149	141		140	3.9	2.8	4.6
2-1	171	156	172	195	187		176	6.8	3.9	6.5
2-3	192	161	187	192	192		185	5.7	3.1	5.1
2-5	199	164	187	200	196		189	6.4	3.4	5.6
2-8	192	169	182	201	190		187	5.4	2.9	4.8
2-10	136	122	133	147	143	131	135	3.6	2.7	4.5
3-1	208	172	178	201	190	172	187	6.0	3.2	5.3
3-3	202	176	179	203	197	173	188	5.6	3.0	5.0
3-4	181	185	170	184	192	188	183	3.1	1.7	2.8
3-8	195	170	187	204	201	176	189	6.6	3.5	5.8
3-10	164	138	155	181	165	141	157	6.6	4.2	7.0
4-1	192	157	175	196	191	168	180	6.4	3.5	5.8
4-3	194	174	185	187	185	170	182	3.6	2.0	3.3
4-8	189	171	192	185	150		177	7.0	4.0	6.6
4-10	127	118	114	121	121	126	121	2.0	1.7	2.8
5-5	186	151	193	193	160	153	173	8.2	4.7	7.8
6-5	169	150	180	147	171	147	161	2.1	1.3	2.1
7-5	132	149	180	131	144	136	145	7.6	5.2	8.6
8-5	141	144	163	138	133	139	143	4.4	3.1	5.1
9-5	135	134	130	123	132	136	132	2.0	1.5	2.5
10-5	182	207	174	182	184	184	185	4.6	2.5	4.1

NOTE: Standard deviation of average for X observations = $\frac{\text{Standard deviation of individual readings}}{\sqrt{X-1}}$

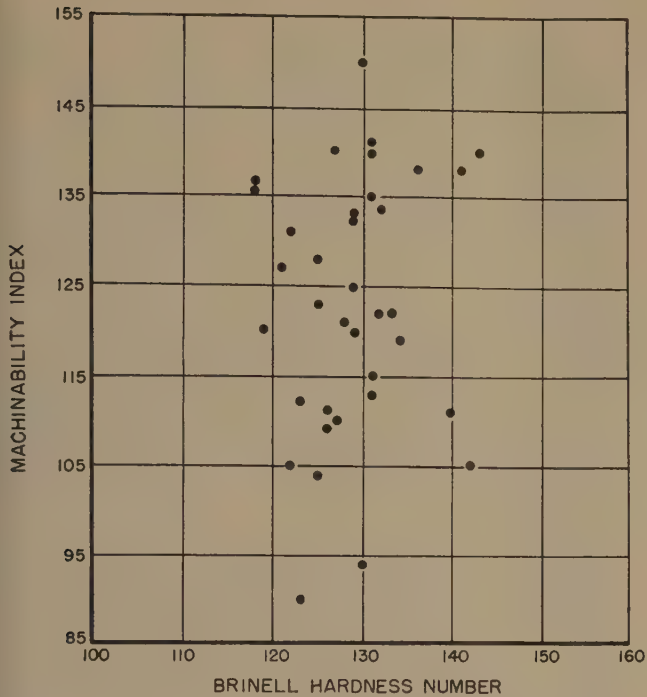


FIG. 5 SCATTER CHART SHOWING LACK OF CORRELATION BETWEEN HARDNESS AND MACHINABILITY INDEX OF 36 HOT-ROLLED SAMPLES

on logarithmic co-ordinates, gives a straight line for each steel. Furthermore, the lines expressing this relationship for the eleven steels tested to date have had the same slope. The trend lines for eleven different steels made an angle of 54 to 55 deg with the horizontal axis when the data for feed and tool load were plotted on logarithmic co-ordinates with equal scales as indicated. Therefore the plots for the different steels varied only in the magnitude of the feed resulting from the feed loads.

Since the data give parallel lines for these eleven steels, it follows that the machinability indexes are constant for any feed load ranging from 25 to 100 lb in turning tests on $7/8$ -in. rounds at 377 rpm. If this be true for all materials, it is permissible to test steels with poor machinability at higher than normal tool pres-

sures, and fast-cutting steels with lower tool loads, and extrapolate the data to make comparisons at standard conditions.

The tool loads or pressures measured during tests with the same weight on the hanger showed slight variations. These variations are attributed partly to errors in measuring the actual load by means of tool deflections. The range in loads varied about ± 3 lb, for tests with 40 and 100-lb loads, for ten readings during a 2-in. cut. The variation is a little greater for different tests. Although the fluctuation in tool pressure is minor, it undoubtedly impairs the reproducibility of the feed values. A slight variation in tool pressure would be expected to result from any differences in friction along the ways, the reaction on the tool when a chip breaks off, or tool chatter.

Effects of Speed on Feeds Resulting From a Constant Tool Pressure. Tests were made on fourteen different steels to determine the effect of speed on the feed resulting from a constant horizontal tool-pressure component. Results indicated that changes in speed had a greater influence on the rate at which high-sulphur steels cut than they did on the performance of low-sulphur steels. The data also indicated that spindle speeds had no effect on the feeds obtained on a particular steel other than that of changing the surface speed, that is, comparable results were obtained at any two combinations of spindle speed and specimen diameter which gave the same surface speed. This means that specimens of different sizes can be compared directly if the spindle settings are selected to give equivalent surface speeds.

The effect of variations in surface speeds on the feeds obtained on five different grades of steel is illustrated in Fig. 7. The tests covered speeds ranging from 5 to 300 mean surface feet per minute. The speeds are those calculated for the average diameter of the bars before and after the turning cut rather than the peripheral speed.

The curves in Fig. 7 show that the steels were rated in the same order by tests made over a wide range of speed. The "index" or relative rating compared to the B1111 standard steel, however, remained approximately constant only over the range from about 120 to 300 surface feet per minute. Below approximately 70 surface feet per minute, there was a pronounced difference in the response of different steels to changes in speed. The Bessemer samples showed a maximum feed at approximately 40 surface feet per minute; the open-hearth steels, on the contrary, showed a lowered feed at this speed. This difference

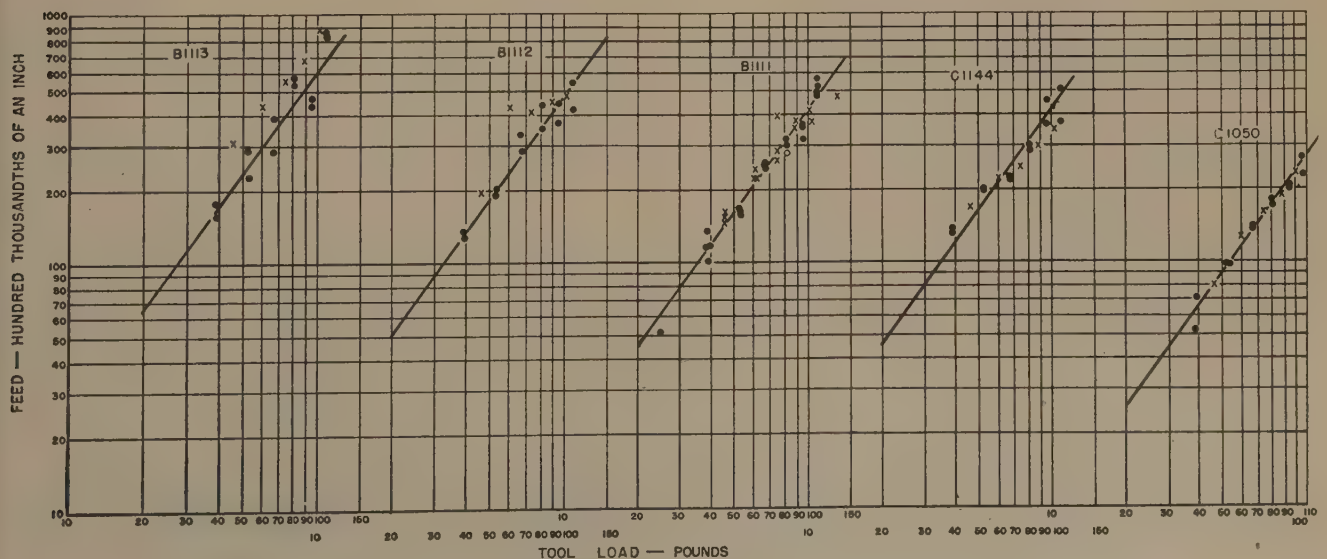


FIG. 6 FEEDS RESULTING FROM VARIOUS TOOL LOADS IN $1/8$ -IN. TURNING CUTS AT 377 RPM

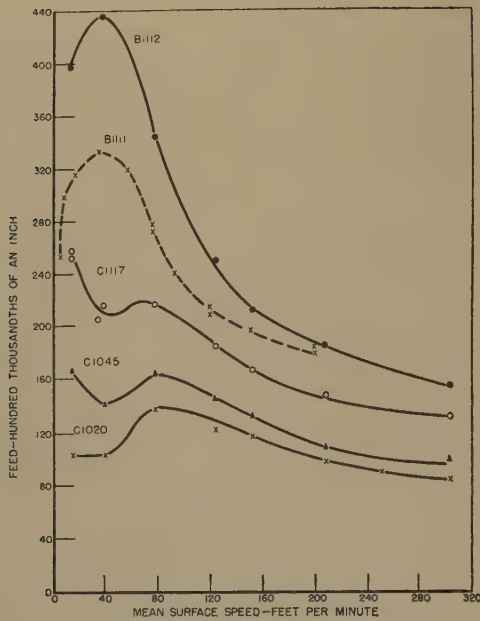


FIG. 7 EFFECT OF SPEED ON FEEDS OBTAINED ON FIVE STEELS WHEN CUTTING WITH A CONSTANT TOOL PRESSURE

between the two types of steel in dry cutting was observed in other samples, but the fundamental explanation for it is unknown.

The data for these tests indicate that the standard speed of testing, 75 mean surface feet per minute, gives a big spread in feeds between the different grades, although it was not quite so sensitive as some slower speeds. This speed gave reproducible ratings which reflected performance at higher speeds with cutting fluids. The speed of 75 surface feet per minute is also a convenient one considering rapidity of testing, long useful tool life, and the avoidance of excessive temperature changes in tool and workpiece.

COMPARISON OF CONSTANT-PRESSURE TEST RATINGS WITH TOOL-LIFE TEST AND COMMERCIAL MACHINABILITY RATINGS

In evaluating the dependability of a new testing method, it is helpful to compare its ratings with those made on similar samples by other methods. Therefore constant-pressure lathe test ratings on various materials were compared with those obtained in tool-life tests and with the results of commercial machining tests. In the laboratory tool-life tests, the machinability rating was the cutting speed in feet per minute resulting in 60-min tool life for dry cutting. The feed, depth of cut, tool material, and tool shape were chosen arbitrarily, and better steels were assumed to be indicated by higher limiting cutting speeds for 1-hr tool life. Comparisons were also made between commercial tool-life tests in normal machining operations and results in the laboratory tests.

Agreement of machinability results obtained commercially and at different laboratories depends on the sampling as well as on the test methods. Obviously, quantitatively correct ratings obtained on samples which are dissimilar or unrepresentative of the ingot or heat will not give concordant results. This section presents the ratings obtained on three series of samples. When the sampling is known to have been satisfactory, because of check analyses and methods employed, good correlation existed between machinability ratings. In the series where analyses showed differences in compositions between specimens and ingot checks, and less was known about the sampling procedure, the

agreement between the three testing methods was not so good. The problem of representative sampling is a serious one and must be considered in evaluating the evidence for agreement between the three methods of testing machinability.

Samples from series "A" made up one of the best groups for comparing machinability ratings obtained by methods used at two laboratories, because check analyses indicated that the samples received by both laboratories were closely similar in composition. Considerable care was taken to insure that duplicate samples were obtained from practically the same ingot locations.

Fig. 8 is a cross-plot of the ratings given by the two laboratories to duplicate samples obtained from the same billet. The constant-pressure test ratings were obtained in turning tests on $7/8$ -in. rounds; the tool-life data were obtained on 3-in. rounds. The fact that the points plot close to the trend line indicates a close agreement between the two types of machinability ratings. There is remarkably little scatter shown by the data, and the order of rating the samples from the ten ingots is practically identical for both methods.

Another series "B" was sampled for tests by the two laboratory methods, and records were taken during commercial machining operations on various lots. The data for the ten different lots can be used for a cross-comparison of ratings by the three testing methods.

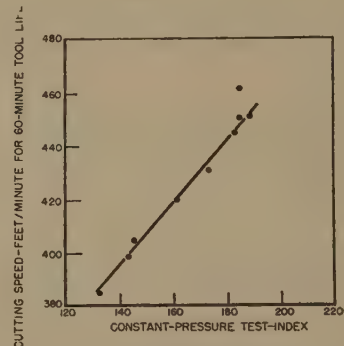


FIG. 8 SHOWING GOOD CORRELATION BETWEEN MACHINABILITY RATINGS BY TWO LABORATORY METHODS ON SAMPLES FROM TEN INGOTS OF SERIES "A"

Table 2 lists the laboratory ratings and the number of parts produced per tool in commercial operations on the different lots. The results of the commercial tests are given as the number of parts per tool because the relationship between tool life and total parts varied considerably in different tests.

The data in Table 2 show poor correlation between any two of the three methods of rating machinability. This is believed to indicate that the specimens received by the two laboratories differed significantly and that neither set represented the lots very closely. Apparently the lots were not sufficiently uniform in composition to permit reliable sampling. In spite of the differences believed to result from sampling difficulties, it is still possible to make general comparisons. All three methods of testing indicate that lots Nos. 8, 6, and 4 were in the better half of the series in respect to machinability; similarly, all tests placed lots Nos. 1 and 2 in the poorer half of the group.

Samples were also supplied to the laboratory from another series of steels used for comparative commercial machining tests. The series consisted of seven B1112 and two B1111 steels. Table 3 lists the commercial results and ratings of these materials based upon feeds obtained at a constant horizontal tool pressure. The average number of parts produced per tool was also chosen as the best parameter for commercial ratings.

TABLE 2 MACHINABILITY RATINGS FOR SAMPLES FROM EACH OF TEN LOTS OF SERIES "B" IN LABORATORY TESTS AND COMMERCIAL OPERATIONS

Lot no.	Rating in tool-life test ^a	Rate based on feed at constant tool pressure ^b	Commercial tests ^c	
			No. of tests	Average parts per tool
8	510	200	2	7360
6	482	186	3	9123
4	473	187	2	12194
3	486	179	4	7189
5	461	150	4	6289
7	510	180	3	4167
9	470	206	5	2244
10	470	199	5	2241
2	461	189	5	2435
1	450	188	5	1863

^a Cutting speed in feet per minute for 60-min tool life, hot-rolled steel, size unknown.

^b Machinability rating compared to a "standard" B1111 steel based on 1/8 in. turning cuts on 1/8 in. bars in hot-rolled condition.

^c Drum nuts were made at 315 surface feet per minute from 11/16 in. cold-drawn stock in 4-spindle automatics.

TABLE 3 COMMERCIAL AND LABORATORY MACHINABILITY RATINGS FOR NINE LOTS OF BESSEMER STEEL

Item no.	Constant-pressure test index ^a	Average parts per tool ^b
1	111	10,120
2, 3	112	10,020
4	125	10,402
5	113	8,530
6	119	10,337
7	134	14,075
8	95	7,362
9	119	11,923

^a Average for turning and facing tests.

^b Feeds were 0.0078 in.; tests at 1719 and 1547 rpm were given equal weight.

The commercial and laboratory tests agree in rating items 7, 9, 6, and 4 as the four best, and items 8, 5, 1, 2, and 3 as the worst steels in the group. Fig. 9 shows a cross-plot of the commercial and laboratory ratings on this series. The worst scatter from the trend line occurred for the values for items 4, 5, and 9. Reference to Table 4 shows that the average tool life of these steels in commercial operations was affected in an unusual manner by the change in spindle speed from 1547 to 1719 rpm. Item 9, which plots on the high side of the line in the chart, had a longer tool life at the higher speed which is out of line. Items 4 and 5 showed the largest drop in tool life with increasing spindle speed and gave points farthest from the line on the low side. Therefore the data show that the biggest discrepancies between laboratory and plant ratings occurred for the three materials which gave questionable results in commercial tests. In general, this series shows a good agreement between the laboratory and plant-test ratings.

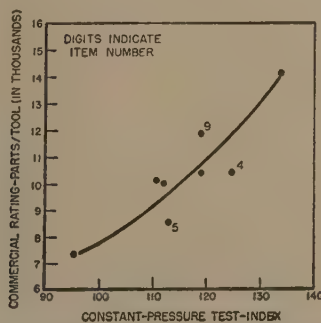


FIG. 9 SHOWING GENERAL AGREEMENT BETWEEN COMMERCIAL DATA AND LABORATORY MACHINABILITY RATINGS IN A SERIES OF EIGHT STEELS

Occasionally, groups of specimens were received upon which commercial machining ratings had been obtained. Each group usually consisted of two or three bars from lots considered to be giving good, normal, or poor performance in specific operations. The specimens in the different groups were tested to determine

TABLE 4 AVERAGE TOOL-LIFE VALUES IN COMMERCIAL TESTS ON ITEMS LISTED IN TABLE 3

Item no.	Average tool life in hours ^a	
	1547 rpm	1719 rpm
1	13.3	6.1
2	11.0	6.0
3	11.2	
4	12.0	2.1
5	15.4	1.1
6	15.7	4.8
7	17.9	8.3
8	6.9	6.7
9	9.1	11.6

^a Making the same part with the same feed, 0.0078 in.

whether or not the laboratory test rated the materials in the same order as the customer's report.

Table 5 lists the machine-shop ratings, hardness values, and laboratory ratings for eight groups of such samples. These eight groups comprise all the specimens obtained in this manner. Comparisons should be restricted to specimens in the group because the groups differ in grade, plant origin, bar size, and type of machining operation involved.

The heats represented by the bars in Group 1 of Table 5 were all considered unsatisfactory by the machine operator; consequently, the variation in laboratory ratings cannot be interpreted. Obviously, if this job required a faster-cutting grade, the best of these B1111 steels would be unsatisfactory, even though it had normal machinability for its grade.

TABLE 5 LABORATORY MACHINABILITY RATINGS FOR MISCELLANEOUS SAMPLES REPUTED TO VARY IN SPECIFIC COMMERCIAL MACHINING OPERATIONS

Group no.	Code no.	Bar size	Commercial observations	Bhn	Constant-pressure test index
1 (c)	C40	3/8 in. rd	Unsatisfactory	190	104
	C47	3/16 in. rd	Unsatisfactory	188	91
	C44	1/2 in. rd	Unsatisfactory	195	84
2 (a)	1	7/16 in. hex	23 hr tool life	..	116
	2	7/16 in. hex	12 hr 3-sec cycle	..	105
	3	7/16 in. hex	12 hr 3.5-sec cycle	..	100
3 (b)	C41	15/16-in. rd	Satisfactory	164	94
	C42	1 in. rd	Satisfactory	142	102
	C43	1 in. rd	Unsatisfactory	117	84
4 (c)	C45	3/4 in. rd	Medium	179	95
	C48	19/32 in. rd	Medium good	180	96
	C46	11/16 in. rd	Unsatisfactory	179	84
5 (b)	C70	1 1/8 in. rd	Satisfactory	134	58
	C71	1 1/8 in. rd	Unsatisfactory	126	52
6 (d)	C72	7/16 in. hex	C.D. 17 hr tool life	217	93
	C73	7/16 in. hex	8 hr tool life	189	134
	C74	7/16 in. hex	5.5 hr tool life	167	150
7 (d)	C57	7/16 in. hex	13 hr tool life	207	125
	C24	7/16 in. hex	poor tool life	212	117
	C93	7/16 in. hex	poor tool life	..	78
8 (a)	C26	1 1/8 in.	Better	130	125
	C27	1 1/8 in.	Poorer	134	100

NOTES: (a) These samples were tested quite early in program and compared only among themselves. All other ratings are based upon performance of a standard steel.

(b) Laboratory ratings based upon turning tests on 7/8-in. bars.

(c) Laboratory ratings based on facing tests on 0.445-in. rounds.

(d) Laboratory ratings based on facing tests on 7/16-in. hexagons.

In view of the difficulties of selecting one short specimen to represent lots ranging up to several tons, remarkably good agreement was obtained between laboratory ratings and commercial results. The relative order of the specimens in laboratory machinability tests is identical with commercial ratings for six of the seven groups for which comparisons can be made. The specimens listed as Group 6 were the only exceptions to a perfect correlation between machinability ratings in the plant and laboratory.

CONCLUSIONS

1 In lathe cutting, under conditions giving tool pressures with a constant component in the horizontal direction, the feed varies depending upon the properties of the material.

2 This permits simple and rapid evaluations on small samples of materials to indicate their probable performance in commercial machining operations.

3 Machinability indexes obtained by the test method described agree adequately with ratings in conventional laboratory tool-life tests and commercial machining tests.

4 With this method of testing, differences of 5 per cent in feed or "machinability index" are probably significant. The sensitivity and reproducibility of the method are sufficient to disclose differences in cutting quality between samples of the same grade of free-cutting steel.

5 The machinability indexes do not appear to be altered by changes in magnitude of the horizontal component of tool pressures used for testing.

6 The feed resulting from a tool pressure with a constant horizontal component varies with the speed of testing. For the steels studied, the feed decreased with increasing mean surface speeds in the range from 70 to 300 fpm. In tests at speeds ranging from 5 to 70 fpm, different steels exhibited different speed-feed relationships. Consequently, the speed of testing can have an important effect on the machinability rating assigned to a steel by this test method, and needs to be standardized.

7 The machinability indexes obtained by this test method do not depend upon the hardness of the free-cutting steel when it varies within the normal range for the grade.

ACKNOWLEDGMENT

This machinability test method was developed during an investigation sponsored by the Carnegie-Illinois Steel Corporation. The authors are grateful for the support and encouragement offered by many members of that organization, particularly Mr. M. W. Lightner, Dr. M. A. Grossmann, and Mr. L. W. Oswald. They also wish especially to thank Messrs. C. E. Sims and S. L. Case of Battelle Memorial Institute, for their counsel and advice on which great reliance was placed.

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Discussion

W. W. AUSTIN.⁶ Although numerous laboratory methods have been developed for evaluating the elusive property of metals known as machinability, many of these tests are so complex as almost to preclude their use in any except the best-

equipped laboratories and machine shops. Hence the need for simple short-term machinability tests such as the one described in this paper is very definite.

The comparison of the authors' constant-pressure-test ratings with tool-life tests is particularly valuable, since it should dispel any doubt that may arise as to the validity of such a short-term test as compared with the longer tool-life tests. In the writer's recent experience with the machinability testing of gray cast iron, similar correlation between a simple drill-penetration test and recognized tool-life measurements has been observed. This would indicate that satisfactory evaluations of machinability can be made on comparatively small representative specimens through utilization of a reliable short-term test.

Although correlation of the laboratory results with commercial tests (Table 2 of the paper) showed rather poor agreement, it is hoped that additional work with more nearly representative specimens and carefully controlled commercial operating conditions will show closer agreement.

The writer would like to inquire of the authors as to their opinion regarding the applicability of their constant-pressure lathe test to the machinability testing of gray cast iron. Would any significant changes in testing conditions be indicated; and if so, what conditions are recommended?

O. W. BOSTON.⁶ This paper gives a very good correlation of the longitudinal force in cutting with other machinability data. The method has been used many times before, even by the writer, but he has never felt that it is as reliable as the results of the present paper show. This may be due to the fact that the materials tested were of the same general types, that is, confined to the free-cutting steels rather than including a wide variety of carbon and alloy steels.

The authors have advanced a basis for using the longitudinal force test for measuring the machinability of free-cutting steels and then have proceeded to present the data on this subject and correlate them with other factors, such as the tool life represented by V_{60} . Their results showing the relation of hardness to their machinability index are not as good as might be expected, but certainly their data showing the relationship between the feed under constant pressure and the tool load, Fig. 6, show very favorable correlation, as does the relationship between constant-pressure index and V_{60} , as shown in Fig. 8 just referred to.

The writer is surprised to see the variation in the machinability index for the various tests up to 40, as shown in Fig. 4, for a ground tool and a tool ground and honed. He is surprised that the ground tool would give the variation in tool life, V_{60} , as is indicated in this chart.

The curves shown in Fig. 7 of the paper giving a relation between the cutting speed and the machinability index for two Bessemer and three-plain-carbon steels, is interesting. This would indicate the 80-fpm cutting speed is more or less critical, as below that value the results change in general character from those of high speed. These data conform with many other similar results, in that, the cutting force becomes erratic at relatively low cutting speeds. For this reason it seems desirable to keep the cutting speed in any constant longitudinal force test above the 80-fpm mark.

The writer has used similar constant-force test procedures in sawing and has found that the results obtained, so far as hacksaw-blade wear with different cutting fluids, corresponds with commercial practice.⁷

⁶ Chairman, Department of Metal Processing, College of Engineering, University of Michigan, Ann Arbor, Mich. Fellow ASME.

⁷ "Performance of Cutting Fluids When Sawing Various Metals," by O. W. Boston and C. E. Kraus, Trans. ASME, vol. 56, 1934, pp. 527-531.

⁶ Metallurgist, Southern Research Institute, Birmingham, Ala.

In applying the same principle however, to twist drills, that is, determining the penetration for a given time for a given drill under a constant load or thrust, the machinability rating has been found for a wide variety of materials not to agree satisfactorily with corresponding unit-horsepower ratings obtained by drilling, milling, and planing.⁸

It is hoped that the writers will continue this method of test and will apply it to material of quite different character, so as to broaden the scope on which the satisfaction of this method of test is based. From the data presented, as well as from the writer's past experience, it seems that the rate of feed for a given longitudinal force, or the longitudinal force developed for a constant feed, is equally satisfactory.

CHARLES A. DAVIS.⁹ The long period of time and arduous effort through the years that have accompanied previous attempts to define and evaluate the still rather vague term, "machinability," were evidenced in this paper by a number of references. It has been stated elsewhere there have been over 4000 pertinent articles published during the complicated growth and history of many decades of related research. This is proof of the widespread interest in the general subject.

Evidence was presented to show the sensitivity and reproducibility of the test, as well as its comparability to V_{90} tool-life-test results. These features are certainly most essential, but perhaps there is also room for further development in the direction of simplicity, and in an effort to correlate results, in a general way at least, with the higher speeds generally encountered in the best modern practices.

The test seems certainly to be worthy of the close attention of the metal-cutting research fraternity. The authors' approach to the problem may well be such that more laboratories should gather data by the same method for comparative purposes. Certainly the need for a standard test and definition of machinability, correlated as these two must be, is a great need.

Strict reproducibility of shop-production results need not be a criterion, for most will agree these are difficult to reproduce in themselves. It does seem essential, however, that any test for evaluating machinability should be such that results could be translated through a sufficient range of speeds, into predictions for some (if not all) other machine operations under controlled laboratory conditions.

It would be a further amazing, but nevertheless useful, quality if such results could possibly be brought into simple relationships to such properties as tensile strength, yield-strength ratio, hardness, and impact, an expedient our authors wisely advise we shun for the moment.

G. H. ENZIAN¹⁰ AND E. J. PALIWODA.¹⁰ The test described by the authors is an interesting approach to the extremely difficult problem of evaluating the turning quality of free-cutting steels in reliable, quantitative terms on a laboratory basis. The small amount of material employed and the rapidity with which the test can be run are distinct advantages from an investigational standpoint. However, because of the complexity of the problem of machinability, any laboratory test on small specimens also has certain disadvantages, and the test results must be interpreted with caution.

The machinability of steel, under given cutting conditions, seems to be influenced to a considerable extent both by the physical characteristics of the steel and by the presence of abrasive silicate

or aluminate inclusions. The term "physical characteristics" of the steel includes such factors as hardness, microstructure, chemical composition, sensitivity to cold work, shear strength, etc.; these are factors which determine how the steel reacts to the cutting action, and the ease with which it may be machined. On the other hand, it seems likely that the more important effect of abrasive inclusions is on the cutting tool, and thus the presence of such inclusions affects the machinability of the steel in a somewhat more indirect manner.

The quantitative influence of abrasive inclusions on the machinability of steels has not, as yet, been satisfactorily determined. Ramsey and Graper¹¹ have ascribed considerable importance to the influence of steel "cleanliness" on machinability. Merchant and Zlatin¹² have taken a similar view but showed that the presence of abrasive inclusions did not materially alter the geometry of tool forces during orthogonal cutting.

The test described in this paper should prove valuable in the analysis of the effect of the physical characteristics of the steel on turning performance. However, it would be of interest to know if the authors have observed whether the presence of varying amounts of abrasive inclusions affects their test results materially.

We have been interested in the machinability problem for some time, and in the past several years have made extensive use of the test developed by Cruickshank.¹³ This test differs somewhat from that described in the present paper in that the machinability rating is based upon tool life using a special tool made from a hot-die steel of moderate alloy content and hardness. Briefly, the test consists of determining the highest speed, in surface feet per minute, that a standard test length of 870 surface feet can be cut without appreciable tool wear. Our experience has shown that, with this relatively "soft" test tool, the presence of even small amounts of abrasive inclusions in the steel usually results in low test values. We believe that this action of the inclusions on the test tool is an accelerated indication of their more gradual effect on a high-speed tool in commercial machining operations.

The authors indicate (Table 2), that they obtained poor correlation between the feed test index, the rating in a 60-min tool-life test, and commercial performance for a series of steels. We would agree that the principal reason for this lack of correlation probably lies in the difficulty in obtaining representative test samples, because we have encountered similar difficulties in our work. Quality spreads of considerable magnitude have occasionally been found within ingots; furthermore, it appears that the quality of the steel is not necessarily associated with the position of the test within the ingot.

It would seem, therefore, that the difficulty in obtaining test specimens which are representative of relatively large amounts of steel, and satisfactory interpretation of test results, are paramount considerations in the successful application of laboratory machinability tests.

J. H. FRYE.¹⁴ The rapid duplication of interchangeable parts is the very foundation of our industrial success and living standards. One of the primary methods of mass duplication is that of machining, and especially as it pertains to automatic-screw-machine operations.

This investigation is an important step toward arriving at a

⁸ "Metal Processing," by O. W. Boston, John Wiley & Sons, Inc., New York, N. Y., 1941, fig. 16, p. 12.

⁹ La Salle Steel Company, Chicago, Ill. Mem. ASME.

¹⁰ Metallurgical Research Division, Jones & Laughlin Steel Corporation, Pittsburgh, Pa.

¹¹ "Observations in the Making and Use of Sulphite-Treated Steels," by E. L. Ramsey and L. G. Graper, Trans. AIME, vol. 150, 1942, pp. 127-130.

¹² "Basic Reasons for Good Machinability of Free-Machining Steels," by M. E. Merchant and N. Zlatin, ASM Preprint no. 21, 1948.

¹³ "Method for Testing the Machinability of Free-Cutting Steels," by A. Cruickshank, U. S. Patent no. 2,269,360, Jan. 6, 1942.

¹⁴ General Manager of Sales, Columbia Steel & Shafting Company, Pittsburgh, Pa.

means of measuring and evaluating the complex factors which influence the production of machined parts. It has been said that there are 100,000 variables entering into machining characteristics of steels, and although this sounds fantastic, it will be agreed that there are a great many of them. Obviously, any attempt to measure or compare machinability must eliminate as many variables as possible, and the ingenious procedures adopted in this investigation have accomplished this in splendid fashion. The Carnegie-Illinois Steel Corporation is to be congratulated on its foresight and keen interest in this important subject, as well as the researchers for their method of attack and thoroughness of their exploration.

A laboratory method of evaluating the machining characteristics of steels is vital to every basic producer of free-machining steels, if improvement in machining qualities is to continue. However, the writer thinks it is important to differentiate between research and practical test procedures. From a practical point of view, machinability is best measured by the number of pieces per day made on a machine. This of course involves a complexity of factors other than the steel, including those real, intangible, and psychological. Also, an evaluation of machining by a turning test does not necessarily—and probably will not—apply to end working operations (drilling, reaming, etc.), or to thread cutting or rolling, knurling, and others. It should be repeated that the several laboratory methods of testing for machinability, including this fine addition to our knowledge, are of inestimable help in the development of free-machining steels, but as yet have not satisfied the need for a practical test for commercial purposes. The wide discrepancies in the laboratory and shop values shown in Tables 3 and 4 of this paper are indicative of that statement.

Fig. 9 of the paper shows graphically "general agreement between commercial data and laboratory machinability ratings in a series of eight steels." The writer would hesitate to term these results as being in general agreement. The difference between 8300 and 10,000 pieces, or between 10,200 and 11,900 pieces is too great a variation between commercial and laboratory results. Those differences are more than a difference between profit and loss in a jobbing screw-machine shop.

The authors do not indicate chemistry, "as-rolled" grain size, sulphide distribution, hardness, method of processing (hot-rolled or cold-drawn bars), and some of the other factors normally considered to influence machining characteristics. It is assumed that these factors were investigated, but there is some question in the writer's mind regarding the influence of one or more of these factors and their effect on the variations in some of the test results. Among these chemical and mechanical factors consideration should be given to the yield-tensile ratio, as experience has indicated, and at least partially confirmed by the work of Ernst and Merchant, that with low-carbon steels the machinability improves as the yield strength more closely approaches the tensile strength. Cold-drawn bars machine much better than hot-rolled bars of the same composition and, largely, in the writer's estimation, because of the increased hardness and the much higher yield as compared to the tensile strength.

The authors' comments on the effect of hardness on machinability are contrary to the writer's experience in industry. Considering only low-carbon (0.23 maximum carbon) steels, sulphurized or not, the greater the hardness the better the piece production. Such steels literally cannot be too hard (except by heat-treating methods) for machining purposes. However, Fig. 5 of the paper indicates that only low hardness values, and in a narrow hardness range (120–140 Bhn), were involved. The level of hardness values and the range of hardness are too restricted for any outstanding difference. It would have been interesting if the authors had included specimens of a steel which had widely different hardness values. As an example, the writer would be in-

terested in their findings on specimens of B-1112 steel with hardness values of 140, 160, 180, and 200 Bhn. These higher values are produced regularly in nominal sizes by cold-drawing and are regularly used in production screw-machine shops.

It would be of interest and possible value to have this work continued on an expanded basis, possibly to cover such variations as grain-size effect, sulphide distribution, yield-tensile ratio, and hardness levels. However, it is the writer's opinion that this testing procedure is a laboratory tool only and not generally suitable for commercial applications.

T. M. GARVEY.¹⁵ The consistent and uniform changes in feed resulting from corresponding changes in tool pressure are an indication of the suitability of the constant-pressure lathe-type machinability test for measuring machinability. As the authors noted, the observed correlation between feed and tool pressure among the various steels provides a basis for determining relative ratings, regardless of the tool load used in individual experiments.

When it had been determined that this test provided data which were in substantial agreement with performance data obtained in commercial machining operations, the writer's company installed similar equipment at its research laboratory. Results to date have confirmed that this is a reliable method for evaluating machinability.

As evidence of the correlation existing between performance in automatic screw machines and the Carnegie-Battelle machinability ratings, the following case is presented as an example: Three steels, similar in composition and processing, responded quite differently in a carefully controlled testing program conducted on a four-spindle machine. That is to say, these three steels resulted in different tool performances at a given cutting speed. The research laboratory of the writer's company was requested to evaluate the machinability of these three steels in the fixed pressure test. The ratings obtained in this test are compared with the tool-life values in the commercial test in the following table:

Identification	Tool life, hr	Machinability indexes ^a
Steel A	10.5	185
Steel B	6.2	171
Steel C	3.8	158

^a Based on hot-rolled B1111 steel as a standard.

It is apparent that the laboratory method placed these steels in the same relative order as the commercial test. Subsequent metallographic examination disclosed that the differences in microstructure among the steels were of such nature as to indicate qualitatively that they would respond to turning operations in the order noted.

Recently it has been reported to us that a number of free-machining steels which appeared from tests conducted at Battelle to possess superior machining properties had likewise exhibited excellent machinability in a variety of applications at several shops. On the basis of such verification of test results, it appears that this test provides a reliable index of machinability. What is also significant, moreover, is that the test consumes so little time and so little material, thus permitting us to appraise the machinability of various steels fairly rapidly.

E. K. HENRIKSEN.¹⁶ The method described in the paper has something in common with the old loaded-drill test, but the new method avoids that drawback of having continuously varying

¹⁵ Research Associate, Carnegie-Illinois Steel Corporation, Pittsburgh, Pa.

¹⁶ Professor in Charge, Department of Materials Processing, Sibley School of Mechanical Engineering, College of Engineering, Cornell University, Ithaca, N. Y.

cutting speed all along the cutting edge which was present in the old method, and for that reason this new method is much more reliable.

The writer agrees fully with the authors that machinability, in general, is more closely related to feed pressure than to tool pressure in the direction of the cut. He strongly emphasizes, however, the necessity for considering the effect of cutting speed upon tool pressure as one of the various factors that constitute "machinability rating." Several investigations, back to the days of Stanton and Hyde's research¹⁷ show a pronounced effect of cutting speed upon tool pressure and in some cases even the curves intersect so that the order of machinability rating of two different materials may be reversed.

Some of the curves in Fig. 7 show a hump in the region of 50 to 80 fpm, and it is interesting to compare these curves with some curves for tool pressure in relation to cutting speed which were obtained in the writer's former laboratory at the Technical University of Denmark, Copenhagen, by his assistant K. V. Olsen (Fig. 10 of this discussion).

Details of the cutting conditions are given in the illustration, Fig. 10. Two groups of tools were used, the difference being the angle of the cutting edge to a horizontal plane; in the first case the cutting edge was horizontal, in the second case the angle was -5° deg. All the curves show a disruption of the regular course in the region of 60 to 150 fpm cutting speed. Personally, the writer is

¹⁷ "Cutting Tools Research," by T. E. Stanton and J. H. Hyde, *Engineering*, vol. 119, Jan. 30, 1945, pp. 148-150.

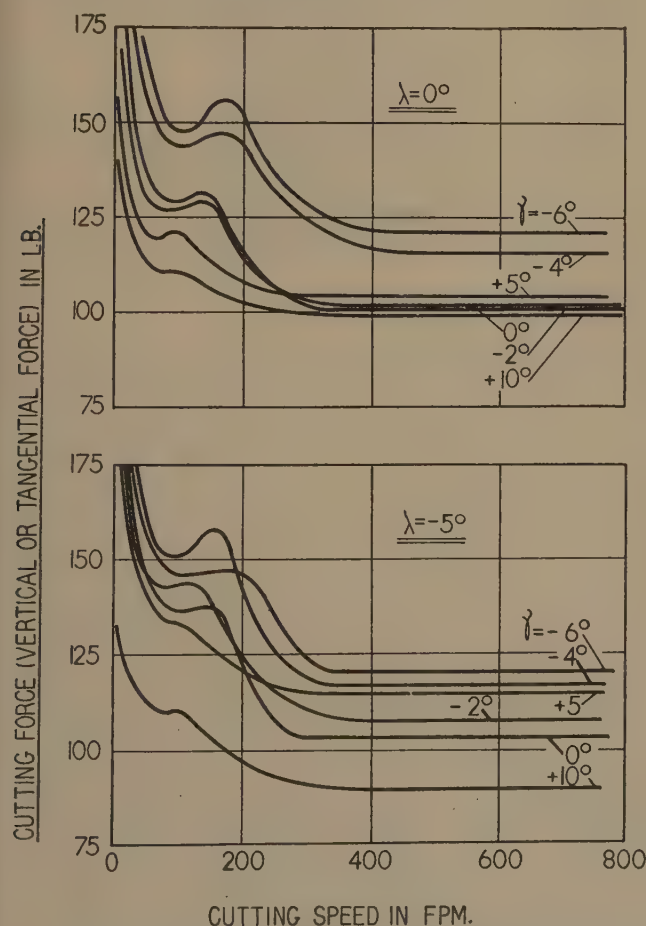


FIG. 10 EFFECT OF CUTTING SPEED UPON CUTTING FORCE
(Material: 0.23 per cent carbon steel. Tool shape: Side-cutting-edge angle 45° deg; nose radius 0.020 inch; back rake angle 0° deg; normal rake angle γ and edge inclination angle λ shown on the graphs.)

inclined to connect these phenomena with the gradual disappearance of the built-up edge but agrees with the authors that we do not yet have a full fundamental explanation of their origin.

M. E. MERCHANT.¹⁸ The authors are to be congratulated on developing such a sensitive test for measuring the machinability of free-cutting steels. The question might naturally be asked as to why the measurement of horizontal tool force at constant feed, rather than of rate of feed at constant horizontal tool force as in the paper, would not provide just as sensitive a measure of machinability. The answer is found in the well-known empirical law which relates horizontal tool force to feed when all other variables are held constant. This law, stated in approximate form, is

$$F_t = K \sqrt{f} \dots \dots \dots [1]$$

where

F_t = horizontal tool force (longitudinal component of tool force)

K = constant of metal being cut (when depth of cut and tool form are held constant)

f = feed per revolution

Thus if feed is held constant, the horizontal force is proportional to the constant K of the metal being cut. However, Equation [1] herewith can also be written in the form

$$f = \frac{1}{K^2} F_t^2 \dots \dots \dots [2]$$

by squaring both sides of Equation [1], and solving for f . It can then be seen that if horizontal tool force is held constant, as in the test described by the authors, the resulting feed will be proportional to the reciprocal of the "square" of the constant K of the metal being cut. Thus if one metal has a value of K three times as great as another, for instance, then in the case of a constant-feed test (Equation [1]), the horizontal force will also be three times as great. However, in the case of a constant-pressure test (Equation [2]), on the same metals, the metal having a value of K three times that of the original metal will cut with a feed rate only $1/9$ as great. In other words, feed rate is much more sensitive to the metal properties than is longitudinal force. The test developed by the authors therefore is one very sensitive to the properties of the workpiece.

Other questions then naturally arise. Just what, basically, does the constant-pressure test measure? What is the work material property that is being measured by this test? The experimental quantity which the authors observe in testing by this method is the value of feed required to give a constant horizontal tool force. In doing this they use a cutting tool having no side-cutting-edge angle and a relatively small value of rake angle. Under these conditions, the constant horizontal tool force is practically equivalent to a constant force of friction between the cutting tool and the chip. This can be seen from an equation coming out of an analysis of the mechanics of metal cutting,¹⁹ namely

$$F = F_t \cos \alpha + F_c \sin \alpha \dots \dots \dots [3]$$

where

F = force of friction between tool and chip

F_t = horizontal (longitudinal) tool force

F_c = tangential tool force

α = true rake angle of cutting tool

¹⁸ Senior Research Physicist, Cincinnati Milling Machine Company, Cincinnati, Ohio. Mem. ASME.

¹⁹ "Basic Mechanics of the Metal-Cutting Process," by M. E. Merchant, *Journal of Applied Mechanics*, Trans. ASME, vol. 66, 1944, p. A-168.

Equation [3] is a general relation which holds true for all types of chips found in metal cutting; it is not dependent upon chip type.

The equation can be applied directly to a turning operation when using a tool with no side-cutting-edge angle, as in the authors' test. The true rake angle in this case is then merely the side rake. Since, in the present test this rake angle is small, the second term of Equation [3] is quite small compared to the first term and therefore can be neglected for purposes of a first approximation. It is then evident that the horizontal tool force F_t is directly proportional to, and virtually equal to, the friction force F . In other words, this test measures essentially the feed required to produce a certain friction between chip and tool. Thus the test is essentially a very sensitive measure of chip friction.

As a sensitive measure of chip friction, the constant-pressure lathe test should be quite suitable for indicating the machinability of "free-cutting" steels. We are glad to see that the authors limited their paper to free-cutting steels in the title. Apparently, all of the steel samples reported on were of the free-cutting variety. However, there is no indication in the body of the paper to suggest that the test might not be suited to general machinability measurements on all types of steels. That the test is not so suited seems fairly certain, for the following reasons: It has been shown by recent studies²⁰ that free-cutting steels (i.e., sulphurized or leaded steels) owe their free-cutting properties to the reduced friction between chip and tool, produced by the action of the sulphur or lead. Their machinability (in terms of tool life) is proportional to the amount of this friction reduction. Thus machinability is found to correlate directly with the present test, which measures chip friction, as long as free-cutting steels are used.

However, in the case of ordinary steels, machinability (i.e., tool life) is not nearly so dependent on chip friction. With carbon or alloy steels which do not contain free-machining additives such as sulphur or lead, the abrasive action of the steel on the cutting tool seems to exert at least as important an influence on tool life as does chip friction. This abrasive action of the steel on the cutting tool does not show up in terms of chip friction at all, but is very evident in terms of tool wear. It is dependent on such factors as the bulk hardness of the steel, its strain hardenability, and in particular, its microstructure.

As an example of this behavior, consider the typical case of two bars of SAE 3150-1 and 3150-2. Both came from the same heat of steel but the 3150-1 had a spheroidized structure while the 3150-2 had a pearlitic structure. When tool-force measurements were made on these materials, the two steels showed identical chip-friction values, as well as identical tool-force values. However, the cutting speed for a 60-min tool life was quite different for the two samples of steels. For the 3150-1, having the spheroidized structure, this value was 155 fpm, whereas for the pearlitic 3150-2 the cutting speed for 60-min tool life was only 132 fpm. It is evident that in spite of the fact that chip friction was the same in both cases, the pearlitic structure was more abrasive to the cutting tool than the spheroidized structure.

Numerous similar cases of pairs of steels exhibiting identical tool forces and identical chip friction, but giving quite different values of tool life because of the difference in their abrasive action on the tool, have been observed in our laboratory. Thus while the test described by the authors is excellent for determining the machinability of free-cutting steels, it appears to be basically unsound for rating machinability in terms of tool life when applied to ordinary steels which are devoid of free-machining additives.

G. P. WITTEMAN.²¹ It is well known that industry is sorely in need of a simplified short-time method of measuring machinability which would evaluate specific commercial machining operations. The authors are to be commended for this approach to the problem. Their data indicate significant correlation of constant-pressure index with commercial observations. This kind of information should give encouragement to continue the project. In the event that further work is contemplated, it is suggested that consideration be given to the relationship of tool life to the following:

- (a) Smoothness of machined finish.
- (b) Constancy of tolerance.

Specifically, mass production of parts on single- and multiple-spindle automatic machines is sometimes appraised on such quality standards.

Coincidental with the publication of this paper, the writer's company has used the constant-pressure method on a commercial scale. The Bethlehem Plant No. 2 machine shop designed and built a special double-end horizontal drilling machine capable of handling bars from 4 to 12 in. diam up to 40 ft long. The bars are chucked in a driving head located at the center of the bed. This driving head operates independently from a 50-hp variable-speed motor. The right- and left-hand drill-mandrel carriages are sustained hydraulically at constant-pressure feeds by two conventional cylinders and piston rods. The capacity of the hydraulic system is 8 tons maximum for each drill-mandrel carriage, and the increments of constant-pressure feeds are regulated by appropriate metering valves and limit switches which control the motors on the hydraulic pumps.

The machine is adequately instrumented which provides for the following data:

- 1 Constant-pressure feed recorder located in the relay control panel.
- 2 Direct-reading column fluid gages which indicate the rate of feed in inches per hour for each drill mandrel.²²
- 3 Dial pressure gages, reading in tons of constant pressure for each boring bar.²²
- 4 Speedometer reading in revolutions per minute for the work material.²²

The drill-mandrel heads are designed for a simple fixture ground spade-type cutter, made from 6-6 high-speed steel, hardened to R.C. 63-65, which can be changed rapidly by means of a single set-screw. The point angles are varied from 10 deg to 20 deg, or 160 deg to 140 deg included angles. The cutting edges are provided with necessary chip breakers. A water-emulsion cutting fluid is directed to the cutter at 150 psi pressure through a 3/4-in. pipe line which is flush with the outside diameter of the mandrel. This copious flow of cutting fluid facilitates in washing out chips and has ample cooling capacity to prevent heating of the chips.

In production studies of 2 1/4-in.-diam deep-hole drilling of 4142 and 4145 modified, annealed, and heat-treated, the rate of feed varied from 4 to 55 in. per hr at a constant pressure of 2 1/2 tons for a peripheral cutting speed of 70 rpm or 41 fpm.

To determine if metallurgical inconsistencies caused variables in the rate of feed, disk sampling of work material was impossible as the specified length of the bar would have been rendered useless. As an alternative, it was decided to sample the chips, which was accomplished by noting the fluctuations on the feed gages. The chips were readily identified with the rate of feed by differences in thickness, otherwise referred to as chip load.

In an attempt to evaluate the work material, four chips of

²⁰ "Basic Reasons for Good Machinability of 'Free Machining' Steels," by M. E. Merchant and N. Zlatin, Trans. ASM, vol. 41, 1949.

²¹ Assistant Metallurgical Engineer, Bethlehem Steel Company, Bethlehem, Pa. Mem. ASME.

²² These instruments are located at the driving head panel.

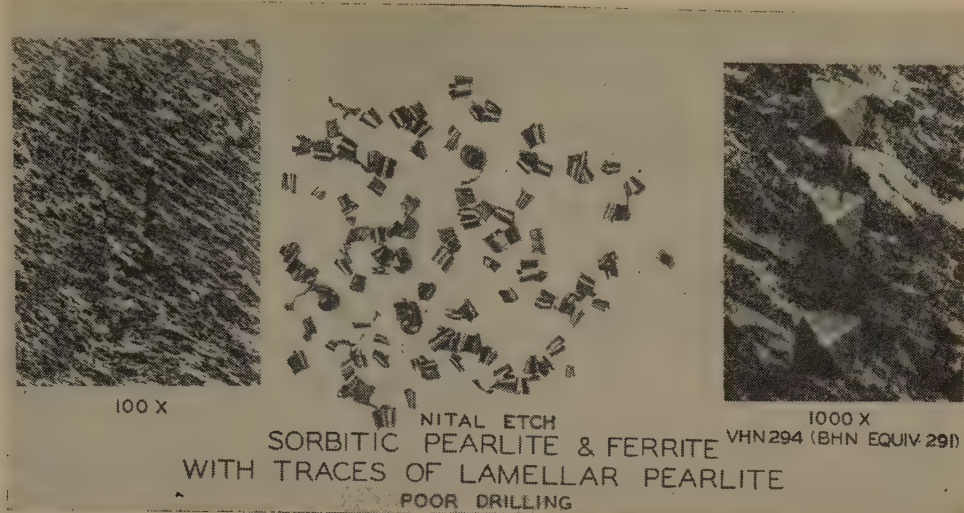


FIG. 11 CHIPS FROM HEAT CH695, CODE S17, GRADE 4142, SIZE 5 $\frac{3}{4}$ IN. DIAM ANNEALED, FEED 4 IN. PER HR.

Location of specimen left end 10 ft. 9 in., microhardness Vhn 294. Companionate specimen right end 10 ft. 1 in., microhardness Vhn 330.

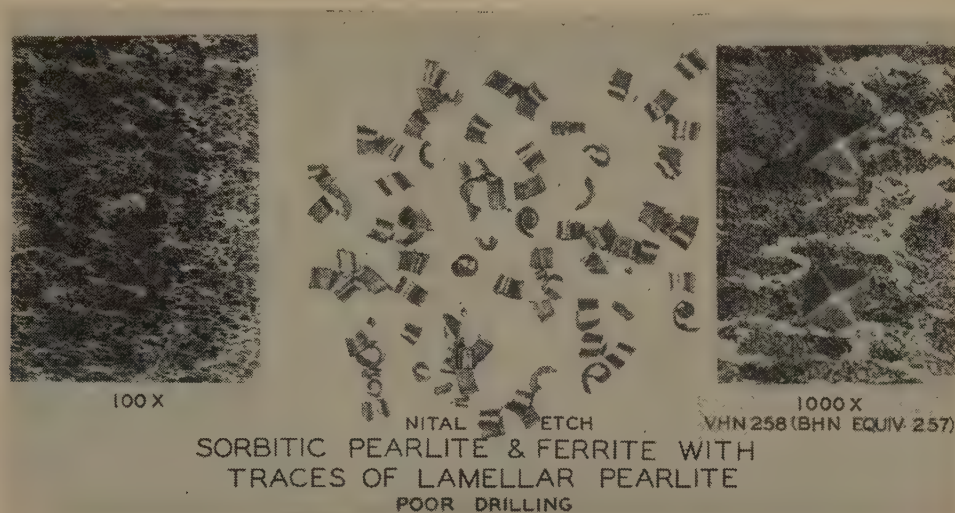


FIG. 12 CHIPS FROM HEAT 6H695, CODE S17, GRADE 4142, SIZE 5 $\frac{3}{4}$ IN. DIAM ANNEALED, FEED 28 IN. PER HR.

Location of specimen left end 11 ft. 6 in., microhardness Vhn 258. Companionate specimen right end 12 ft. 1 in., microhardness Vhn 258.

uniform thickness were selected from each group which indicated feed variables. These chips were mounted in plastic, polished, and etched for microstructures at $\times 100$ and $\times 1000$. Included were microhardness determinations which, for an average of ten readings on each chip, checked within 2 per cent. The over-all sampling consisted of fifteen sets of chips taken from five heats of steel annealed or heat-treated to various mechanical properties. For the purpose of this discussion details of three heats and chips thereof are shown in Table 5 herewith. In conjunction are illustrations of the chips with respective microstructures and microhardness results identified in the caption, Figs. 11, 12, 13, and 14.

The writer would summarize the results of this work as follows:

1 As the project dealt with actual production of relatively large masses of work material, the data could not be confirmed by retests.

2 The sensitivity of the constant-pressure hydraulic system is rather significant as shown by the rates of feeds, especially within the same bar. Moreover, there was a noticeable change in rates of feeds due to tool conditions. During one of the observations,

cutting started at 34 in. per hr, left end at 6 in., and dropped to 28 in. per hr at 20 in. The pressure gage wavered between 2.5 and 2.6 tons, and upon examination of the cutter it was found that the cutting edge on one angle had spalled for $\frac{3}{8}$ in. The cutter was replaced, after which the rate of feed was maintained at 38 in. per hr.

3 There is not sufficient duplication of data to set forth definite conclusions; however, from what is available certain trends are indicated. These appear to have some correlation with the authors' experience as associated with differences in microstructure, composition, and hardness. Some of these trends are:

(a) Rates of feeds are affected by the nonuniformity of structures for annealed 4142 heat 6H695, as compared with more uniform structures for normalized and drawn 4145 modified, heats 6H634 and 2H341.

(b) Hardness results of the three heats did not reveal any criteria as to the difference in rates of feeds. However, it is interesting to note that the annealed 4142 imparts greater proportionate work hardness in the chips.



Fig. 13 CHIPS FROM HEAT 6H634, CODE S10, GRADE 4145 MODIFIED, SIZE $4\frac{5}{8}$ IN. DIAM NORMALIZED AND DRAWN (Feed 30 in. per hr, uniform length 30 ft, microhardness Vhn 348.)

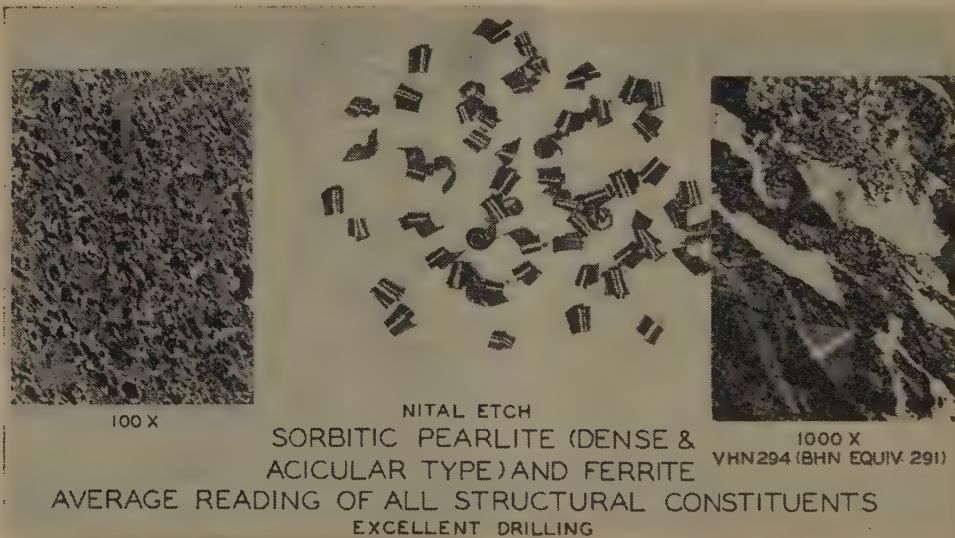


Fig. 14 CHIPS FROM HEAT 2H341, CODE M5, GRADE 4142 MODIFIED, SIZE $6\frac{1}{4}$ IN. DIAM NORMALIZED AND DRAWN, FEED 55 IN. PER HR Location of specimen left end 6 ft, microhardness Vhn 294. Companionate specimen right end 6 ft, feed 45 in. per hr, microhardness Vhn 332.

TABLE 6 DETAILS OF WORK MATERIAL 4142 AND 4145 MODIFIED

Analysis	Heat 6H695	Heat 6H634	Heat 2H341
Carbon.....	0.40	0.46	0.46
Manganese.....	0.89	1.04	0.97
Phosphorus.....	0.022	0.026	0.017
Sulphur.....	0.020	0.020	0.022
Silicon.....	0.27	0.23	0.28
Nickel.....	0.18	0.13	0.08
Chromium.....	1.04	0.99	0.84
Molybdenum.....	0.19	0.29	0.30
Copper.....	0.10	0.11	0.09
Grade.....	4142	4145 mod	4145 mod
Size (diam), in.....	$5\frac{3}{4}$	$4\frac{5}{8}$	$6\frac{1}{4}$
Treatment.....	1600 F, FC 30 deg F per hr to 900 F	1550 F, AQ, draw 930 F	1600 F, AQ, draw 800 F
Machinability.....	Poor	Good	Excellent
Drilling rate, in. per hr	28 and 4	30	55 and 45
Chip load (cal).....	4 in./0.0009 in. and 28 in./0.0065 in.	0.007 in.	0.013 in.
Chip hardness Vhn..	4 in.-294 and 28 in.-258	348	294 and 332
Work material hardness Bhn.....	179/207	302	285/302
Structure.....	Sorbitic pearlite and ferrite with traces of lamellar pearlite	Sorbitic pearlite and ferrite	Sorbitic pearlite (dense and acicular type) and ferrite

(c) Composition, mass effect, and heat-treatment temperatures (Table 6 of this discussion) are contrasted for heats 6H634 and 2H341. The latter produced an acicular-type structure shown at $\times 100$, Fig. 14, because of the higher austenitizing temperature and the lower draw temperature. This condition gave the highest machinability rating which, theoretically, may have been due to the embrittling effect of the acicular constituent.

NORMAN ZLATIN.²³ The constant-pressure lathe test developed by the authors certainly offers an expedient means of evaluating tool life. It overcomes some of the objectionable features of standard tool-life tests in that a minimum of time and material are consumed. The authors should be congratulated for developing this clever test procedure.

However, as in most test methods, there are certain limitations to which an analysis of the results must be confined. The results of the authors' tests indicate only tool life and not surface finish or

²³ Metcut Research Associates, Cincinnati, Ohio.

energy consumption. Hence the test only measures one of the three main factors which determine the machinability of a material. A single machinability index is not adequate unless it can evaluate completely all three of these machining characteristics. As yet, to the writer's knowledge, no single index having this requirement has been published.

Perhaps the most important limitation of this test method is the fact that it does not analyze the machining characteristics of the materials, but only rates them on the basis of tool life. From the results of these tests one could not tell whether the low machinability index, resulting from cutting a particular steel, was due to the presence of hard abrasive inclusions in the material, or its capacity to work-harden during cutting, or the high coefficient of friction between the sliding chip and the tool.

There is another feature of this test method which should be considered while interpreting the results. The authors state: "The laboratory lathe test is based upon finding that steels of different machining quality cut at different rates, for constant surface speed, when a fixed lateral pressure is applied to the tool." It has been the writer's experience that tool life is influenced by the magnitude of the feed rate. In this constant-pressure lathe-test method, it seems that in reality a comparison of the various tool lives is made using different feeds on each test material. Should not this be taken into account when evaluating the test results?

AUTHORS' CLOSURE

The authors wish to thank all those who contributed their time and effort to the discussion of this test method. The number of discussions and their quality indicate the great interest in the problems of machinability and machinability testing. It is only through such interest that progress can be made in the complicated study of the machining of metals.

It is interesting to learn that Mr. Austin has obtained a correlation between simple drill-test ratings on cast iron and commercial tool-life measurements. A limited number of tests have indicated that the constant-pressure lathe test may be used in evaluating specimens of cast iron. More work, however, would have to be done to determine the tool material and tool shape for a long useful life, as well as the best testing conditions.

Dr. O. W. Boston points out that the constant-pressure method has been used successfully for sawing tests but with less success in drilling tests. We have found that drilling and sawing tests may give confusing results in some cases. For instance, the relative ratings of various steel samples can be altered by changing from a fine- to a coarse-tooth saw blade. Difficulties in sharpening drills identically and in chip removal from the hole may explain the disappointing results in drill tests. These difficulties are much less serious in lathe operations, using single-point tools.

Fig. 4 of the paper refers to the effect of tool preparation and use on the reproducibility of the feed resulting from a constant horizontal pressure. It does not refer to "V60" tool life. The data for the ground tool can be interpreted as showing that little, if any, wear occurred. The data show that the tool cut faster in tests 21 to 40 than it did in previous tests 8 to 19. The objection to the ground tools is based on their erratic performance rather than on wear or tool life.

Dr. Boston also mentions that the correlation between hardness and the constant-pressure machinability index is "not so good as might be expected." Some workers expect harder steels to cut with greater difficulty, others expect them to have better machinability. The range in hardness of the free-cutting steels under discussion was small and had less influence than other factors in which the steels differed. When the variations in hardness between samples are large, hardness does affect the feed resulting from a constant tool pressure.

We agree with Mr. Davis, who recognizes the human element as an important variable in shop-production tests. A number of tests have indicated that constant-pressure tests at 75 mean sfm seem to reflect the relative performance of various free-cutting steels at surface speeds comparable to those encountered in most machining practices.

All the points raised by Messrs. Enzian and Paliwoda appear pertinent. We agree that all test results must be interpreted with caution, and that is one reason the title restricts the test to free-cutting steels. There seems little doubt that one of the principal difficulties in studying machinability is that of obtaining samples representative of large lots. This sampling problem can be circumvented, but in our work we were not ordinarily interested in rating heats or ingots. Usually the problem consisted of testing small quantities of stock and determining the metallurgical cause for variations in machinability ratings.

The question raised by Enzian and Paliwoda about the effect of abrasive inclusions is an interesting one. As they point out, the importance of abrasive inclusions has not been determined satisfactorily by any investigators. The tests reported in this paper do not provide much information on this point, since very little, if any, tool wear occurs during the constant-pressure lathe tests. The fact that ratings by this method correlate well with laboratory tool-life tests and also in commercial operation, can be interpreted in either of two ways. Perhaps the effect of abrasive particles has been overestimated, or the steels we tested contained comparable quantities of such inclusions. This may be a good place to inject a note of caution, because most of our tests were made on hot-rolled Bessemer steels. We have few test data on other grades and can only speculate about materials and testing conditions other than those discussed in the paper.

Mr. Frye, in his discussion, hesitates to term the results in Fig. 9 of the paper as having a general agreement. As was indicated in the text, the points which deviate the farthest from the trend line are for samples representing lots that gave questionable results in commercial tests. Perhaps the discrepancies resulted from the difficulties in shop tests which Mr. Frye considers to involve "a complexity of factors . . . real, intangible, and psychological." We agree with him that the constant-pressure lathe test is a laboratory method, and that behavior in a turning test does not necessarily indicate the results to be expected in other machining operations. Two steels do not rate in the same order in all types of metal-cutting. The obvious differences in feeds, speeds, and tool pressures involved may account for disagreements. Frequently, the anomalies may be explained by the fact that two steps are involved in machining:

- 1 Cutting metal from the workpiece.
- 2 Removing the chips from the scene of the cutting operation.

Mr. Frye's assumption that the metallurgical factors and mechanical properties affecting machinability were studied is correct. However, the information is not germane to the present paper.

The data and remarks by T. M. Garvey are especially valuable because he is closely connected with recent machinability studies at Carnegie-Illinois Steel Corporation. It is heartening to know that he considers the test simple, useful, and reliable after employing the constant-pressure lathe for a considerable length of time in the Carnegie-Illinois laboratories.

It is interesting to note the similarity between the curves presented in Professor Henriksen's Fig. 10, and the curves for open-hearth steels in Fig. 7 of the paper. The authors feel that the hump shown in the figures may be the result of cutting temperature. In Professor Henriksen's experiments, the cutting temperature probably changes with the tool angle used.

Dr. Merchant's exposition of the reasons for the sensitivity of the test method and the necessity for caution in applying the

test to other grades of steel adds considerably to the publication. Dr. Merchant finds good theoretical reasons for considering the test method a sensitive one for comparing the machinability of free-cutting steels. The test method was of course developed and used only for studying materials of that kind. He may be right in his opinion that it would be less useful when applied to steels without additions to improve their machinability. However, a number of constant-pressure tests made on nonfree-cutting steels have disclosed significant differences. For instance, the feeds resulting from a constant horizontal tool pressure have been found to vary with grain size and microstructure when a steel is tested in different conditions of heat-treatment. Tests on a certain C1045 steel gave heavier feeds when the structure consisted of lamellar pearlite than when the specimens had been spheroidized. Consequently, we believe that the question of the suitability of the method for testing steels without special additives is still unsettled.

It was surprising to learn that Mr. Witteman's group is using a constant-pressure test for studying machinability. The fact that he obtains useful information by drilling at conventional speeds using a cutting fluid, conditions quite different than our experiments, indicates that the assumptions underlying the test are justified.

His data seem to confirm our experience that microstructure and composition affect the feed resulting from a constant tool pressure. Apparently the original hardness of the workpiece is not a reliable indication of machinability. These conclusions seem reasonable. The correlation between feed and increase in hardness of the chips shown by his tests on alloy steels is striking. The authors did not investigate this point.

It is probably true, referring to Mr. Zlatin's discussion, that no single test method provides all the information desired about the

cutting quality of a material. There is good reason to believe that a steel which performs well in one type of machining operation, such as lathe turning, may not be as satisfactory for other operations such as drilling. Therefore a laboratory test should simulate the critical operation for which the material is to be used.

The constant-pressure lathe test seems to give reliable indications of the performance of free-cutting steels in screw machines. As Mr. Zlatin points out, the laboratory ratings are in good agreement with commercial tool-life results. Contrary to his interpretation, the authors believe the constant-pressure test ratings do indicate energy consumption, but no experiments were made to check this point. As he points out, the test ratings do not provide information on the finish to be expected in conventional machining operations. This objection is common to most laboratory tests.

Although the reliability of the constant-pressure lathe test was evaluated by some comparisons with tool-life data, we disagree with Mr. Zlatin's opinion that the test gives ratings only "on the basis of tool life." Practically no tool wear occurs during the test. The feed at which a steel cuts in the test depends upon the energy required for work-hardening and shearing the chip, and in overcoming the friction between the chip and tool. The feed is probably an integrated indication of these factors for the cutting conditions employed, although the data do not permit separate determinations of the different characteristics.

The answer to Mr. Zlatin's last question is that the test results are evaluated on the basis of the different feeds. The tool lives are not determined for different feeds. The variation in feeds between materials results from a constant tool pressure under cutting conditions resulting in practically no tool wear.

Aerodynamic Heating and Convective Heat Transfer—Summary of Literature Survey¹

By H. A. JOHNSON² AND M. W. RUBESIN³

The analytical and experimental information appearing in the literature for convective heat transfer at high velocities is summarized. Particular emphasis is given to heat transfer for submerged flow over flat plates, wedges, cones, and cylinders with a single reference to flow in pipes. The evidence presented supports, as a fundamental heat-transfer relation, the modified Newton's law of cooling

$$q = hA[t_w - (t_o + ru_o^2/2gJc_p)]$$

where the term $ru_o^2/2gJc_p$ accounts for the influence of frictional dissipation and is readily evaluated when the recovery factor r is known. A second and very important result is the evidence given to show that the heat-transfer coefficient h , is independent of the magnitude of the frictional-dissipation term. Data are presented for the recovery factor and heat-transfer coefficient for high-speed subsonic- and supersonic-flow velocities. It should be noted, however, that heat transfer by radiation is not considered here as it would be treated in the usual manner for heat transfer without frictional dissipation.

NOMENCLATURE

The following nomenclature is used in the paper:

- a = speed of sound, fps
- A = surface area for heat transfer, sq ft
- c = specific heat, Btu/(lb)(deg F)
- c_p = specific heat at constant pressure, Btu/(lb)(deg F)
- C_{Dz} = local skin-friction drag coefficient, dimensionless
- D = diameter, ft
- g = gravitational constant (32.2), ft/sec²
- h = heat-transfer coefficient (unit thermal conductance), Btu/(sq ft)(deg F)
- h_x = local heat-transfer coefficient at location x , Btu/(sq ft)(deg F)
- h_m = mean heat-transfer coefficient, Btu/(sq ft)(deg F)
- J = mechanical equivalent of heat (778), ft-lb/Btu
- k = thermal conductivity, Btu/(sq ft)(hr)(deg F/ft)
- L = length, characteristic dimension, ft
- L_{tr} = length at transition from laminar to turbulent flow, ft
- M = Mach number, u/a , dimensionless
- m = exponent for potential flow over wedges, dimensionless
- N_{Nu} = Nusselt modulus, dimensionless
- N_{Nu_z} = local Nusselt modulus, dimensionless
- N_{Num} = mean Nusselt modulus, dimensionless
- n = viscosity-conductivity temperature exponent, dimensionless, i.e., $\mu/\mu_o = k/k_o = (T/T_o)^n$

- p, P = absolute pressure, psf
- P_o = pressure coefficient, dimensionless
- N_{Pr} = Prandtl modulus, dimensionless
- q = heat-transfer rate, Btu per hr
- r = local recovery factor, dimensionless
- r' = local-free stream recovery factor, dimensionless
- r_o = over-all recovery factor, dimensionless
- r_h = recovery factor based on velocity and temperature after shock, dimensionless
- N_{Re_s} = local Reynolds modulus, ux/ν , dimensionless
- N_{Re_L} = Reynolds modulus, uL/ν , dimensionless
- N_{Re_D} = Reynolds modulus, uD/ν , dimensionless
- R = ideal gas-law constant, ft-lb/(lb)(deg R)
- t = thermodynamic-state temperature of fluid, deg F
- t_{aw} = adiabatic-surface (wall) temperature, deg F
- t_o = free-stream-state temperature, deg F
- t_t = total or stagnation temperature, deg F
- t_1 = state temperature at edge of boundary layer, deg F
- t_w = surface (wall) temperature, deg F
- Δt_{ad} = total adiabatic stagnation-temperature rise, deg F
- T = absolute temperature, deg R
- T' = absolute temperature for evaluation of fluid properties, deg R
- u = local velocity, fps
- u_1 = local velocity at edge of boundary layer, fps
- u_o = free-stream velocity, fps
- w = specific weight, lb per cu ft
- x = distance along surface from leading edge, ft
- y = distance normal to surface, ft
- μ = absolute viscosity, lb-sec per sq ft
- ν = kinematic viscosity, $\mu g/w$, sq ft per sec
- ρ = mass density, lb-sec/ft⁴
- α = total wedge angle, radians
- γ = isentropic exponent, dimensionless

INTRODUCTION

Frictional heating, the heating produced by fluid-friction forces, has acquired increased importance with such developments as the gas turbine and high-speed flight. The need for evaluating heat-transfer behavior under these conditions is apparent when it is observed that aerodynamic heating (frictional heating in air) can cause a (adiabatic) surface temperature to rise 57 deg F above that of the ambient air for flight speeds of 600 mph and 1940 F for rocket speeds of 3600 mph (attained by the V-2). Realizing that an adiabatic (insulated) surface, one from which there is zero heat transfer when at thermal equilibrium, takes on a temperature in excess of the ambient fluid, the familiar concept that the rate of convective heat transfer is proportional to the difference between the surface and ambient fluid temperature obviously is absurd. Some connection between the rate of high-speed heat transfer and the temperature obtained by an insulated surface must exist.

Convective heat transfer like fluid (skin) friction is considered a boundary-layer phenomenon, and it is held (a), that heat passes to the fluid by molecular conduction through a laminar layer which is always present immediately on the surface; and (b), that for the moment the heat rate is proportional to the tem-

¹ Reference (1). Numbers in parentheses refer to Bibliography at end of paper.

² Associate Professor of Mechanical Engineering, University of California, Berkeley, Calif., on leave with the General Electric Company, Schenectady, N. Y. Mem. ASME.

³ Aeronautical Research Scientist, NACA Ames Laboratory, Moffett Field, Calif.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society. Paper No. 48–A-39.

perature difference between the surface and the ambient fluid (usual treatment). Thus the rate of convective heat transfer is expressed by

$$q = -kA \left(\frac{\partial t}{\partial y} \right)_{y=0} = hA(t_w - t_o) \dots [1]$$

From Equation [1] the heat-transfer coefficient may be defined by

$$h = \frac{-k(\partial t / \partial y)_{y=0}}{(t_w - t_o)} \dots [2]$$

However, for the purpose of correlating heat-transfer data in dimensionless form, Equation [2] is written in terms of the Nusselt modulus

$$N_{Nu} = \frac{hL}{k} = \frac{-\left(\frac{\partial t}{\partial y} \right)_{y=0}}{(t_w - t_o)/L} \dots [3]$$

Using this definition of the Nusselt modulus, it can be shown (2, 3) from the steady-flow energy of a fluid and for a given geometrical system that

$$N_{Nu} = f_1[N_{Re}, N_{Pr}, \mathbf{M}, \Delta t_{ad}/(t_w - t_o)] \dots [4]$$

Of the five dimensionless variables in Equation [4], only the first three, N_{Nu} , N_{Re} , and N_{Pr} normally appear for heat transfer because $\mathbf{M}$ and $\Delta t_{ad}/(t_w - t_o)$ are substantially zero for low-velocity flow. Consequently the Nusselt modulus and in turn the heat-transfer coefficient h , are independent of the temperature potential $(t_w - t_o)$, which is a necessary requirement in order that Equation [1] conveniently may represent the rate of heat transfer. At high velocities, however, the temperature factor, $\Delta t_{ad}/(t_w - t_o)$, in Equation [4] is finite, and N_{Nu} is dependent upon the temperature potential, thereby destroying the convenience of using Equation [1].

The reasoning that leads to the deduction of a heat-transfer equation in which the temperature potential and heat-transfer coefficient are independent of each other follows. Suppose that a thermally insulated plate is placed parallel to the direction of flow in a high-speed fluid stream. The fluid flowing over this plate is retarded as a result of the viscous forces which act in creating the flow boundary layer. The combined effect of damming, work done by the viscous forces, and conduction of heat through the fluid tend to create a temperature distribution in the thermal boundary layer, the shape of which is dependent on the relative magnitudes of the afore-mentioned processes. For instance, for laminar flow, the resulting temperature distribution is that of Pohlhausen (4), a typical curve of which is indicated in Fig. 1.

Since the plate is thermally insulated, the rate of heat flow

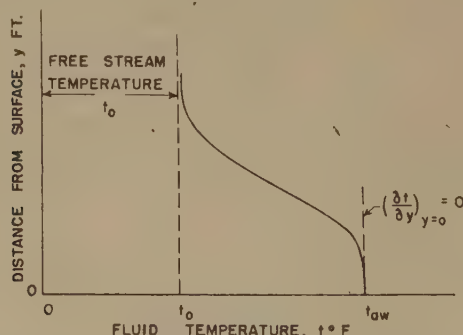


FIG. 1 TEMPERATURE DISTRIBUTION IN BOUNDARY LAYER OF A THERMALLY INSULATED PLATE AT HIGH FLUID VELOCITIES

and consequently the temperature gradient with respect to the normal to the surface must be zero. The temperature which the plate (and the fluid in contact with the plate) assumes is designated as the "adiabatic surface temperature,"⁴ (t_{aw}).

Suppose now that the plate is heated so that its surface temperature t_w is greater than t_{aw} . The temperature gradient with respect to the normal is no longer zero as heat is being transferred to the fluid, see Fig. 2. The converse is true if t_w were less than

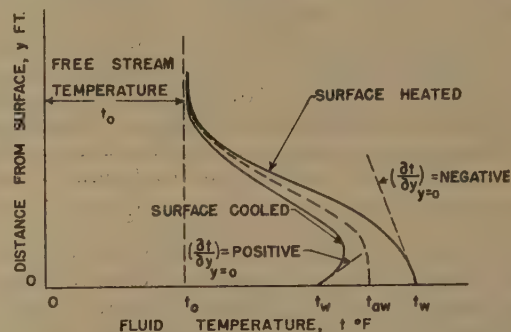


FIG. 2 TEMPERATURE DISTRIBUTION IN BOUNDARY LAYER OF HEATED AND COOLED PLATES AT HIGH FLUID VELOCITIES

t_{aw} . It is logical then to postulate that the convective heat transfer with frictional heating can be expressed as

$$q = \bar{h}A(t_w - t_{aw}) \dots [5]$$

where $\bar{h}$ = heat-transfer coefficient with frictional heating, Btu/(sq ft)(hr) (deg F). In order to establish the worth of Equation [5] it is necessary to investigate the relationship between the adiabatic surface temperature t_{aw} and the coefficient $\bar{h}$. Considering t_{aw} first, it can be seen from Equations [3] and [4] that when the plate is insulated, i.e., $t_w = t_{aw}$ and $(\partial t / \partial y)_{y=0} = 0$, Equation [4] reduces to

$$\frac{t_{aw} - t_o}{\Delta t_{ad}} = \frac{t_{aw} - t_o}{t_i - t_o} = \frac{t_{aw} - t_o}{u_o^2 / 2gJc_p} = f_2(N_{Re}, N_{Pr}, \mathbf{M}) \dots [6]$$

or defining the temperature difference ratio as the "recovery factor," r

$$r = \frac{t_{aw} - t_o}{u_o^2 / 2gJc_p} = f_2(N_{Re}, N_{Pr}, \mathbf{M}) \dots [7]$$

A knowledge of the recovery factor r is therefore prerequisite to a determination of the modified temperature potential $(t_w - t_{aw})$ and is treated independently of the heat transfer.

The heat-transfer coefficient with friction $\bar{h}$ has been investigated by theoretical analyses for laminar flow (5), and turbulent flow (6, 7) over flat plates, which show conclusively that it is both independent of the modified temperature potential and is identical with that for low speeds as evaluated for the same Reynolds and Prandtl modulus, i.e., $\bar{h} = h$. Some support for these statements has been obtained experimentally for high-speed subsonic air flow in pipes (8), gas-turbine-blade cooling (3), and supersonic flow over cones (26). Thus the high-speed convective heat-transfer equations can be summarized as

$$q = hA(t_w - t_{aw}) = hA \left[t_w - \left(t_o + r \frac{u_o^2}{2gJc_p} \right) \right] \dots [8]$$

$$N_{Nu} = \frac{hL}{k} = f_1(N_{Re}, N_{Pr}, \mathbf{M}) \dots [9]$$

⁴ The temperature defined here as the adiabatic surface temperature is referred to in the literature as the "inherent, natural, unheated or insulated surface, braking plate thermometer, and boundary-layer temperature."

$$r = \frac{t_{aw} - t_o}{\Delta t_{ad}} = \frac{t_{aw} - t_o}{u_o^2 / 2gJc_p} = \frac{t_{aw} - t_o}{t_i - t_o} = f_2(N_{Re}, N_{Pr}, M) \quad [10]$$

It now remains to evaluate Equations [9] and [10] for various-shaped bodies and several types of flow.

RECOVERY FACTORS

The recovery factor is defined in terms of the adiabatic surface temperature by Equation [10]. It should be noted, however, that there are occasions requiring the use of one or more of three different recovery factors, dependent upon the shape of the body. The most common, referred to simply as the recovery factor, is the "local" recovery factor, based upon local values of fluid velocity, temperature, i.e., local values at the edge of the flow boundary layer, and on local values of the adiabatic surface temperature. If the undisturbed free-stream fluid velocity, temperature, and the mean surface temperature are used, the result is called the "over-all" recovery factor. A third type, based on the undisturbed fluid velocity, temperature, and the local adiabatic surface temperature, is termed the "local-free stream" recovery factor. In the following, the extent of existing knowledge on recovery factors is presented for various shaped bodies and several types of flow. Emphasis is given to submerged flow and, unless otherwise specified, to the local recovery factor. Considerable attention is given to flow over the flat plate because the results are applicable to supersonic flow over wedges and cones for which, like the flat plate, the static pressure distribution is constant over the flow surface.

Flat Plate. For laminar flow along the length of a flat plate the recovery factor has been evaluated analytically by several investigators (see references 4 and 9 to 14, inclusive). A review of these references (15), stressing those solutions which consider variable fluid properties and including numerical results for selected values of Prandtl modulus from 0.72 to 1.2, Mach number from 0 to 10, and n , the temperature exponent for viscosity and thermal-conductivity variation from 0.5 to 1.25, shows that the recovery factor is independent of Reynolds modulus and Mach number (see Equation [10]) and is well represented by the square root of the Prandtl modulus

$$r = \frac{t_{aw} - t_o}{u_o^2 / 2gJc_p} = \sqrt{N_{Pr}} \quad [11]$$

Because the Prandtl modulus was maintained as an independent parameter in the various solutions, no means of determining the reference temperature at which to evaluate N_{Pr} is possible from these solutions. At subsonic velocities Equation [11] has been verified experimentally by Hilton (16), who measured the adiabatic surface temperature for air flow over thin plates, and by Eckert and Weise (17) for flat plates and axial flow along cylindrical probes (see Fig. 3, for the apparent laminar-flow range of Reynolds modulus, i.e., less than 5×10^5).

At supersonic velocities, Equation [11] has been substantiated somewhat by the data of Eber (26) who performed experiments for flow in the transition region over cones. It was possible to extend these cone data to flat plates (0 deg cone) because the method of reducing data, using the potential flow velocity and temperature behind the attached shock wave, was such that the local recovery factor is shown to be independent of cone angle (see Fig. 8). The data seem questionable, however, for although the Reynolds moduli indicate transition on the cone surface (verified by heat-transfer data), the results show a constant recovery factor along the surface that is the same as for laminar flow over flat plates.

For the turbulent boundary layer with subsonic flow over a flat plate, Hilton (16) noticed that the adiabatic temperatures

in the rear portions of his plates were higher than those given by Equation [11], and ascribed them to the turbulent boundary layer. Similar results obtained by Eckert and Weise (17), Fig. 3, show the recovery factor increasing with Reynolds modulus

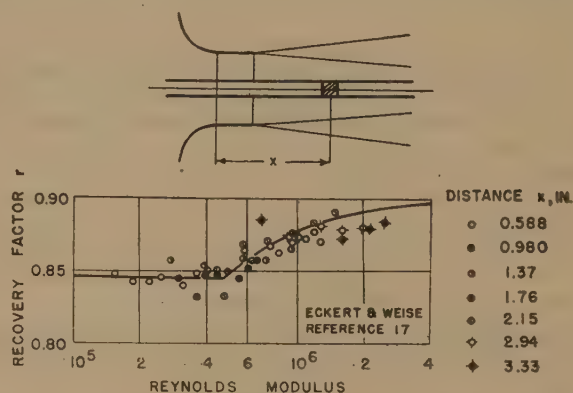


FIG. 3 VARIATION OF RECOVERY FACTOR WITH REYNOLDS MODULUS FOR AXIAL PROBE IN AIR AT SUBSONIC VELOCITIES

and approaching a limiting value of 0.89. However, experimental values for supersonic velocities which are about 7 per cent higher are reported by Kraus (18), suggesting a Mach-number effect as well as Reynolds and Prandtl moduli dependency.

While there are no turbulent boundary-layer analytical solutions comparable to those for laminar flow, some analyses have been made including frictional dissipation. Ackermann (19) obtained an expression for gases at large Reynolds modulus which, with Prandtl modulus between 0.5 and 2, is approximated by

$$r = N_{Pr}^{1/2} \quad [12]$$

a result which agrees with the limiting value of 0.89 shown in Fig. 3. Seban (7), postulating that the turbulent boundary layer of a flat plate could be represented by Nikuradse's velocity-distribution data for turbulent flow in pipes, evaluated the heat transfer with dissipation in the laminar sublayer, buffer layer, and turbulent layer and obtained

$$r = 1 - (4.71 - 4.11 B - 0.601 N_{Pr}) N_{Re}^{-0.2} \quad [13]$$

where

$$B = \frac{N_{Pr} (5 N_{Pr} - 7)}{2 (5 N_{Pr} + 1)}$$

or for air with $N_{Pr} = 0.72$

$$r_{0.72} = 1 - 0.869 N_{Re}^{-0.2} \quad [14]$$

By neglecting the buffer layer, the solution is the same as that of Shirokow (20), namely

$$r = 1 - 4.55 (1 - N_{Pr}) N_{Re}^{-0.2} \quad [15]$$

or for air with $N_{Pr} = 0.72$

$$r_{0.72} = 1 - 1.25 N_{Re}^{-0.2} \quad [16]$$

These results are compared in Fig. 4 and, except for Equation [12], it is observed that the predicted values for the turbulent boundary layer lie substantially above the experimental results from Fig. 3, and also approach unity rather than $N_{Pr}^{1/2}$. As mentioned, values from 0.91 to 0.98 are reported for supersonic velocities and attributed to a Mach effect, which would not be indicated by the foregoing incompressible flow analyses.

Wedge. Analytical solutions are available for laminar, incom-

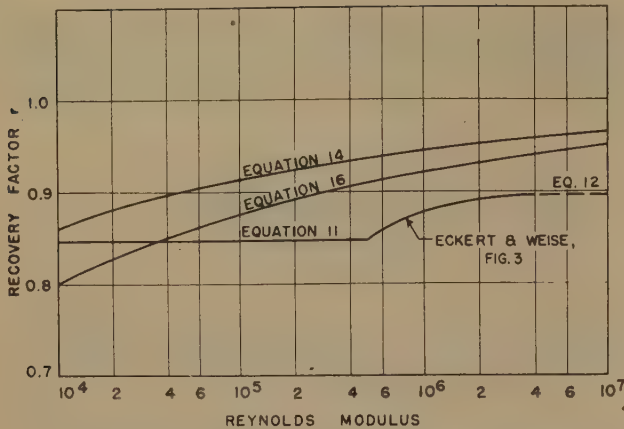


FIG. 4 COMPARISON OF FLAT-PLATE RECOVERY FACTORS FROM TURBULENT BOUNDARY-LAYER ANALYSES FOR $Pr = 0.72$

pressible, subsonic flow over wedges at a zero angle of attack. The local velocity at the outer edge of the flow boundary layer has been found from potential flow to be

$$u_1/u_o = (x/L)^m \dots \dots \dots [17]$$

where

- u_1 = local velocity at edge of flow boundary layer, fps
- u_o = free-stream fluid velocity, fps
- x = distance along surface from leading edge, ft
- L = distance along surface from the leading edge where local velocity substantially equals free-stream velocity, ft
- $m = \alpha/(2\pi - \alpha)$
- α = total wedge angle, radians (note $\alpha/\pi = \beta$)

Falkner and Skan (21), and Hartree (22) integrated the incompressible-flow equations to determine the velocity distributions in the boundary layer, and Fig. 5 shows the calculated ve-

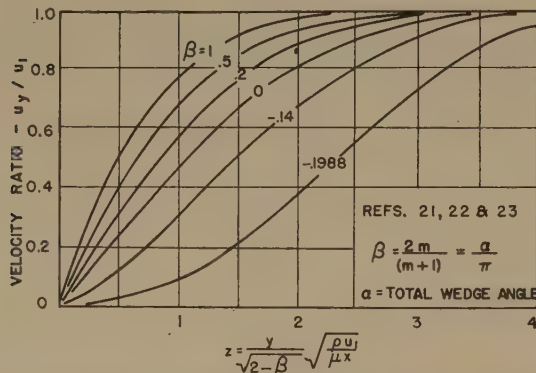


FIG. 5 VELOCITY DISTRIBUTION IN BOUNDARY LAYER FOR FLOW OVER SURFACE OF AN INFINITE WEDGE

locity profiles for various wedge angles. Employing these velocity distributions in the energy equations and postulating all material properties constant, including density, Eckert and Drewitz (23) obtained the temperature profiles shown in Fig. 6 for thermally insulated wedges. (Note that T_1 is, like u_1 , a local value at the outer edge of the boundary layer.) It should be noted that in the calculations of Eckert and Drewitz it is presupposed that the temperature equalization along the wedge surface is low.

Fig. 6 indicates that the local recovery factor at the surface of a flat plate, i.e., temperature difference ratio at $y = 0$ for a wedge of zero apex angle identified by $\beta = 0$, is 0.84 or $N_{Pr}^{1/2}$, and that

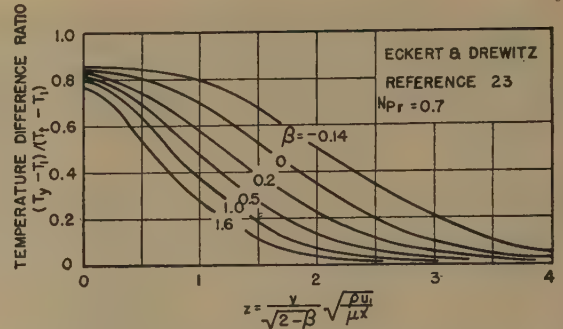


FIG. 6 TEMPERATURE DISTRIBUTION IN BOUNDARY LAYER FOR FLOW OVER SURFACE OF AN INSULATED WEDGE

for the other extreme of a plate normal to the airstream, i.e., $\beta = 1$, r is 0.78. Similar results, $r = 0.84$ for $\beta = 0$, and $r = 0.72$ for $\beta = 1$, were obtained by Tifford (24). The reason for the discrepancy between results for $\beta = 1$ is not clear but is probably due to differences in precision of the numerical calculation upon which both results are based. Noting in Fig. 6 that the local recovery factor is independent of the distance x and is within either 5 or 15 per cent of $N_{Pr}^{1/2}$ for all wedge angles, use of the flat-plate recovery factor is justified for the usual wedge angles (60 deg or less). Experimental verification of these results is not available at present.

Solutions of supersonic flow over an infinite wedge (similarly for a cone) at zero angle of attack indicate that the velocity, state temperature, and pressure along the outer edge of the boundary layer are constant with distance along the surface from the apex, when the Mach number is sufficiently large to insure an attached shock wave at the apex. These conditions are identical with those for a flat plate and therefore indicate use of flat-plate recovery factors, see Fig. 8.

Cylinder, Normal to Flow Direction. Experiments have been performed to determine the local-free stream recovery factors around the circumference of right-circular cylinders placed normal to the fluid stream. The most elegant of these experiments at subsonic free-stream velocities, that of Eckert and Weise (17), determined the change in adiabatic surface temperatures along the periphery at intervals of 2 to 3 deg. The data are presented (see Fig. 7), both in terms of the local-free stream recovery factor

$$r' = \frac{t_{aw} - t_o}{t_t - t_o} = \frac{t_{aw} - t_o}{u_o^2 / 2gJc_p} \dots \dots \dots [18]$$

and the local recovery factor

$$r = \frac{t_{aw} - t_1}{t_t - t_1} = \frac{t_{aw} - t_1}{u_1^2 / 2gJc_p} \dots \dots \dots [19]$$

where subscript o refers to the free-stream conditions and subscript 1 to local values⁵ at the edge of the boundary for the same angular position as for t_{aw} .

It is seen from Fig. 7 that the local-free stream recovery factor, r' , is unity at the leading edge of the cylinder, indicating that the true stagnation temperature is attained there. It then drops continuously to a value near zero at the rear stagnation point with a slight inflection at the point of flow separation where the pressure starts to rise. The values obtained in the rear of the cylinder are inconsistent and are not correlated systematically

⁵ The local values are determined by the relations

$P_1 = P_o - \frac{1}{2} P_o u_o^2 / P_o$, $T_1 = T_o (P_1 / P_o)^{\frac{\gamma-1}{\gamma}}$, $u_1^2 / 2gJc_p = t_t - t_1$ where γ is the isentropic exponent.

with Reynolds or Mach moduli. Similar conclusions were noted by Hilton (16) as a result of his measurements.

An important conclusion is drawn from the local recovery factor r (dotted line) which is constant with the peripheral angle and approximately equal to 0.84 ($r = \sqrt{N_{Pr}}$). Since experiment has shown that laminar flow exists on the leading edge of a cylinder up to the point of separation (between 70 and 80 deg meas-

ured from the upstream stagnation point), it is significant that this local recovery factor r , is identical with that for laminar flow on a flat plate. It is concluded then that the local recovery factor r , along the leading edge of a cylinder, defined by Equation [19], is the same as that for laminar flow on a flat plate up to the point of separation, or, if transition occurs, up to the start of the turbulent boundary layer. The case of transition to turbulent flow ahead of separation, along with the state of flow in the region behind the separation point, has not been clarified as yet.

For supersonic speeds, the flow conditions over the surface of a cylinder normal to the flow direction have not been clarified as yet and the local recovery factor r , described earlier for the leading edge of the cylinder cannot be obtained. There are data, however, for the over-all and local free stream recovery factors for both subsonic and supersonic speeds.

The over-all recovery factor r_o , for subsonic speeds is reported for small wires (diameters 0.003 to 1.02 in.) by Hottel and Kalitinsky (25), for small cylinders (0.08 to 1.2 in.) by Eckert and Weise (2), and for large cylinders (1 to 2.8 in.) by Cavigli and Swanson (27). The results show over-all recovery factors which range from 0.55 to 0.7 for Mach number less than 0.5, and from 0.55 to 0.82 for Mach number between 0.5 and 1.0, with no systematic method for correlating these data. If, however, a single diameter of 0.5 mm is considered, consistent data are given by Eckert and Weise (2), and Eber (26) for a range of free-stream Mach numbers from 0.4 to 3.4, Fig. 9. The dip in the curve

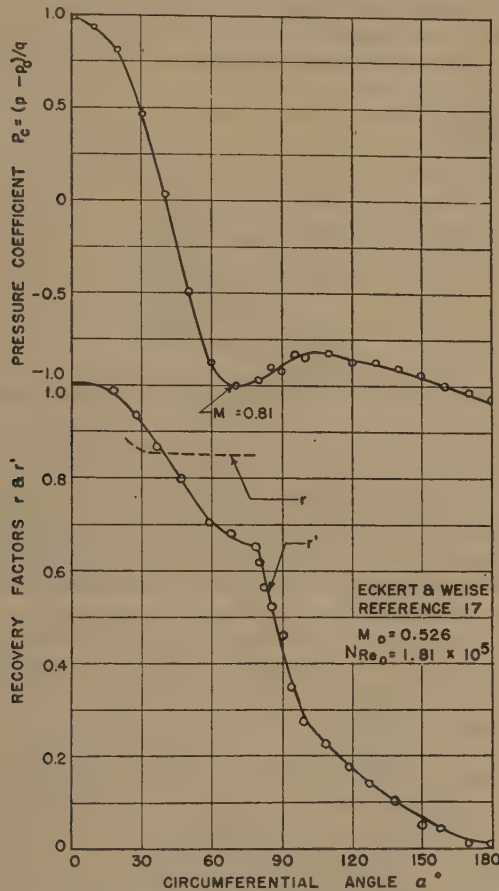


FIG. 7 TYPICAL CIRCUMFERENTIAL PRESSURE AND RECOVERY-FACTOR DISTRIBUTION FOR AIR FLOW ACROSS A 2-CM-DIAM CYLINDER

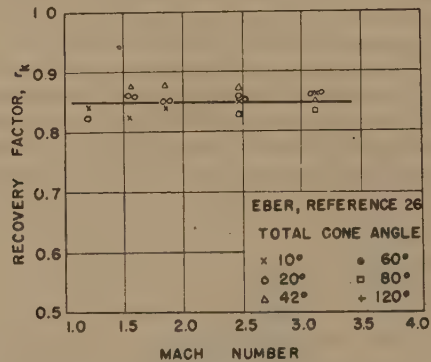


FIG. 8 RECOVERY FACTORS FOR CONES, BASED UPON CONDITIONS BEHIND AN ATTACHED SHOCK WAVE

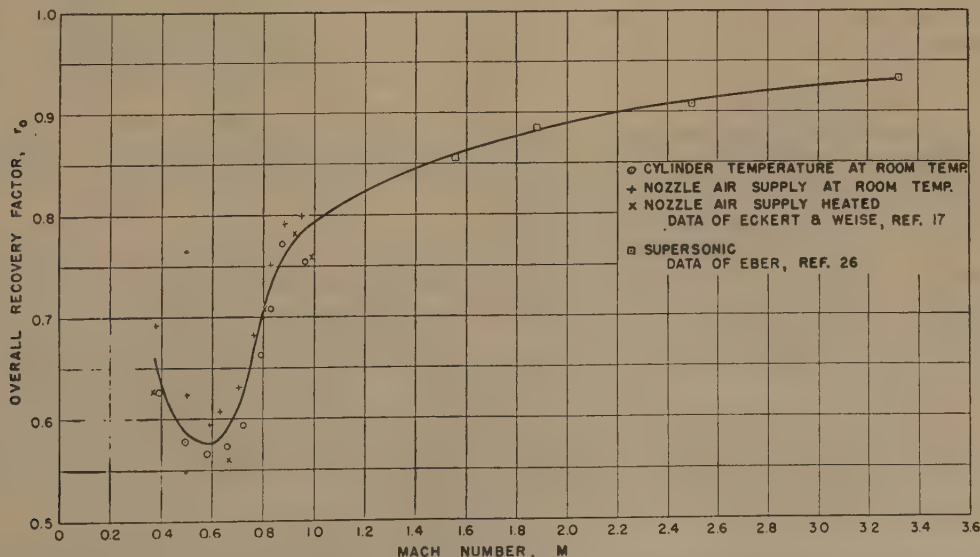


FIG. 9 OVER-ALL RECOVERY FACTOR FOR A 0.5-MM-DIAM CYLINDER NORMAL TO DIRECTION OF AIR FLOW

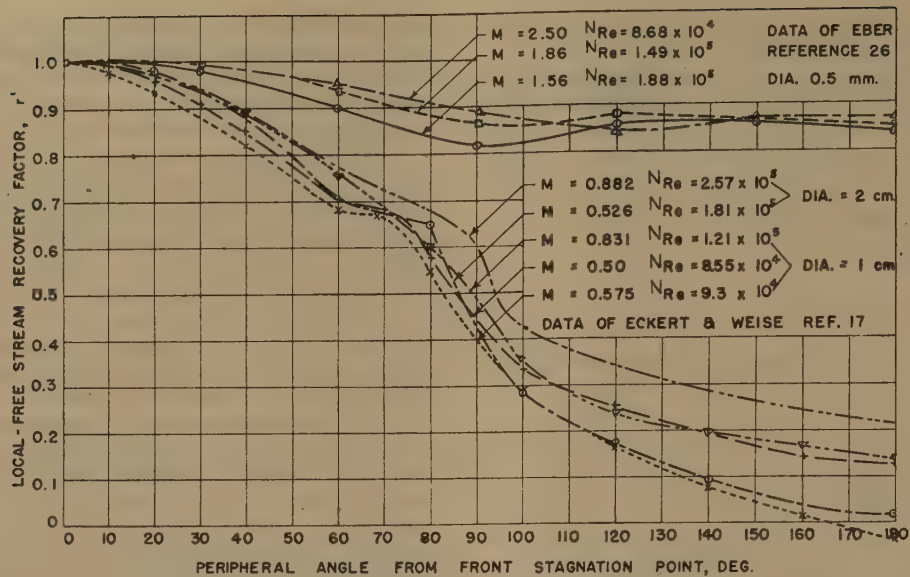


FIG. 10 CIRCUMFERENTIAL VARIATION OF LOCAL FREE STREAM RECOVERY FACTOR FOR AIR FLOW NORMAL TO CIRCULAR CYLINDERS

at Mach number near 0.5 is attributed to the effect of a local sonic Mach number at some point on the cylinder periphery, an effect which does not occur for free-stream Mach numbers less than 0.5 or greater than 1.0.

Local-free stream recovery factors r' , for subsonic and supersonic air flow normal to cylinders, shown in Fig. 10, indicate substantial increases on the rear portion of the cylinder as the free-stream Mach number exceeds 0.6, and so accounts for the rising over-all recovery factors given in Fig. 9, as the Mach number increases above 0.6. Also, from Fig. 10 it is observed that:

- The adiabatic stagnation temperature (recovery factor = 1) is attained at the front stagnation point for subsonic and supersonic speeds.
- The local-free stream recovery factor at supersonic speeds does not decrease as greatly with distance around the circumference as it does for subsonic speeds. Also, the value at the rear stagnation point increases from about 0.1 to 0.87, as the Mach number changes from 0.5 to 2.5.
- The inflection point, due to separation of flow, for subsonic speeds is delayed for the larger of the two cylinders, and no sharp inflection is noticed for supersonic speeds; however, the latter may have been masked by the 30-deg test interval used by Eber.

HEAT TRANSMISSION

It was pointed out earlier that the analytical studies and some experimental work which have been done for heat transfer indicate that the heat-transfer coefficients with and without frictional heating are identical, provided the temperature potential in Newton's law of cooling is defined as the difference between the surface temperature and the so-called adiabatic surface temperature (Equation [5]). Under these circumstances, the following available information for correlating and predicting heat-transfer coefficients without frictional heating is of equal importance to that which refers specifically to heat transfer at high speeds:

Flat Plate. The heat-transfer characteristics of the laminar boundary layer are treated in detail in reference (15). In general, the results for local heat-transfer coefficient can be expressed by the following dimensionless equation

$$N_{Nu} = \frac{(C_{Dx} \sqrt{N_{Re}})}{2} N_{Re}^{1/2} N_{Pr}^{1/3} \dots \dots \dots [20]$$

where the subscript x refers both to a characteristic length x , measured along the length of the plate from the leading edge, and to local values of all quantities represented in the equation. All of the properties, including density in the defining equation of local drag, are based on the same temperature, not necessarily the free-stream temperature (usually specified for the drag coefficient).

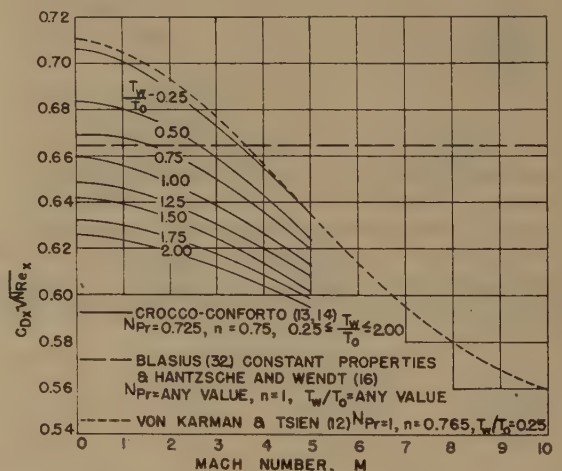


FIG. 11 FLAT-PLATE DRAG COEFFICIENT FOR LAMINAR FLOW OF A GAS WITH HEAT TRANSFER FOR PROPERTIES EVALUATED AT FREE STREAM TEMPERATURE

The results of the cited references are plotted in Fig. 11 in terms of the bracketed product ($C_{Dx} \sqrt{N_{Re}}$) which has a constant value of 0.664 for the Blasius solution. It is seen that this quantity and consequently the Nusselt modulus in Equation [20]⁶ is dependent upon Mach number M , the wall or plate temperature T_w , and the viscosity-conductivity temperature exponent n . By a cross-plot of Crocco's results, it was found that the effects of the parameters appearing in Fig. 11 were substantially eliminated if all properties are evaluated at a fluid temperature T' defined by

⁶ By definition, if values of $C_{Dx} \sqrt{N_{Re}}$ are taken from Fig. 11, all properties in Equation [20] are evaluated at the free-stream temperature.

$$\frac{T'}{T_0} = 1 + 0.032 M^2 + 0.58 \left(\frac{T_w}{T_0} - 1 \right) \dots [21]$$

The small variation in the quantity $C_{Dx} \sqrt{N_{Re,x}}$, as based upon properties evaluated at the temperature defined in Equation [21] is illustrated in Fig. 12. Thus for laminar flow of an ideal gas, Equation [20] reduces to one with a constant coefficient (0.664), replacing the quantity $C_{Dx} \sqrt{N_{Re,x}}$, making it identical to that obtained by Pohlhausen for constant properties, provided the fluid properties are evaluated at the temperature T' , defined by Equation [21].

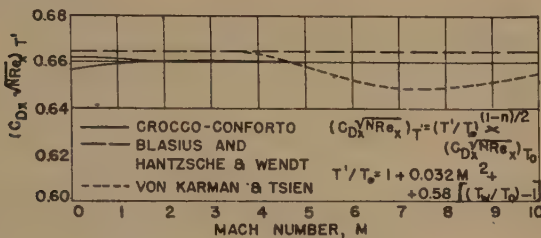


FIG. 12 FLAT-PLATE DRAG COEFFICIENT FOR LAMINAR FLOW OF A GAS WITH HEAT TRANSFER FOR PROPERTIES EVALUATED AT TEMPERATURE T'

Experimental verification of Equation [20] without frictional heating, is given by the data of Fage and Falkner (28), and Jakob and Dow (29). While the results of Fage and Falkner are about 13 per cent high, attributed to the nonuniform temperature of the plates used by the experimenters, the constant-temperature cylindrical-probe results of Jakob and Dow agree within the accuracy of their experiments.

For turbulent flow over a flat plate, the local heat-transfer coefficient without frictional heating is represented by the equation

$$N_{Nux} = 0.0296 N_{Re,x}^{0.8} N_{Pr}^{1/3} \text{ (turbulent flow)} \dots [22]$$

This equation results from frictional drag for turbulent flow on a flat plate, as correlated by Colburn (30), using the momentum-heat transfer analogy. Latzko (31) obtained the same result analytically (for Prandtl modulus at unity), by applying the seventh-power-law distribution of temperature and velocity, based upon the measurements of Elias (32). Equation [22] was also verified by the data of Jürges (33), and Jakob and Dow (29), which deviate from it by plus 6 per cent and minus 14 per cent, respectively.

With frictional heating and turbulent flow, analyses by

Crocco (6), Shirokow (20), and Seban (7), based on the momentum-heat transfer analogy and the modified temperature potential, indicate that the heat-transfer coefficient without frictional heating applies. In fact, the heat-transfer coefficients obtained by Shirokow and Seban are identical with those given by Prandtl and von Kármán, respectively, where frictional heating was negligible. A comparison of these results is given in Equation [23] and Fig. 13.

$$\frac{N_{Nux}}{N_{Re}^{0.8} N_{Pr}^{1/3}} = 0.0296 \text{ (Colburn)} = \frac{0.0296 N_{Pr}^{2/3}}{1 + 2.13 N_{Re}^{-0.1} (N_{Pr} - 1)} \text{ (Prandtl)} \dots [23]$$

$$= \frac{0.0296 N_{Pr}^{2/3}}{1 + 0.775 N_{Re}^{-0.1} [N_{Pr} + \ln(1 + 5 N_{Pr})] - 2.17} \text{ (von Kármán)}$$

Considering that the results shown in Fig. 13 do not deviate excessively for Prandtl's modulus near 1, Equation [22] is recommended for its simplicity. Questions as to Mach-number effect (see Equation [9]), and the temperature at which to evaluate the fluid properties cannot be answered by the cited analyses. A temperature equal to the surface temperature is suggested, considering that the laminar sublayer in turbulent flow may be controlling.

The mean heat-transfer coefficient for a plate of length L is considered as follows: Usually laminar flow exists on the forward portion of the plate which "transits" to turbulent flow when the Reynolds modulus for transition N_{Retr} is exceeded. If it is postulated that the local heat-transfer coefficient for the turbulent portion of the plate is the same at a given value of x as it would be if turbulent flow had started at the leading edge, i.e., Equation [22] is valid, then the average heat-transfer coefficient for a plate of length L will be

$$h_m = \frac{1}{L} \left[\int_0^{L_{tr}} h_{x, \text{lam}} dx + \int_{L_{tr}}^L h_{x, \text{turb}} dx \right] \text{ (transition flow)} \dots [24]$$

where L_{tr} is the transition length or distance along the plate to the point $N_{Retr} = N_{Re, \text{tr}}$.

Substituting Equations [20] (for constant property case) and [22], Equation [24] becomes

$$N_{Nux} = 0.037 (N_{ReL}^{0.8} - N_{Retr}^{0.8} + 18.0 N_{Retr}^{0.5}) N_{Pr}^{1/3} \text{ (transition flow)} \dots [25]$$

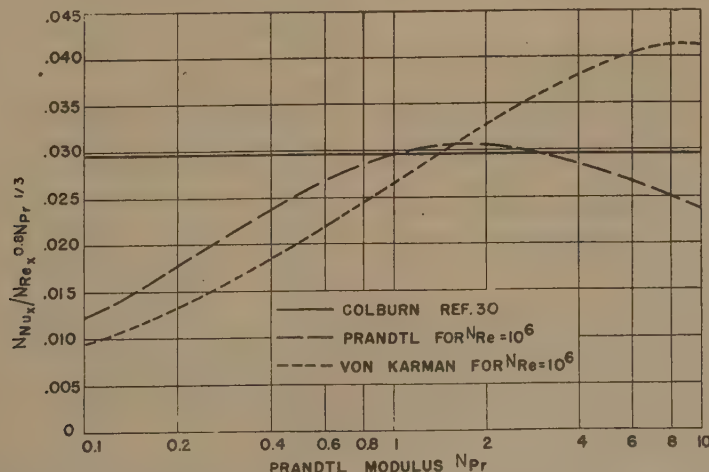


FIG. 13 COMPARISON OF TURBULENT BOUNDARY-LAYER HEAT-TRANSFER ANALYSES

or with N_{ReL} taken at 5×10^5 , a value often reported for heat-transfer data

$$N_{Num} = 0.037 (N_{Re}^{0.8} - 23,100) N_{Pr}^{1/3} \text{ (transition flow) } \dots [26]$$

Note that for laminar flow only

$$N_{Num} = 0.664 (N_{ReL})^{1/2} (N_{Pr})^{1/3} \dots [27]$$

and for turbulent flow only

$$N_{Num} = 0.037 (N_{ReL})^{0.8} (N_{Pr})^{1/3} \dots [28]$$

Wedge. In the discussion of recovery factors for supersonic flow over wedges it was pointed out that potential flow over the wedge surface for an attached shock wave is the same as for a flat plate. Thus, as for the recovery factor, the potential flow behind the oblique attached shock is determined, and the flat-plate heat-transfer coefficients are applied.

On the other hand, the heat-transfer coefficients for subsonic flow are of interest because it has been recommended (34) that the local velocity at the outer edge of the boundary layer be used in the Reynolds modulus of the flat-plate equation so that those equations will apply approximately to surfaces of large radii of curvature. Also Eckert (35) indicates that the solution for wedges may be so applied as to permit predictions for other shaped bodies. For subsonic flow then, the heat transmission from a wedge in a laminar-fluid stream at zero angle of attack has been determined analytically by Eckert and Drewitz (23), and Tifford (24). In these calculations it was postulated that the fluid properties and the surface temperature are constant; the results are

$$N_{Nuz} = \frac{\sqrt{(m+1)/2}}{\int_0^\infty \exp\left(-N_{Pr} \int_0^{\bar{n}} \bar{f} d\bar{n}\right) d\bar{n}} (N_{ReL})^{1/2} = C_w (N_{ReL})^{1/2} \dots [29]$$

where

$$N_{Nuz} = h_x x / k, N_{ReL} = u_1 x / \nu$$

$\bar{f}$ = nondimensional function, representing velocity distribution in boundary layer

$\bar{n}$ = nondimensional function, correlating local Reynolds modulus and geometry of system

$m = \alpha/(2\pi - \alpha)$, where α is total apex angle of wedge in radians

C_w = multiplier, defined by Equation [29], which is a function of Prandtl modulus and wedge apex angle (see Fig. 14)

To investigate the recommendation that the flat-plate equations, based upon the local potential-flow conditions at the edge of the boundary layer, be used for wedges, the multiplier C_w in Equation [29] is plotted for Prandtl modulus 0.7 (air) in Fig. 14. Since this multiplier for a flat plate, i.e., $C_w = 0.294$ from Equation [20], is only 10 per cent less than the value in Fig. 14 for a 40-deg wedge, the suggested approximation is supported for laminar flow over wedges of less than 40-deg apex angle. As there are no similar analyses for the turbulent boundary layers, the method is arbitrarily extended for turbulent flow.

Cones. Since the flow boundary conditions of a cone in supersonic flow are identical with those for flow over a flat plate, Hantzsche and Wendt (36) showed that the laminar-flow flat-plate heat-transfer-coefficient equations apply when modified by a multiplying factor determined by the geometry of the flow. The factors are $\sqrt{3}$ and $(2/3)\sqrt{3}$ for the local and mean heat-transfer coefficients, respectively, i.e., from Equations [20] and [27]

$$N_{Nuz} = 0.575 N_{Re}^{1/2} N_{Pr}^{1/3} \dots [30]$$

$$N_{Num} = 0.767 N_{ReL}^{1/2} N_{Pr}^{1/3} \dots [31]$$

Slight verification of these equations has been obtained experimentally in supersonic air flow. One set of experiments was

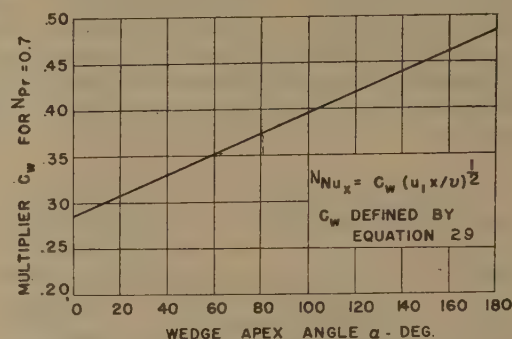


FIG. 14 REYNOLDS-MODULUS MULTIPLIER FOR SUBSONIC-LAMINAR-FLOW HEAT TRANSFER FROM WEDGES

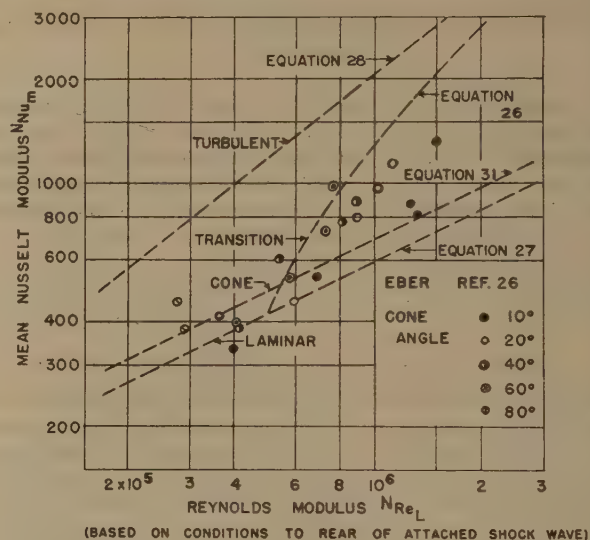


FIG. 15 SUPersonic-FLOW HEAT-TRANSFER DATA FOR CONES COMPARED WITH FLAT-PLATE EQUATIONS

conducted by Eber (26), who performed short-duration tests on small cones in a blowdown wind tunnel. The data are correlated as shown in Fig. 15, based on flow conditions over the cone surface aft of the shock wave with viscosity arbitrarily evaluated by Eber at the adiabatic surface temperature. In Fig. 15 it appears that these data confirm the application of the flat-plate equations in that the results are independent of cone angle and are also well represented by predictions based on those equations. It should be noted that the agreement shown in Fig. 15 is cited as evidence that there was transition flow on the cone surface. Curves in the turbulent and transition region, based on cone equations, could not be presented as the geometry factor relating flat plates and cones is not known at present for the turbulent boundary layer. Another set of experiments by Fischer and Norris (37) indicate that flat-plate equations apply approximately to the conical tip and side plates on a V-2 missile in flight when the fluid properties are evaluated at the temperature T' of Equation [21].

SUMMARY

Analytical and experimental investigations of convective heat

transfer from bodies in submerged flow at high speeds, where frictional heating is important, have so far supported the modified Newton's law of cooling (Equation [8]) as a fundamental heat-transfer relation. These investigations also indicate that the heat-transfer coefficients with the modified temperature potential and frictional heating are identical to those without frictional heating. Thus the procedures differ from that normally followed for heat-transfer calculations only in the evaluation of the adiabatic surface temperature which replaces the state temperature of the fluid.

The recovery factor (Equation [10]), which by definition determines the adiabatic surface temperature, is well represented by $\sqrt{N_{Pr}}$ for laminar flow over various-shaped bodies, even for large Mach number, provided the temperature equalization in the surface is low. The recovery factor for transition and turbulent flow is not established but apparently depends on Mach number and is greater than that for laminar flow, exceeding the suggested value of $N_{Pr}^{1/2}$.

Heat-transfer measurements with frictional heating are at present limited to one set of experiments for supersonic flow over cones and a single V-2 flight test, the results of which support the extrapolation of flat-plate heat-transfer coefficients for that purpose. The use of flat-plate heat-transfer equations is indicated for wedges and cones at supersonic velocities when the shock wave is attached. For other shapes of large radius of curvature and for high-speed subsonic flow over wedges and cones, estimates of the heat transfer might be made based on local conditions at the edge of the flow boundary layer applied to the available analytical solution for wedges (Fig. 14), or again to the flat plate.

The temperature at which to evaluate fluid properties, as determined from the analytical solutions for laminar flow of gases over flat plates, is given by Equation [21]. For turbulent flow, this reference temperature is much in doubt and will be determined eventually only by test; however, until such data are available the surface temperature is believed to be more suitable than the free-stream temperature.

ACKNOWLEDGMENT

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Supersonic Convective Heat-Transfer Correlation From Skin-Temperature Measurements on a V-2 Rocket in Flight¹

BY W. W. FISCHER² AND R. H. NORRIS,³ SCHENECTADY, N. Y.

Coefficients of convective heat transfer have been evaluated by analysis of the skin-temperature measurements made at several points on the nose of a V-2 (German A-4) rocket during flight in New Mexico. On one side, the surface of the nose was smooth but on the other side, 4 in. from the nose tip, a transverse ridge about 1/8 in. high, was purposely provided as a turbulence promoter.

The test results are compared in dimensionless form with various proposed correlations for both laminar and turbulent conditions of the boundary layer. The test results, thus correlated, seem to justify the following conclusions: (1) Boundary-layer transition seems to be more dependent, at supersonic speed, on Mach number than on Reynolds number. (2) The theoretical heat-transfer correlation for a laminar layer is confirmed by the appropriate sets of test points to the estimated order of accuracy of the test. (3) For turbulent layer conditions, the subsonic correlation for a flat plate provides better agreement with the supersonic test results than does Eber's correlation, but there still remains a range of uncertainty as to the best choice of temperature for evaluation of the air properties.

NOMENCLATURE

The following nomenclature is used in the paper:

- a = velocity of sound, fps
 C_f = friction coefficient = $\frac{\tau}{\rho V^2/2g}$, dimensionless
 c = specific heat, Btu/(lb)(deg F)
 h = coefficient of heat transfer = $\frac{q/S}{T_{adw} - T_w}$, Btu/(sec)(sq ft)(deg F)
 J = mechanical equivalent of heat, 778 (ft-lb)/Btu
 j = heat-transfer factor = $\left(\frac{h}{c_p \rho V}\right) \left(\frac{c_p \mu}{k}\right)^{2/3}$, dimensionless
 k = thermal conductivity of the air, Btu/(sec)(sq ft) (deg F/ft)
 M_0 = Mach number, V_0/a_{am} , relative to the undisturbed air, dimensionless
 M_1 = Mach number, V_1/a_1 , locally, just outside the boundary layer, dimensionless

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³ General Engineering and Consulting Laboratory, General Electric Company. Mem. ASME.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society. Paper No. 48-A-54.

- n = exponent in relation that μ is proportional to T^n , dimensionless
 p = absolute pressure, psf
 q = heat flow rate, Btu per sec
 R = Reynolds number, = $\frac{\rho V x}{\mu}$, for position x , dimensionless
 S = surface area, sq ft
 T = absolute temperature, deg R (deg F abs)
 t = time, sec
 V = velocity of air, relative to surface, fps
 x = distance along surface, measured from tip of cone or nose unless otherwise specified, ft
 δ = thickness, see Equation [8], ft
 ϵ = radiation emissivity of skin, dimensionless
 μ = viscosity of air, lb/(sec ft)
 ρ = density, lb per cu ft
 σ = Stefan-Boltzmann constant, = 0.483×10^{-12} Btu/[(deg R)⁴ (sq ft)(sec)]
 τ = shear stress on surface due to friction, psf
Subscripts:
 adw = adiabatic wall, as defined for T_{adw} in Equation [7].
 am = "ambient" conditions in undisturbed atmosphere (static values)
 c = value of cement layer, see Equation [8]
 m = mean value, by integration over surface from tip of nose, cone or plate, to point x
 o = value (of V or M) referring to the undisturbed air, relative to surface.
 p = value at constant pressure, see definition of c
 s = value of stainless-steel skin, see Equation [8]
 w = value at wall surface, or of material of wall itself
 x = "local" value at point x (as distinguished from mean value, denoted by subscript m)
 1 = value for air just outside boundary layer, at location x
 $'$ = value at a temperature T' , defined by Equation [2]

INTRODUCTION

One of the important problems of supersonic flight is the prediction of convective heat transfer between the surface of a body and the air through which it moves. The available theory for supersonic speeds, reviewed by Johnson and Rubesin (6, 13)⁴ is limited to laminar boundary-layer conditions. Furthermore, the test data heretofore available for confirmation of this theory and extension of the correlation to other conditions seem inadequate. Although some results of wind-tunnel tests have been available (1), the uncertain effects of nonuniformity of the heat transfer over the surface make the interpretation of those results questionable.

Another method of test is to measure the skin temperature versus time during the flight of a large supersonic rocket. The large size reduces the lengthwise heat conduction so that temperature at one spot should be dependent only on the heat-

⁴ Numbers in parentheses refer to the Bibliography at the end of the paper.

transfer conditions at that spot, without serious error from heat conduction from other parts of the rocket having different temperatures. The method of evaluating the convective heat transfer from the measured temperature data is explained later.

A flight test of a V-2 (German A-4) rocket was made at White Sands Proving Ground, New Mexico, on October 9, 1947, for the specific purpose of obtaining skin-temperature measurements on a nose of appropriate configuration. Skin-temperature measurements have been made on V-2 rockets on other occasions, but for other purposes requiring less accuracy.

For the present purpose, in order to obtain even fair accuracy in the heat-transfer coefficient, relatively high accuracy of the skin-temperature measurements was desired. The reason for this is that not only the temperature itself is needed but also its time rate of change. This rate is not uniform but varies widely with time. Furthermore, there are many other sources of error, as explained later, which limit the accuracy of the evaluation of the convective heat-transfer correlation. The present method was expected to be sufficiently accurate, however, for the following purposes:

- (a) To indicate whether the boundary layer was laminar or turbulent.
- (b) To indicate whether there was any serious error in the heat-transfer theory for laminar layers.
- (c) To justify for turbulent layers some decision as to what temperature basis to use for evaluation of the air properties when applying to supersonic speeds the subsonic correlation for convective heat transfer. It was recognized that such a decision would be only approximate and tentative.

THEORETICAL CORRELATIONS

For laminar boundary layers, most of the theoretical analyses have been for flat plates, not cones. However, these analyses can be adjusted so as to apply to cones. The relation for this purpose has been shown theoretically (8) to be valid for supersonic as well as subsonic velocities. It is also necessary in the case of cones to consider, in place of the ambient temperature, the local static temperature T_1 and velocity V_1 of the air (relative to the cone surface) at a point close to the cone just outside the boundary layer.

Among the various forms of the theoretical heat-transfer correlation for a flat plate with laminar layers at supersonic speed, the form most convenient for comparison with test results is the one presented by Johnson and Rubesin (6). For the present purposes, their correlation has been modified slightly, in the interest of simplicity, by using T' , instead of T_w , for evaluating c_p and $c_p \mu / k$ (the Prandtl number); the difference in the results is negligible for the present tests. When adjusted, as just explained, so as to apply to a cone, the resulting correlation can be written in the form

$$\left(\frac{h_x}{c_p \rho' V_1} \right) \left(\frac{c_p \mu'}{k'} \right)^{2/3} = 0.575 \left(\frac{\rho' V_1 x}{\mu'} \right)^{-1/3} \dots [1]$$

where the primed quantities are evaluated at the following temperature

$$T' = 0.42 T_1 (1 + 0.076 M_1^2) + 0.58 T_w \dots [2]$$

For turbulent boundary layer on flat plates, no theoretical solution for either heat transfer or friction is available for supersonic speeds. Thus in the absence of any conclusive test evidence, it is not unreasonable to consider whether the usual semi-empirical correlation for subsonic flow over flat plates (7) will remain valid at supersonic speeds. This relation can be written in the form

$$\left(\frac{h_x}{c_p \rho V} \right) \left(\frac{c_p \mu}{k} \right)^{2/3} = \frac{(C_f)_x}{2} \dots [3]$$

where $(C_f)_x$ can be evaluated from a relation (15) involving $\log R(C_f)_x$. An approximation to this relation, which is very close for Reynolds number from 100,000 to 4,000,000 and only a little too low for still higher values is

$$(C_f)_x = 0.056 \left(\frac{\rho V x}{\mu} \right)^{-0.2} \dots [4]$$

This factor 0.056 used for $(C_f)_x$ can be shown to correspond to the factor 0.070 in the analogous equation for the more customary quantity $(C_f)_m$. Combination of Equations [3] and [4] yields

$$\left(\frac{h_x}{c_p \rho V} \right) \left(\frac{c_p \mu}{k} \right)^{2/3} = 0.028 \left(\frac{\rho V x}{\mu} \right)^{-0.2} \dots [5]$$

It can be seen from Equation [2] that the T' temperature is an intermediate temperature which gives 58 per cent weight to the surface temperature T_w , and 42 per cent weight to a temperature about halfway between the local static temperature T_1 , and the adiabatic-wall temperature, T_{adw} . Therefore this T' temperature seems a reasonable temperature to pick for a trial correlation of the turbulent-layer data as well.

In applying the flat-plate correlation to cones, the local values T_1 and V_1 seem appropriate as in the case of the laminar layer. It remains uncertain whether the factor 0.056 of Equation [4] will require adjustment for the case of cones, but in the absence of evidence to the contrary, the correlation is here considered without attempting any adjustment.

EXPERIMENTAL EQUIPMENT

The skin temperature versus time data were recorded on the ground by means of a telemetering system. Such a system



FIG. 1 NOSE CONE SHOWING THE SMOOTH SIDE

consists of electronic equipment aboard the missile which automatically measures and transmits the data, by means of radio waves, to ground receivers which record these data on microfilm. The telemetering system (23) of the authors' company, used in this flight, sampled each of its 27 channels 30 times per sec.

General Arrangement of Temperature-Measurement Equipment. Temperature measurements were made at six locations on the skin of the missile. Four locations were on a special nosedpiece, shown in Fig. 1, installed on an old-type NRL warhead (10). The other two locations were on the left door of the instrument section of the missile; one of them is shown in Fig. 2.

Stations A, C, and G were located one behind another, respectively, 2½ in., 6 in., and 12 in. from the tip of the nose cone. Station H was located 12 in. from the tip of the nose cone but diametrically opposite station G. Stations K and M were located at 84.4 in. and 121.4 in., respectively, from the tip of the nose cone. Distance to all stations was measured along the skin.

The special nose cone was constructed with a total included angle of 45 deg. The first ¾ in. of the cone, measured axially, was machined from solid stainless steel. A support ring was located 4 in. from the tip, also measured axially. The tip and ring were supported by brackets from a 1-in. axial tube. Supports were designed to minimize heat conduction. The section between the tip piece and the support ring together with a half-circular part of the remainder of the cone was formed of a single piece of 1/32 in. nominal thickness type 302 stainless-steel sheet.



FIG. 2 OUTSIDE OF INSTRUMENT-SECTION DOOR, SHOWING STATION K SKIN INSERT AND DOOR-TOP EDGE FAIRING STRIP

Stations A, C, and G were on the line of symmetry of this sheet. The other half of the base section was formed from a 1/16-in. nominal thickness type 302 stainless-steel sheet. The base of the cone was machined from steel stock.

The instrument-section door stations were constructed by modifying the original door. The modification consisted of cutting out rectangular sections of the skin and replacing them with 5-in. (axially) × 7-in. type 302 stainless-steel inserts 1/32 in. thick. Steel strips approximately ¾ in. wide and 3/32 in. thick were spot-welded to the inner surface of the door around the opening, forming a lip to support the stainless-steel insert. The insert was then spot-welded to the strip, making the outside surface of the insert flush with the outside surface of the original skin.

To minimize radiation effects, particularly at the high skin temperatures, the stainless-steel nose cone and instrument-section door inserts were polished to a mirror finish. Next, they were plated with 0.5 mil of nickel, followed by 0.1 mil of rhodium. Light buffing of the rhodium surface completed the operation. Rhodium was used because its emissivity remained unchanged (judged by appearance of the surface) when exposed to air at a temperature over 700 F for 6 hr. The nose cone, as well as the inserts, were polished shortly before the launching.

Thicknesses, including the plating, at the various stations were as follows:

A	C	G
0.0295 in.	0.0294 in.	0.0300 in.
H	K	M
0.0637 in.	0.0324 in.	0.0324 in.

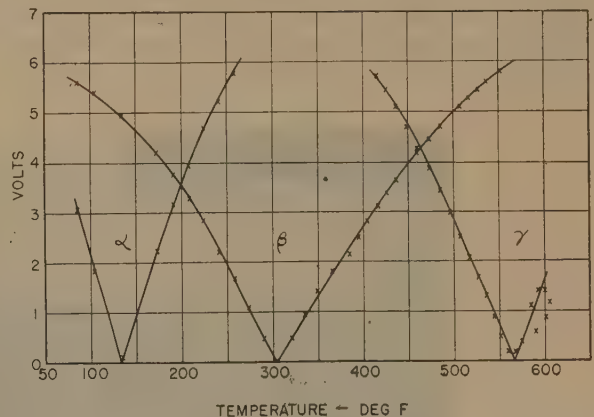


FIG. 3 OUTPUT VOLTS VERSUS TEMPERATURE AT A TYPICAL STATION (Ranges of α , β , and γ bridges are shown.)

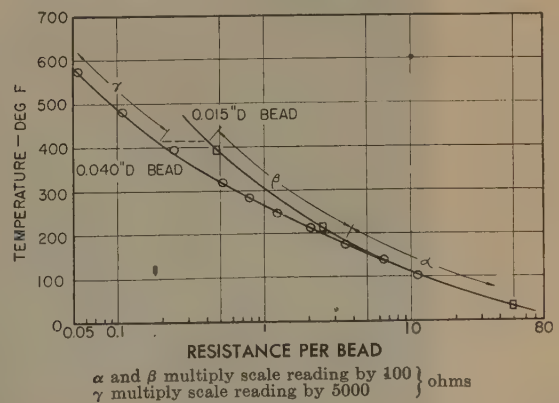


FIG. 4 TYPICAL TEMPERATURE VERSUS RESISTANCE CURVE FOR THERMISTOR BEADS (Ranges of α , β , and γ bridges are shown.)

Temperature-Sensing Elements. The temperature-sensing elements were Western Electric "thermistor" beads (22), connected as one pair of bridge arms. The other two arms were fixed resistances. By proper choice of the resistances and the size and number of beads installed, such a bridge was made to balance at any selected temperature. Over a range near the balance point, the output versus temperature relation was V-shaped with almost linear response on each side as shown in Figs. 3 and 4. Usable output was 0 to 5 volts although the output reached 14 volts for conditions of very large unbalance.

The bridges were selected to provide three temperature ranges, thereby providing higher accuracy over the entire range covered. The nominal balance points of these were alpha bridge 120 F, beta bridge 300 F, and gamma bridge 570 F.

The alternating-current input to the bridges was supplied by the 10-volt 10-kc regulated generator contained in the telemetering system. The bridge elements and voltages were selected to reduce to a negligible amount the heating of the thermistors by the arm current. The design criterion was a maximum temperature rise of 2.5 deg F based on the power at the point of crossover from one bridge to another.

The ratio-arm resistors, transformers, and output circuits were mounted remotely from the temperature-sensing beads inside of metal containers. Attention was paid to shielding and elimination of common leads in consideration of the 10-kc frequency.

Mounting of Thermistor Beads. The thermistor beads were embedded in the stainless-steel skin to reduce the thermal time lag of response. This was accomplished by engraving to a depth of 20 mils either a trough or a wedge-shaped groove whose nominal dimensions were 0.042 in. $\times$ 0.075 in., and 0.020 in. $\times$ 0.040 in., respectively. One bead was mounted in each engraved hole, the larger holes being used for the beads of the gamma bridges. A

typical station consisted of 16 beads, of which two were for the gamma bridge (type No. V668 0.040 in. diam), eight were for the beta bridge (type No. D170575 0.015 in. diam), and six were for the alpha bridge (type No. D170575 0.015 in. diam). All beads at a particular station were the same distance from the tip of the missile.

Prior to mounting the beads, the engraved area of the skin was

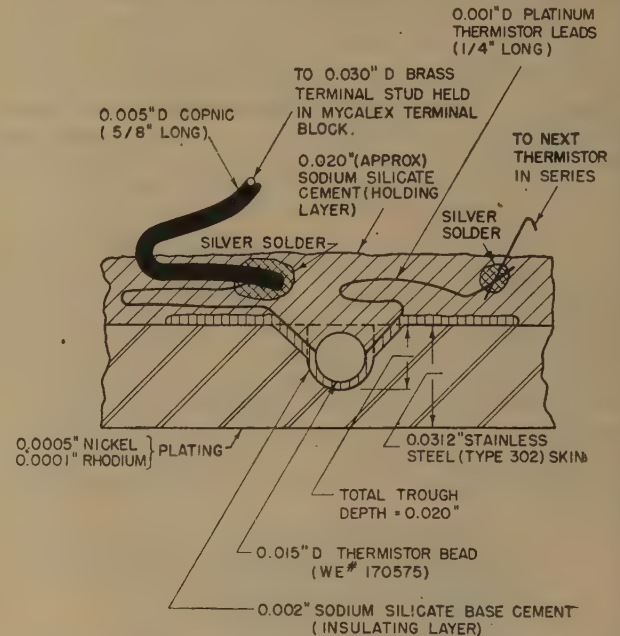


FIG. 5 TYPICAL THERMISTOR MOUNTING

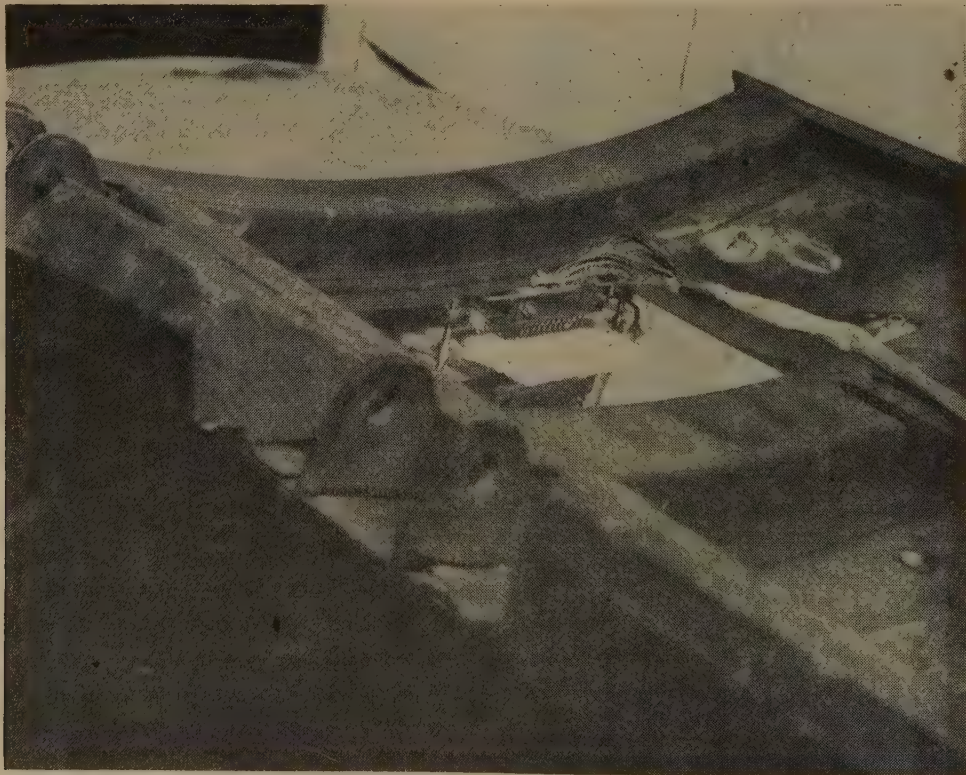


FIG. 6 INSIDE OF INSTRUMENT-SECTION DOOR AT STATION M, SHOWING LOCATION OF THERMISTORS, CEMENT, LEAD WIRES, AND TERMINAL BLOCK

coated with a layer of cement (type GE-2155, sodium silicate base) approximately 2 mils thick. This was found to be necessary to avoid danger of short circuits from the bead leads to the skin.

After the skin insulation had set, the beads were placed in the grooves and troughs and cemented in place by means of a layer of the same cement. Figs. 5 and 6 show the details of a typical installation. The delicate 1-mil leads were embedded in the cement to prevent later damage from vibration. During assembly, in the effort to hold these leads in place, this second cement layer unfortunately was allowed to reach an average thickness of about 20 mils. The undesirability of so thick a layer was not appreciated until too late, after all the beads had been cemented. Lack of time before the scheduled flight date prevented a second attempt.

Calibration Provisions for Thermal Equipment. To allow for calibration of the bridges, each station was fitted with two thermocouples. These couples were mounted in line with the thermistor beads; one couple at each end of the line. The couples were constructed by spot-welding a 5-mil iron-copernic junction to the inside surface of the stainless-steel skin. These leads were $\frac{1}{2}$ in. in length and were silver-soldered to the longer 20-mil iron-copernic leads.

A conical oven for heating the nose cone was constructed, using distributed coils of Calrod, arranged in three circuits to provide more uniform temperature distribution.

Electrical hot-plate elements were used in an oven constructed for heating stations K and M. A $\frac{1}{8}$ -in.-thick copper plate was mounted between the heater element and the door to secure a more even distribution of heat across the thermistor location.

Advantages of Thermistors. Thermistors were chosen as the temperature-sensing elements primarily because of the relatively high voltage output of the thermistor bridge circuit, which allows input directly to the telemetering system without the use of intermediate amplification, thereby providing a desirably stable system. A second outstanding advantage of the thermistors is the ease with which the sensitivity of response (volts per degree) and the nominal range can be adjusted to desirable values, see Fig. 4.

Furthermore, the size of the thermistor is small, thus allowing local readings to be obtained easily. It is recognized that the thermal-insulating property of the electrical-insulating cement

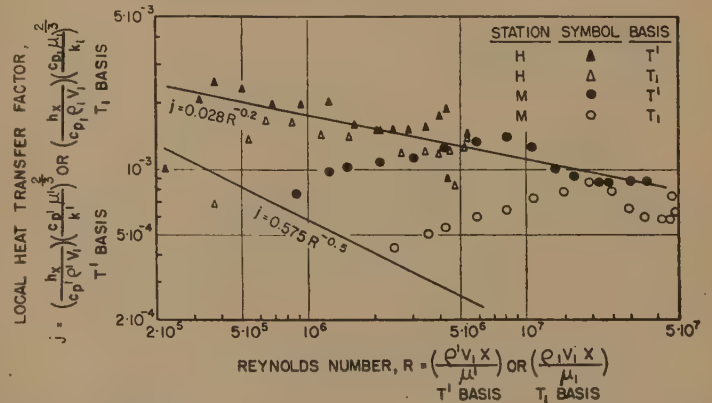


FIG. 8 COMPARISON OF LOCAL HEAT-TRANSFER FACTOR VERSUS REYNOLDS NUMBER USING TWO TEMPERATURE BASES FOR EVALUATION OF AIR PROPERTIES

(Data shown for stations H and M.)

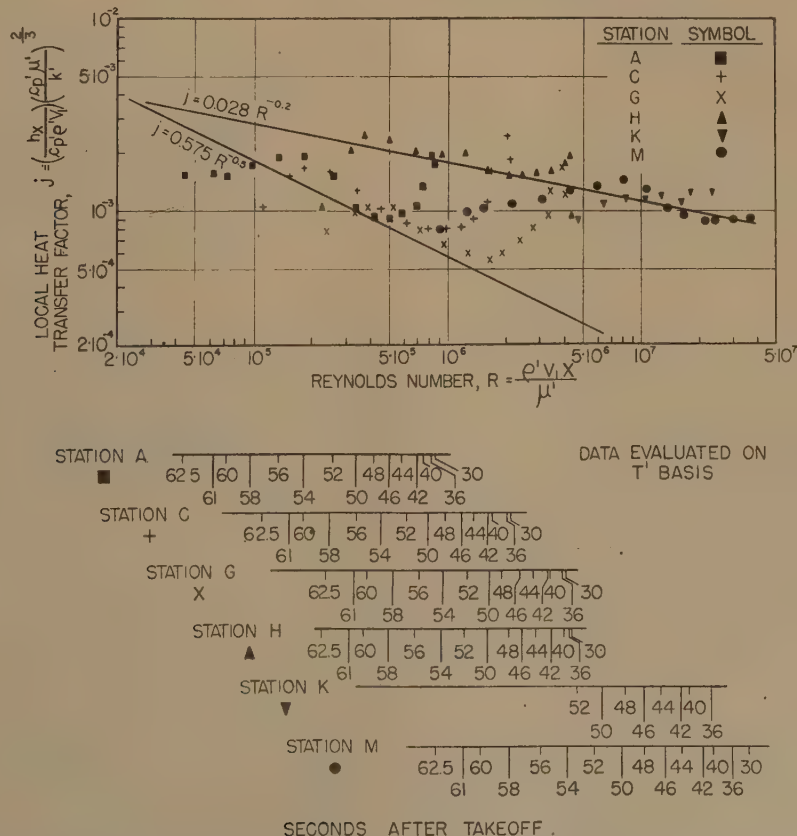


FIG. 7 LOCAL HEAT-TRANSFER FACTOR VERSUS REYNOLDS NUMBER FOR V-2 FLIGHT OF OCTOBER 9, 1947

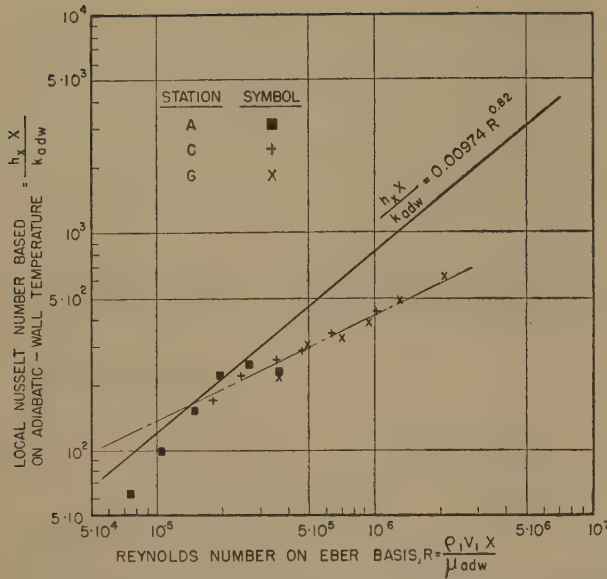


FIG. 9 EBER'S CORRELATION: LOCAL NUSSELT NUMBER VERSUS REYNOLDS NUMBER; LAMINAR-LAYER TEST POINTS

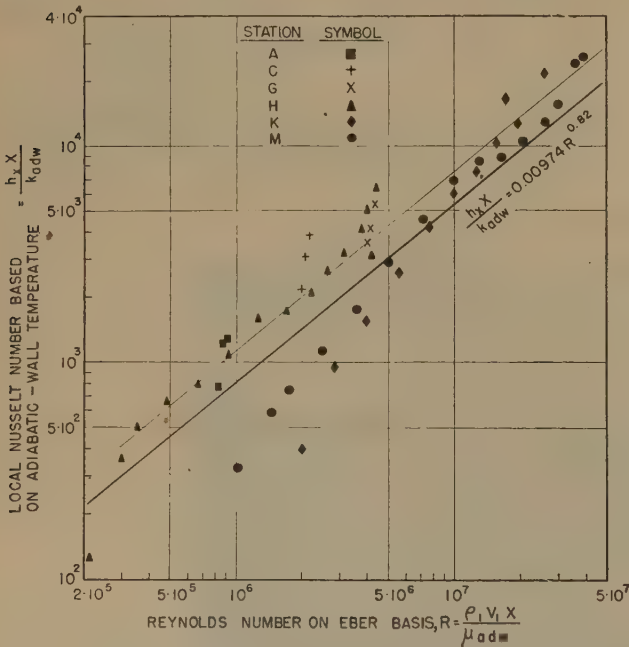


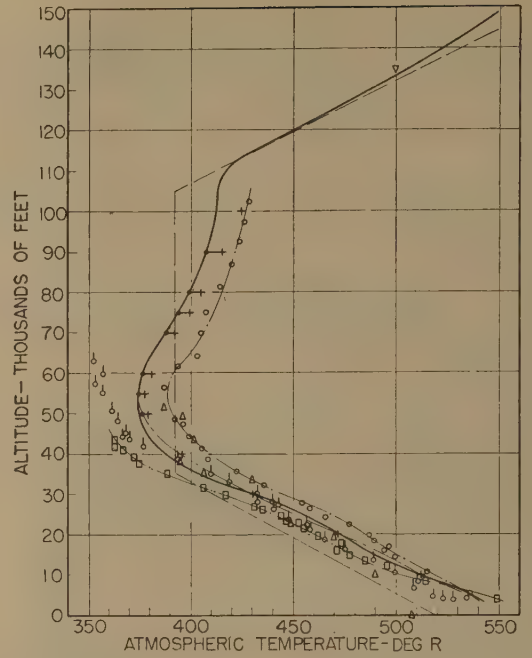
FIG. 10 EBER'S CORRELATION: LOCAL NUSSELT NUMBER VERSUS REYNOLDS NUMBER; TURBULENT-LAYER TEST POINTS

between the thermistor bead and the skin introduces some uncertainty. While use of a thermocouple could eliminate need for insulation, its output is a very small direct-current voltage, thus requiring use of a direct-current amplifier "ahead" of the telemetering equipment. In addition, the fact that a thermocouple is grounded is undesirable.

EVALUATION OF CORRELATIONS FOR LOCAL CONVECTIVE HEAT TRANSFER

In the dimensionless correlations of the test results presented in Figs. 7, 8, 9, and 10, the various factors were evaluated as follows:

The atmospheric temperature T_{am} , was evaluated from Fig. 11,

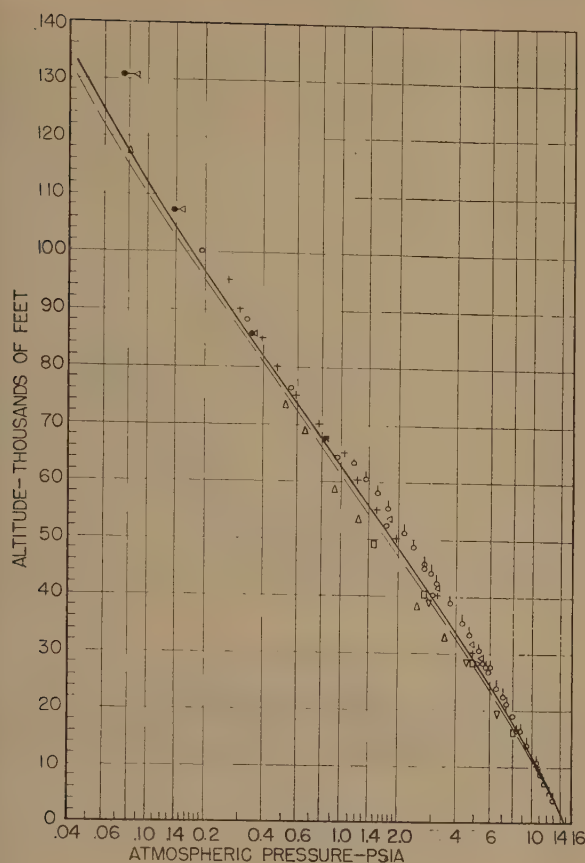


- NACA STANDARD ATMOSPHERE
SELECTED CURVE
BALLOON FLIGHTS
RADAR TRACED
AVERAGE OF 6 FLIGHTS APR 15 TO
JULY 29, 1947 [SEE NOTE(1)]
OCT 10, 1947 - 1030 MST
NOT TRACKED
OCT 9, 1947 - 0949 MST [SEE NOTE(2)]
OCT. 9, 1947 - 1427 MST [SEE NOTE(2)]
MAR 7, 1947 - NRL(10)
CALCULATED FROM SLOPE OF PRESSURE CURVE
MAR. 7, 1947 - NRL(10)
NOTES:
(1) SYMBOL $\bullet +$ DESIGNATES COMBINATION OF
(A) UNCORRECTED DATA AS + AND
(B) SHIFT DUE TO CORRECTION FOR RADIATION
AND BALLOON WAKE EFFECTS AS $\bullet$
(2) ALTITUDE FOR EACH TEMPERATURE READING
CALCULATED FROM HYPSONETRIC EQUATION(12)
APPLIED TO BALLOON FLIGHT TEMPERATURE
AND PRESSURE DATA.

FIG. 11 ATMOSPHERIC TEMPERATURE VERSUS ALTITUDE

using the solid curve except where specified otherwise. This solid curve was chosen to represent a reasonable compromise of the available data, also shown in Fig. 11, with realization that these data were not all equally significant. The pressure values corresponding to this solid line, calculated by means of the "hypsometric equation" (12), are shown as the solid curve in Fig. 12. These pressure values were used for evaluation of p_{am} . Fig. 12 shows also the significance of the data of absolute pressure on the nose-cone tap shown in Fig. 13.

The quantities T_1 , p_1 , M_1 , ρ_1 , and V_1 , were evaluated for conditions of the air immediately outside of the boundary layer at the station considered. For the stations on the nose cone, these quantities were evaluated from T_{am} , p_{am} , M_0 , and V_0 , by applying the theoretical relations based upon Taylor and Maccoll's solution (9) for supersonic flow over a cone at zero angle of attack. For the stations K and M, which were farther aft on the part of the nose having an ogival profile, the local pressure values p_1 , were evaluated from the German wind-tunnel test results (20) for the model of the V-2 rocket. The values of local static temperature T_1 , were then evaluated as the temperatures cor-



NACA STANDARD ATMOSPHERE
PRESSURE HYPSEMETRICALLY DERIVED [SEE NOTE(1)]
SELECTED CURVE OF FIG. 11
BALLOON FLIGHT OCT. 9'47-0949 MST ○
BALLOON FLIGHT OCT. 9'47-1427 MST □
RECORDED PRESSURE FROM BALLOON FLIGHTS } RADAR
OCT. 10, 1947-1030 MST } TRACKED
AVERAGE OF 6 FLIGHTS, APR. 15 TO JULY 29, 1947 +
CALCULATED PRESSURE FROM V-2 NOSE TAP TEST RESULTS
OCT. 9, 1947-1200 MST [SEE NOTES(2) & (3)] ●◄
ROCKET SONDE DATA
NRL(1) OCT. 10, 1946 ▽
MAR. 7, 1947 Δ

NOTES:
(1) CALCULATED BY MEANS OF HYPSEMETRIC EQUATION (12)
BASED ON TEMPERATURES FROM FIG. 11
(2) SYMBOL ●◄ DESIGNATES COMBINATION OF
(A) UNCORRECTED DATA AS ◄ AND
(B) SHIFT DUE TO CORRECTION FOR TIME LAG
IN PRESSURE MEASURING SYSTEM AS ●
(3) ESTIMATED ACCURACY OF NOSE TAP PRESSURE
INDICATING DEVICE = ± 0.2 psi = $\pm 2\%$ AT FULL SCALE.

FIG. 12 ATMOSPHERIC PRESSURE VERSUS ALTITUDE

responding to an isentropic change to the local pressure p_1 , from the ambient pressure p_{am} , after the nonisentropic change through the shock wave from the nose tip was appropriately accounted for. The quantities V_1 and M_1 were evaluated in a similar manner.

The properties c_p' , ρ' , μ' , and k' were evaluated from reference (17) for the temperature T' , defined by Equation [2]. The coefficient h_x was evaluated from

$$h_x = \frac{(q/S)_x}{(T_{adw} - T_w)} = \frac{c_w \delta_w \rho_w \left(\frac{dT_w}{dt} \right) + \epsilon \sigma T_w^4}{(T_{adw} - T_w)} \dots [6]$$

where the constants are, respectively

$$c_w = 0.12 \text{ Btu/(lb)(deg F)}$$

$$\delta_w = \text{equivalent skin thickness (discussed in next section)}$$

$$\rho_w = 496 \text{ lb per cu ft}$$

$$\epsilon = 0.25 \text{ (discussed later)}$$

The adiabatic-wall temperature T_{adw} , was evaluated from the relation

$$T_{adw} = T_1 + \left(\frac{c_p' \mu_1}{k_1} \right)^n \left(\frac{V_1^2}{2 g J (c_p)_{av}} \right) \dots [7]$$

where

$$n = 1/2 \text{ for a laminar boundary layer (by references 6 and 13)}$$

$$n = 1/3 \text{ for a turbulent boundary layer (by reference 13)}$$

$$(c_p)_{av} = \text{difference in enthalpy (by reference 17) between } T_{adw} \text{ and } T_1, \text{ divided by } (T_{adw} - T_1).$$

For the correlation in both Figs. 7 and 8, the choice of n was made as follows: The value $1/2$ was used if the test point proved, upon trial, to be closer to the correlation line for the laminar-layer theory than it was to the line for the turbulent layer; the value $1/3$ was used if it lay closer to the turbulent-layer correlation.

Composite time-temperature plots of the stations are shown in Figs. 13(a) and 13(b). These figures show the points for each bridge at each station. The faired curves were used to evaluate both T_w and dT_w/dt .

Missile altitude and velocity versus time were evaluated from Fig. 14.

Equivalent Thickness of Skin. In order to allow for the thermal capacity of the cement, the effective thickness δ_w , of the skin was evaluated as follows

$$\delta_w = \delta_s + \left(\frac{c_p \rho_c}{c_s \rho_s} \right) \delta_c \dots [8]$$

where the constants are, respectively,

$$\delta_s = \text{tabulated elsewhere in this paper}$$

$$\delta_c = \text{estimated as } 0.022 \text{ in. (0.00183 ft)}$$

$$c_c = 0.20 \text{ Btu/(lb)(deg F)}$$

$$c_s = 0.12 \text{ Btu/(lb)(deg F)}$$

$$\rho_c = 119 \text{ lb per cu ft}$$

$$\rho_s = 496 \text{ lb per cu ft}$$

The second term of the Equation [8] thereby contributed 8.8 mils to the equivalent thickness.

ACCURACY OF RESULTS

Experimental Temperature Data. For the alpha and beta bridges, covering temperatures to 550 F, it has been estimated that the probable errors in the measurement of the temperature are as follows:

	Deg F
Calibration.....	6
Allowance for hysteresis of thermistors.....	10
Telemetry error per bridge (1 per cent).....	2
Reading error per bridge (1 per cent).....	2
Rms error from foregoing sources.....	12

The random scatter of the plotted temperature results in Fig. 13 is satisfactorily small, well within this estimated error, with the following exceptions: For station M at about 200 F, where the crossover from the alpha to the beta bridge occurred, there is an inconsistency of about 30 deg F between the results from the two bridges. For station K , which had only one bridge, there

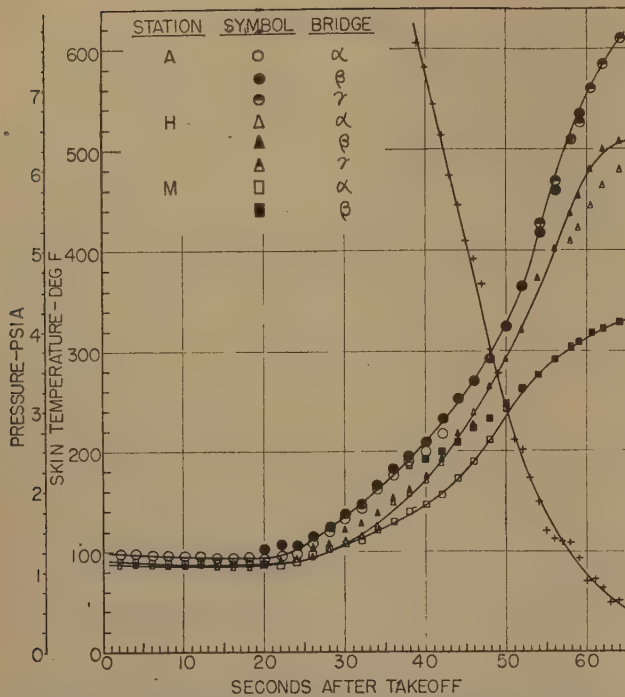


FIG. 13(a) LOCAL SKIN TEMPERATURE AT STATIONS A, H, AND M, AND ABSOLUTE PRESSURE AT NOSE-CONE TAP VERSUS SECONDS AFTER TAKE-OFF FOR V-2 FLIGHT OF OCTOBER 9, 1947

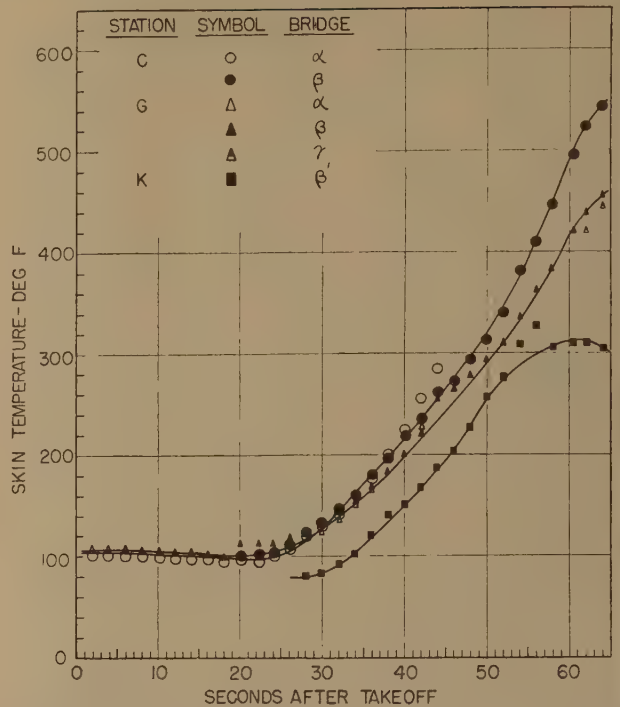


FIG. 13(b) LOCAL SKIN TEMPERATURE AT STATIONS C, G, AND K VERSUS SECONDS AFTER TAKE-OFF FOR V-2 FLIGHT OF OCTOBER 9, 1947

is irregular scatter at about 300 F. Inspection of the calibration of station K showed considerable inconsistency between repeated calibrations around this temperature. Therefore results for station K for times greater than 52 sec have not been included in Fig. 7, although at the lower temperatures they are reasonably consistent with those for station M.

The temperature for the crossover from the beta to the less accurate gamma bridge was 550 F. Thus it was not necessary to use the gamma bridge except for station A points beyond 59 sec, where the temperature appreciably exceeded 550 F.

Sources of Error in Correlation. In the evaluation of the dimensionless correlation from the measured skin temperatures, several additional sources of error deserve comment.

One source is the uncertainty in the atmospheric temperature and pressure. Fortunately, the effect of the temperature uncertainty proves to be relatively negligible, provided the pressure is assumed fixed. For example, even an error of 50 deg F in atmospheric temperature, which is more than seems reasonable, can be shown to produce a change of not more than 4 per cent in either the ordinate or abscissa of Fig. 7.

The probable error in pressure might well be ± 5 per cent or possibly ± 10 per cent. Any error in pressure produces a proportional error on both the ordinate and abscissa of Fig. 7, or Fig. 8, if the temperature is fixed. The effect on any discrepancy between a test point and the correlation line is less than this, however, since an increase in pressure moves a test point both downward and to the right, in a direction nearly parallel to the correlation line.

The error from uncertainty in the assumed value of 0.25 for the emissivity ϵ in Equation [6] is small. If, as extreme values, it were assumed to be 0.50 or 0.00, it would change the heat-transfer results by not more than ± 7 per cent, for the 62.5-sec points. Furthermore, the amount decreases rapidly with decreasing time from takeoff.

Another source of error is in the determination of the slope of the time versus temperature curve at each station. Although much care was exercised in drawing the tangents to the time

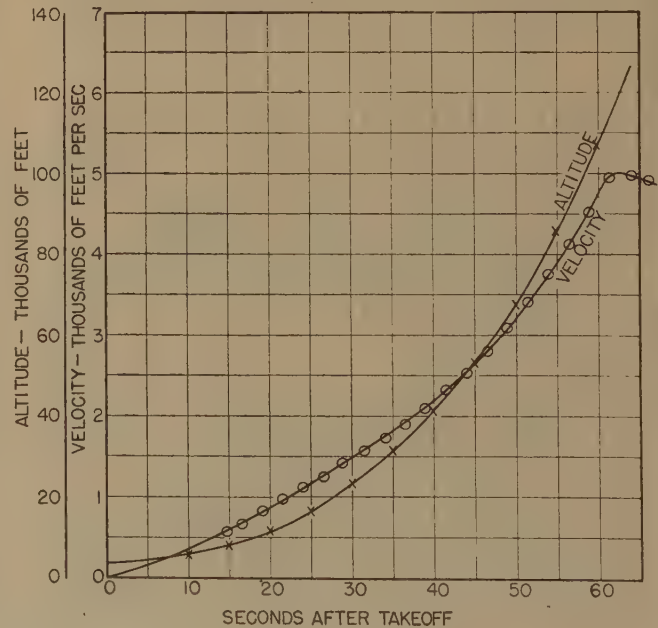


FIG. 14 ALTITUDE AND VELOCITY VERSUS SECONDS AFTER TAKE-OFF FOR V-2 FLIGHT OF OCTOBER 9, 1947

versus temperature curve, it is estimated that the error in the evaluation of the slopes (dT_w/dt) is of the order of ± 5 per cent.

The probable error in the estimate of the cement thickness is estimated as of the order of ± 7 mils. This uncertainty corresponds to 3 mils in the equivalent steel thickness, or about 10 per cent for stations A, C, G, K, and M, and 5 per cent for station H.

The magnitude of the effect on the correlations produced by nonuniformity of skin temperature upstream from any particular station is not known. Most of the skin area upstream from

stations *C*, *G*, and *H* is likely to have been more uniform in temperature than for stations *A*, *K*, and *M*, however. The correlations for *C*, *G*, and *H* do seem the best of the six.

The time-lag effects of the skin temperature versus time data will be discussed later.

Temperature Nonuniformity in the Cement. One of the assumptions used to justify Equation [6] is that the thermal capacity of the cement can be considered as lumped with that of the steel, thus assuming negligible nonuniformity of temperature through the cement.

An approximate analysis indicates that the nonuniformity was of such a magnitude that the temperature halfway through the cement lags behind the steel temperature by about $1/3$ sec. Even at the maximum rate of temperature rise, this corresponds to only about 8 deg F. The thermistor element is much closer to the steel than this mid-point of the cement layer, so the error from the nonuniformity is considered relatively negligible compared to the other errors already discussed.

Over-All Accuracy. The probable absolute error from all the foregoing sources is estimated as of the order of ± 15 per cent on the average. However, cumulative errors for any one point might well raise the error for that point to ± 25 per cent in some cases.

RESULTS

The results are presented in two different dimensionless correlation forms in Figs. 7, 8, 9, and 10. The significant features of the results are discussed in the next section. These results cover a range of free-stream Mach numbers from 1.40 to 4.83. This corresponds to local Mach numbers, just outside the boundary layer, of 1.0 to 3.1 for the nose-cone stations, and 1.3 to 4.2 for stations *K* and *M*, respectively.

CONCLUSIONS

The results of the correlation as plotted in Fig. 7 indicate the existence of three distinct conditions of the boundary layer, namely, laminar layer, transition, and turbulent layer. The results for these conditions will be discussed in turn.

Laminar Layer. The existence of a laminar boundary layer is clearly evident for stations *A*, *C*, *G*, for the period from 48 to about 61 sec. The plotted points for all three stations could be approximated by a single curve about parallel to the theoretical line for a laminar layer, but about 17 per cent above it. This discrepancy does not seem unreasonable compared to the probable test accuracy of about ± 15 per cent. The 62.5-sec point for stations *C* and *G*, and the points for 60, 61, and 62.5 sec for station *A* all drop considerably below the theoretical line. For station *A*, the three points concerned had to be obtained from the gamma bridge which is believed to give less accuracy than the beta bridge, since the calibration, Fig. 3, showed an unusual and unexplainable scatter above 575 F. For stations *C* and *G*, the 62.5-sec point was at the end of the range of the beta bridge where the accuracy is considerably reduced.

The "hump" of the test points near 56 sec is perhaps attributable to an accidental change of 39 deg in the direction of the rocket's flight which occurred between 49 and 59 sec, and to a slow roll of 0.55 revolutions per sec, which began at 56 sec. The average angle of attack aerodynamically required for this change of direction has been calculated, and its calculated effect on the heat transfer is of an order of magnitude which could account for the hump.

Transition. For the "smooth" conical surface, i.e., stations *A*, *C*, and *G*, the location, and the rapid drop, of the j -values between the 40-sec and the 48-sec readings strongly indicate a transition from turbulent to laminar boundary layer.

Furthermore, this nearly simultaneous occurrence of transition

at three widely different Reynolds numbers seems significantly consistent with the recent theory (11) whereby combinations of Mach number and surface-temperature conditions, not Reynolds number, determine the stability of the boundary layer when the Mach number is considerably above sonic. Fig. 7 of reference (11) shows, as a function of surface temperature, the particular Mach number above which a laminar layer becomes theoretically stable for very high (and even infinite) Reynolds number. This Mach number when evaluated for station *A*, *C*, or *G*, is found to be approximately 1.5, which corresponds quite closely to the local Mach number M_1 at the 37-sec point where transition seems to have begun.

The results for station *H*, which was preceded by a turbulence promoter, show no significant evidence of a transition until much later—near 62 sec. Since station *G* definitely did show early transition, evidently the turbulence promoter performed as intended in spite of the theoretical stability of a laminar layer.

An unexpected feature of the transition data is that partial transition is not confined, at any one instant (for example, the 44-sec point), to a small region on the cone, but exists simultaneously at all three stations (*A*, *C*, and *G*). Perhaps the boundary layer, over a large area, pulsates between a turbulent condition and a laminar condition.

Turbulent Layer. In spite of the relative smoothness of the surface at stations *A*, *C*, and *G*, apparently a turbulent layer existed at the beginning of the test period, i.e., at 30 sec after take-off. This is indicated by the approximate consistency between the results, at the higher Reynolds numbers, for these three stations and those for stations *H* and *M*; in particular, in the interval of 30 to 36 sec for *A*, and from 30 to 40 seconds for *C* and *G*.

For the stations *H* and *M*, each of which is aft of a transverse ridge, the turbulent nature of the boundary layer is indicated, as expected, by the trend of the results. It had been hoped that the supersonic results represented by these tests would follow the subsonic correlation for a flat plate with a turbulent boundary layer. The major uncertainty lay in the choice of the temperature basis for evaluation of the air properties. Comparison of the two sets of test points in Fig. 8, representing two considerably different temperature bases, appears to justify the following conclusions:

1 On the T' basis of correlation, the trend of the test points from both station *H* and station *M* is the same; the points lie approximately along the straight line which represents the subsonic correlation, except for the last few seconds of the test period. During these last seconds the boundary layer may have begun to become laminar, and also the accuracy is reduced. Although there is appreciable scatter, probably attributable to limitations of test accuracy, the consistent trend of the points covers an unusually large range of Reynolds numbers and therefore may be considered reasonably significant, although not satisfactorily conclusive.

2 When correlated on the basis of the local static temperature T_1 (just outside the boundary layer), however, comparison of the test points with the curve for subsonic conditions seems even less conclusive. The points for station *M* drop off toward the laminar-layer line earlier in the flight, and more gradually, than those for station *H*, in spite of the much higher Reynolds number at station *M*. This drop might be ascribable to less effect of the turbulence promotion at station *M* than at station *H*, which in turn might result in earlier transition at *M*, but if so, the transition seems surprisingly gradual. Unfortunately, the test points previous to this possible transition seem also to be inconclusive. This is because in this region the difference between the corresponding points for the T' and the T_1 bases is small—of the same order of magnitude as the test accuracy.

3. The slope of the subsonic turbulent-correlation curve seems about right for the T' basis, but apparently a little too flat for the T_1 basis. The coefficient 0.056 in Equation [4], which represents this curve, also seems about right for the T' basis, but a little too high for the T_1 basis, if the slope remains unchanged.

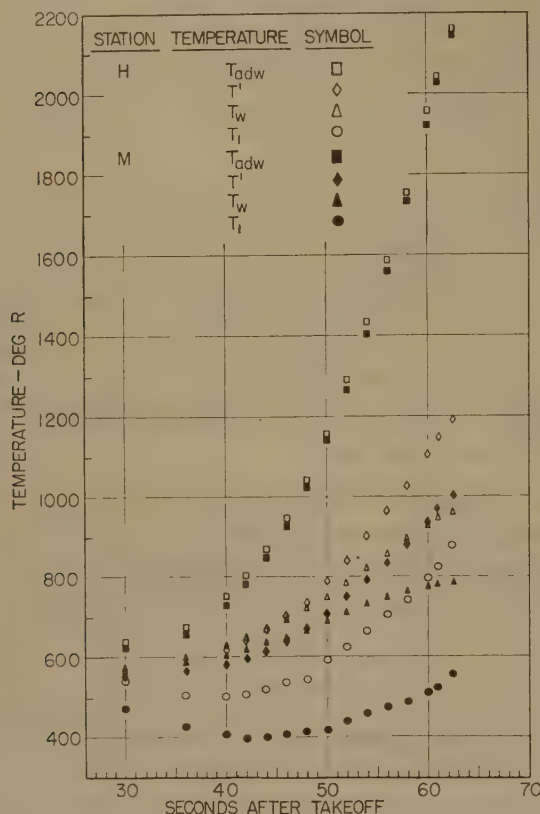


FIG. 15 COMPARISON OF ADIABATIC-WALL TEMPERATURE, TEMPERATURE USED FOR CORRELATION, SKIN TEMPERATURE, AND LOCAL STATIC TEMPERATURE VERSUS SECONDS AFTER TAKEOFF FOR STATIONS H AND M

4. The practical question remains as to what basis should be used for engineering estimates until more conclusive test results become available. In the authors' judgment, the T' basis is the more probable of the two. It seems safe to consider the T_1 basis as one limit to the range of uncertainty of the β -values.

5. The surface temperature T_w was also considered as a possible basis of correlation. It was found to give about the same correlation as T' , since it just so happened in this particular flight that T_w lay fairly close to T' for the test points most significant for the comparison of the alternative temperature bases. This is indicated in Fig. 15.

The Eber Correlation. The empirical correlation proposed by Eber (1) deserves comment, since, heretofore, it has been widely used. When expressed for a cone in terms of h_x instead of h_m , it becomes

$$\frac{h_x x}{k_{adw}} = 0.00974 \left(\frac{\rho_1 V_1 x}{\mu_{adw}} \right)^{0.82} \dots \dots \dots [9]$$

In comparing this relation with the present test results, it is helpful to consider first only those test points which seem, from Fig. 7, to indicate clearly a laminar-layer condition, namely, the points for station A, C, and G for 50 to 60 sec, inclusive. These points are compared with Eber's correlation in Fig. 9. Not only is the line of Eber's correlation too high, but its slope is clearly much steeper than the trend of these test points.

For the turbulent layer it seems appropriate, judging from Fig. 7, to ignore test points which clearly seem to be for transition or laminar layer, and therefore to consider only the test points for station H, K, and M, and the 30 to 40-sec points for stations A, C, and G. These are compared with Eber's correlation in Fig. 10. It is evident that the test points for stations H and the selected points for stations A, C, and G are consistently above the Eber line, by the order of 40 per cent, while the points for stations K and M also start above the Eber line, but decrease more rapidly with Reynolds number, probably thus indicating gradual transition toward the laminar condition.

The test results here reported appear to satisfy the limited objectives listed in the introduction of this paper.

Need remains, however, for further research to provide more accurate test data with turbulent layers, and to establish experimentally the criteria for transition, including the effect thereon of pressure gradient which was believed negligible in the tests here reported. For laminar layers, the authors know of no reason to doubt the validity of the theoretical correlation even though the present confirmation was limited in accuracy.

ACKNOWLEDGMENTS

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Calculations of the correlations of results from the temperature data, Miss Audrey W. Schmidt.

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Meteorological data, U. S. Air Forces, Air Weather Officer, Capt. J. R. Smith.

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Discussion

H. A. JOHNSON.⁵ It is mentioned in the paper that skin temperatures have been measured on V-2 rockets on occasions other than the particular flight reported in the paper. The results of these other measurements would be of considerable interest, even though the accuracy may be limited. It is hoped that the authors can supply these results in their closure.

AUTHORS' CLOSURE

In accordance with Professor Johnson's suggestions, additional figures (Figs. 16-18), are supplied with this closure to provide the desired data on other rockets and to facilitate interpretations of their significance.

Figs. 16 and 17 show the results of correlation of skin-temperature data from these other rockets on the basis of intermediate temperature T' , and on the basis of local static temperature T_1 (just outside the boundary layer), respectively. Information on these rockets is as follows:

V-2 No. 21: Temperatures were measured at two stations on the nose cone, one on a 0.091-in-thick aluminum skin section, the other on a 0.102-in-thick steel skin section, at 18 in. and 32 in. from the apex, respectively. (24)⁶

⁵ General Electric Company, Schenectady, N. Y.

⁶ Bibliography at end of closure.

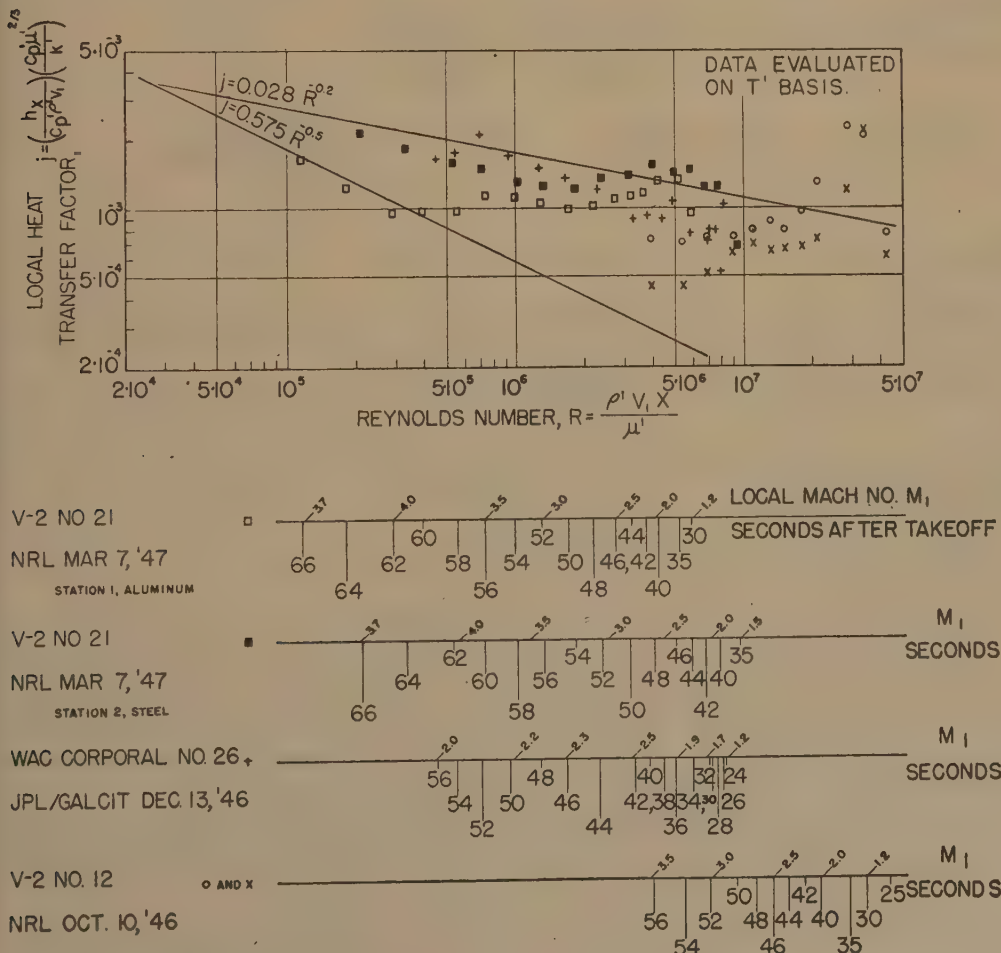


FIG. 16 LOCAL HEAT-TRANSFER FACTOR VERSUS REYNOLDS NUMBER FOR OTHER ROCKETS; DATA EVALUATED ON T' BASIS

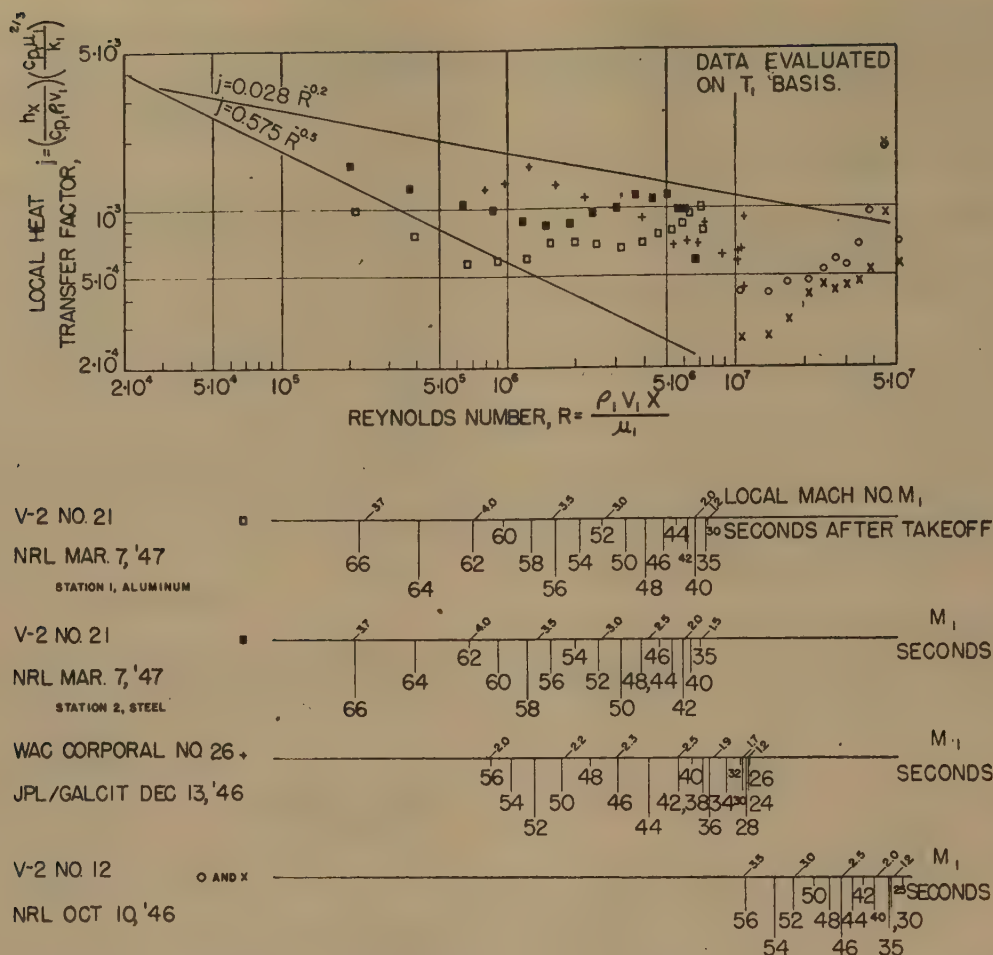


FIG. 17 LOCAL HEAT-TRANSFER FACTOR VERSUS REYNOLDS NUMBER FOR OTHER ROCKETS; DATA EVALUATED ON T_1 BASIS

V-2 No. 12: Temperature was measured at one station where the steel wall was 0.020 in. thick. This station was 109 in. from the apex and had two different types of temperature-sensing elements. For details see reference (10) of the paper and (24) of this closure.

Wac Corporal No. 26: Temperature was measured at one station on the 0.076-in.-thick aluminum nose cone 30 in. aft of the apex. For details see reference (25) of this closure.

It seems evident from comparisons between Fig. 16 and 17 herewith, and of these in turn with Fig. 7 and Fig. 8 of the paper, that the T' basis gives a more consistent correlation than the T_1 basis, particularly for points which seem to represent turbulent-layer conditions. Comparison of the two sets of test points for V-2 No. 21 provide particularly significant evidence, since station 2 was somewhat aft of a joint on the skin which was likely to promote turbulence, whereas station 1 was preceded by no appreciable roughness (except that the very tip of the nose was cut off blunt). Accordingly, where they differ from each other, the points for station 2 and the points for station 1 might be expected to follow the correlation curves for turbulent layer, and laminar layer, respectively. In Fig. 16, this expectation is confirmed fairly well, whereas Fig. 17 shows seemingly erratic deviations between the two sets of points.

Furthermore, the range of scatter of all the test points in Fig. 16 seems less than the scatter in Fig. 17, and the points average somewhat higher and hence closer to the turbulent-layer test-point results in Fig. 7 and Fig. 8 of the paper. The

scatter which is present even in Fig. 16, and which seems worst for V-2 No. 12, probably is not unreasonable in consideration of the limitations of test accuracy.

Additional comment seems warranted also for the wall temperature T_w basis. Fig. 18, herewith, shows the test results on this basis for V-2 No. 19 and V-2 No. 27. The results for the other rockets would show little or no significant difference between the T_w basis and the T' basis, so no separate plot of them on the T_w basis is presented.

Comparison of Fig. 7 and Fig. 8 of the paper, with Fig. 18 seems to warrant the following remarks:

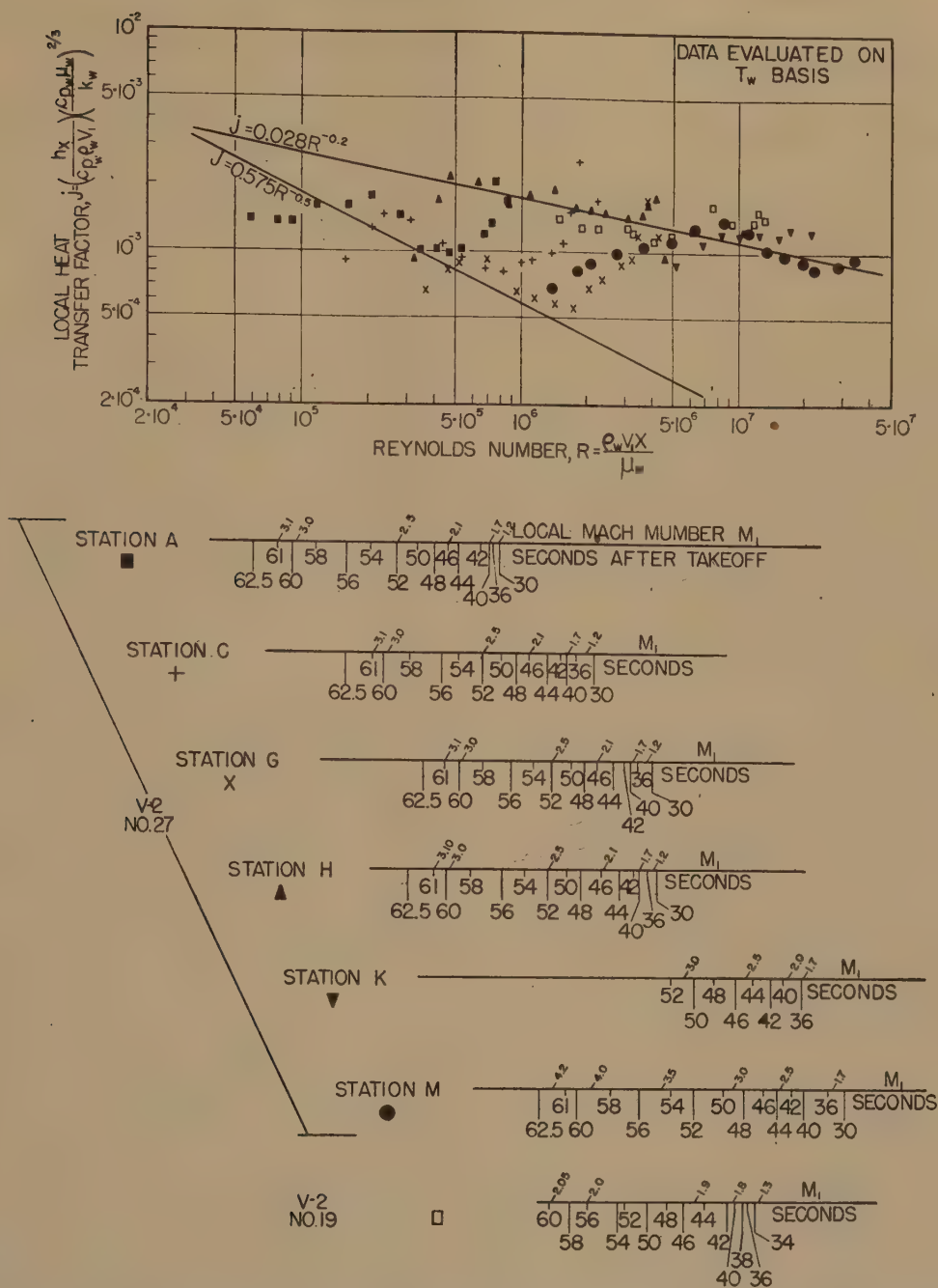
1 The differences between the results on the T_w basis and those on the T' basis are not large enough to be conclusive, but some of them seem to have some significance, as will be explained.

2 Those points for station M which seem to represent conditions of boundary-layer transition show a distinctly steeper slope on the T_w basis, and therefore become more consistent with the corresponding test points for stations A , C , and G .

3 Many of the test points for station H , which are appreciably above the upper correlation line on the T' basis, are lower on the T_w basis so that the trend for all the station H -points approximates that correlation line quite well.

There are also some theoretical grounds (26) which seem to favor the T_w basis for the correlations for the turbulent-layer conditions.

On the whole, therefore, the limited evidence now available

FIG. 18 LOCAL HEAT-TRANSFER FACTOR VERSUS REYNOLDS NUMBER FOR V-2 ROCKETS 27 AND 19; DATA EVALUATED ON T_w BASIS

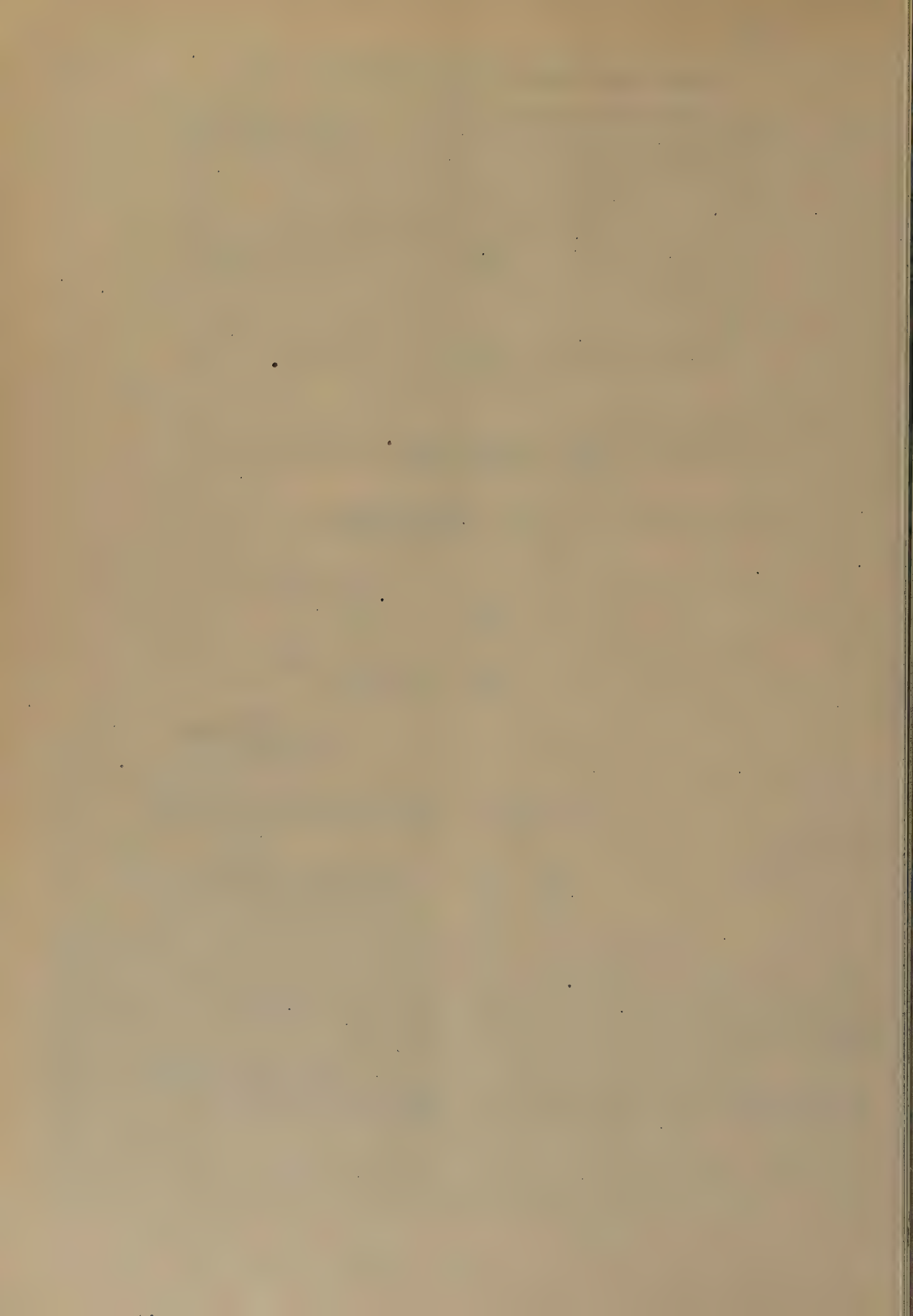
seems slightly—by no means conclusively—in favor of the T_w basis in preference to the T' basis for the turbulent-layer heat-transfer correlation. Both the T_w basis and the T' basis seem preferable to the T_1 basis.

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Correlation of Performance Characteristics of Domestic-Stoker Coals With Their Chemical and Petrographic Composition¹

By R. J. HELFINSTINE² AND G. H. CADY,³ URBANA, ILL.

There is a definite need for information about the value of chemical and petrographic analyses in predicting the behavior of stoker coals in domestic stokers. In order to obtain this information in our Illinois studies, performance characteristics of a large number of coals were determined and correlated with the various items of the proximate and ultimate analyses, heating value, varieties of sulphur, ash analysis, ash-fusion temperatures, free-swelling index, petrographic analysis, and Gieseler plasticity. A brief report on the results, including some graphs of the more useful correlations, are given.

INTRODUCTION

ONE of the most urgent needs in the field of coal combustion is the ability to predict the performance of a coal from knowledge gained from small-scale tests. Numerous types of analyses and tests are conducted on coal, including the proximate and ultimate analyses, heating value, varieties of sulphur, ash analysis, ash-fusion temperatures, free-swelling index, petrographic analysis, and Gieseler plasticity. The Illinois State Geological Survey has made an intensive study of the value of these tests for judging the performance characteristics of the coal as actually utilized. To date, three reports (1, 2, 3),⁴ have been issued, and another is being prepared. The present paper briefly summarizes some of the results found in the studies.

EQUIPMENT USED FOR TESTS

The equipment used for the tests is described in some detail in the publications mentioned. Briefly, it consisted of a domestic stoker, cast-iron boiler, and water-cooled heat exchanger, which were operated as a forced-circulation hot-water system. The entire unit was mounted on scales.

Instruments for recording the performance of the coal included a hot-water meter to indicate the quantity of water flowing through the boiler; a two-pen, mercury-actuated thermometer to record the temperatures of the water entering and leaving the boiler; a chemical-type meter to record the percentage of CO₂ in the stack gases; a pressure gage to record the static pressure in the stoker air duct; and a multipoint potentiometer to record the temperatures in the stack and room, and the opacity of the stack gases. A 16-mm motion-picture camera was available for taking pictures of either the fuel bed or the scale dial.

¹ Published by permission of the Chief, Illinois State Geological Survey.

² Mechanical Engineer, Illinois State Geological Survey.

³ Senior Geologist and Head of Coal Division, Illinois State Geological Survey.

⁴ Numbers in parentheses refer to the Bibliography at the end of the paper.

Presented at the ASME-AIME Coal Conference, White Sulphur Springs, West Va., November 3-4, 1948.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society. Paper No. 48—FU-1.

TEST PROCEDURE

The tests described in this report can be conveniently divided into three phases. For the first phase, a load of 5 or 6 tons of unwashed coal (usually screenings) was obtained from fourteen Illinois mines. These coals were all crushed and screened to a size of 1/2 in. (square hole) by 8 mesh. Combustion tests were made on a part of this raw screened coal. Another portion was passed over a concentrating table to reduce the ash to what was considered to be commercially reasonable, and combustion tests were made on this upgraded coal. The ash content of the remaining coal was reduced to a still lower figure by passing it over the washing table with greater reject, or by standard float-sink procedure. This cleaned fraction was then tested in the stoker-boiler unit. Thus three coals with varying ash content were tested from each mine.

For the next phase reported herein, coals were obtained from four Arkansas mines and tested as received and after passing over the concentrating table with a normal reject. One Illinois coal was tested in the "washed" condition only.

The last phase included tests on commercially prepared stoker coals from Illinois and several other states, and from numerous coal seams. If feasible, they were obtained directly from the mine; if not, they were obtained from a retail coal dealer. There is a possibility of improper identification of the coals obtained in this latter manner, but all reasonable precautions were taken. Table 1 gives information about the source of coals tested in the second and third phase. A previous report (2) gives similar information about the first phase.

TABLE 1 SOURCE OF COALS FOR SECOND AND THIRD PHASE TESTS

State	County	Seam
Illinois.....	Douglas	No. 7
Illinois.....	Grundy	No. 2
Illinois.....	Franklin	No. 6
Illinois.....	Fulton	No. 5
Illinois.....	Knox	No. 6
Illinois.....	Grundy	No. 7
Illinois.....	Christian	No. 6
Illinois.....	Saline	No. 5
Illinois.....	Saline	No. 6
Illinois.....	Randolph	No. 6
Illinois.....	Jackson	No. 6
Illinois.....	Bureau	No. 6
Illinois.....	St. Clair	No. 6
Arkansas.....	Sebastian	Hartshorne (2 samples)
Arkansas.....	Johnson	Hartshorne (Spadra)
Arkansas.....	Logan	Paris
Indiana.....	Vigo	No. 3
Indiana.....	Vigo	No. 4
Indiana.....	Gibson	No. 5
Indiana.....	Greene	No. 6
Indiana.....	Greene	No. 7
Indiana.....	Fountain	Minshall
Indiana.....	Clay	Brazil block
Kentucky.....	Harlan	Darby No. 5
Kentucky.....	Perry	Hazard No. 4
Kentucky.....	Hopkins	No. 11
Kentucky.....	Hopkins	No. 6
Kentucky.....	Letcher	Elkhorn
Missouri.....	Randolph	Bevier
Missouri.....	Macon	Mulky
West Virginia.....	Marion	Pittsburgh
West Virginia.....	Wyoming	Beckley
West Virginia.....	McDowell	Pocahontas No. 4
West Virginia.....	Webster	Sewell
Wyoming.....	Sheridan	Monarch

The sample obtained is not considered as representative of the seam or even of the unnamed mine. Obviously, it was impractical, and of no particular importance, to obtain a representative sample for this study. The coals were obtained from a large number of sources in order to have coals of widely varying characteristics.

Considerable care was used in obtaining a representative sample of the coals as burned for chemical and petrographic analyses. Standard or proposed methods of the American Society for Testing Materials were used wherever they were applicable.

The petrographic composition was obtained for only the Illinois coals in the first phase. Those listed in this report were made by microscopic examination of closely sized fractions. The analyst considered all vitrain bands less than $\frac{1}{10}$ in. thick as part of the clarain.

The combustion-testing schedule included five stoker-operation rates varying from hold-fire to continuous. About 300 lb of coal were burned during each of the operation rates, except hold-fire. No attention was given to the fire during each of these tests with a fixed operation rate.

COMBUSTION-RATING CRITERIA

There are many factors which govern the suitability of coals for domestic stokers, including the following:

- 1 Amount of heat obtained per dollar.
- 2 Attention required by heating plant.
- 3 Ability to maintain the desired temperature in the home.
- 4 Smoke emitted.
- 5 Ability to maintain fire at low rates of operation.
- 6 Cleanliness.
- 7 Appearance of the fuel bed and fire.
- 8 Odors given off by clinkers during their removal.
- 9 Quietness of operation.
- 10 Appearance of the coal.

The foregoing factors vary in relative importance, depending upon the heating system being used and also upon personal preferences of the operator.

Objective measures of all of these factors would be highly desirable. Unfortunately, only the cost of heat can be determined by generally recognized standard tests and this only under arbitrarily fixed conditions. While the cost of heat may be of major importance with large stoker-fired heating plants, it is considered to be of minor importance to many domestic-stoker owners.

Some objective criteria of the first five of the factors listed were devised and determined for all the coals tested. The chief criterion of the attention required is considered to be the quantity of ash which can be obtained from the proximate analysis. The percentage of ash per million Btu is a more precise measure. Other criteria used were the density and friability of the clinker, and the relative amount of ash removed in the form of clinkers. A subjective clinker rating was also made.

The ability of a heating plant to maintain the desired temperature in a house will depend upon many other factors besides the coal being burned. However, certain performance characteristics of the coal are thought to be influential and were determined for all coals tested. These were the uniformity of heat release, ratio of the minimum and average rates of heat release with continuous stoker operation, responsiveness of the fire after a prolonged hold-fire period, the "pickup" after a relatively short "off" period, and the rate of heat output after the stoker shutoff (called "overrun" in this paper).

Although no discussion of smoke and hold-fire ability is given in this paper, objective data were obtained. Motion pictures

were taken of the fuel bed at fixed intervals, and a considerable quantity of color film showing this action is available for study.

RESULTS

Heat Obtained. The amount of heat obtained from each coal tested was very nearly directly proportional to the heating value of the coal, on the as-fired basis, Fig. 1. The solid line in this diagram appears to be the best single line to represent the points shown. All points falling within the dotted lines are within 5 per cent of the value indicated by the solid line.

The heat obtained from the coals tested was also nearly directly proportional to the percentage of carbon as given in the ultimate analysis, Fig. 2. Most of the points are within 5 per cent of the indicated line.

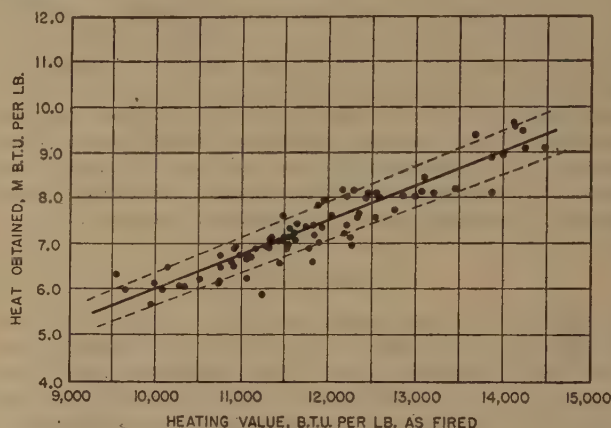


FIG. 1 RELATIONSHIP OF HEAT OBTAINED PER POUND OF COAL TO HEATING VALUE, ON THE AS-FIRED BASIS

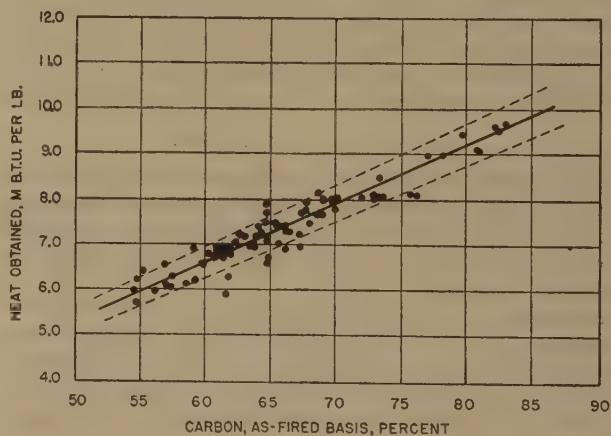


FIG. 2 RELATIONSHIP OF HEAT OBTAINED PER POUND OF COAL TO PERCENTAGE OF CARBON, ON THE AS-FIRED BASIS

A fair correlation between the ash and heat obtained was found, Fig. 3. This relationship is of particular interest because the percentage of ash is also a criterion of the attention required. Ash is not the sole diluent in coal, and as expected, a better correlation with the sum of noncombustibles is shown in Fig. 4.

Numerous other items, and combination of items, gave good correlation with the heat obtained. However, none appears to be as useful as the relationships mentioned, particularly the one with heating value.

Uniformity of Rate of Heat Release. None of the items included in the chemical or petrographic analyses gave a good correlation with the uniformity of rate of heat release. One of the better

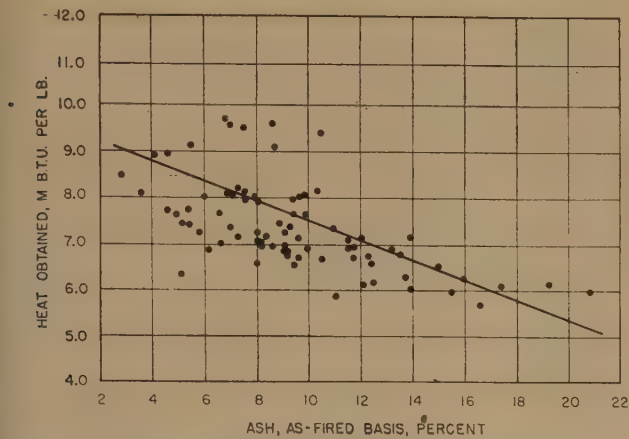


FIG. 3 RELATIONSHIP OF HEAT OBTAINED PER POUND OF COAL TO PERCENTAGE OF ASH, ON THE AS-FIRED BASIS

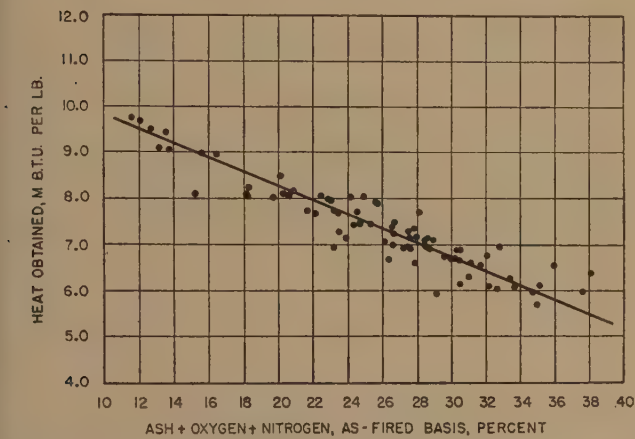


FIG. 4 RELATIONSHIP OF HEAT OBTAINED PER POUND OF COAL TO SUM OF PERCENTAGES OF ASH, OXYGEN, AND NITROGEN, ON THE AS-FIRED BASIS

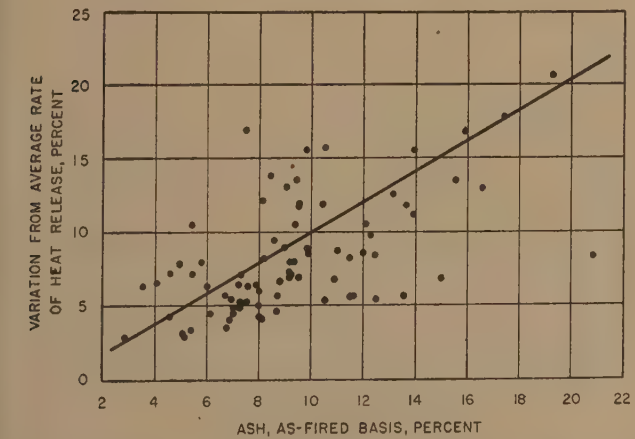


FIG. 5 RELATIONSHIP OF UNIFORMITY OF RATE OF HEAT RELEASE TO PERCENTAGE OF ASH, ON THE AS-FIRED BASIS

correlations appeared to be with the percentage of ash, Fig. 5. The performance of several coals failed to follow the general trend of increasing variability in rate of heat release with an increase in percentage of ash. One example of nonconformity was the Pittsburgh seam coal from West Virginia which had about 17 per cent variation, although its ash content was only

7.5 per cent. An unprepared coal from Wabash County, Illinois, burned with only 8.3 per cent variation, yet its ash content was 20.8 per cent.

Fig. 6 shows the correlation of the uniformity of rate of heat release and the maximum fluidity, in dial divisions per minute, as determined by the Gieseler test. Although the degree of correlation is probably poorer than that exhibited with ash, the Pittsburgh seam coal (abscissa of 6000) conforms to the general relationship shown. The Wabash County (Illinois) coal is not shown because no plastic properties were indicated by the Gieseler procedure.

The coking characteristics of coals are usually thought to exert considerable influence upon the uniformity of combustion in

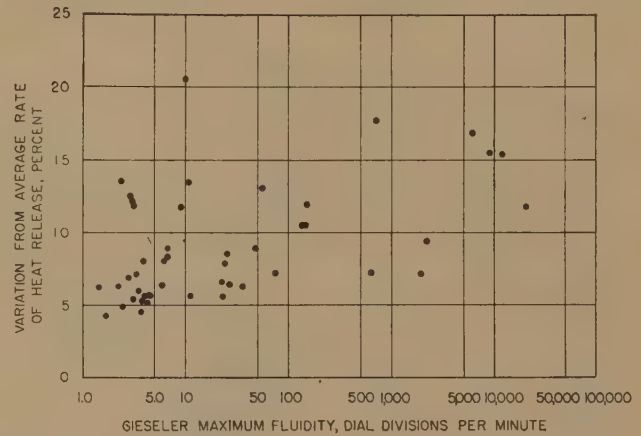


FIG. 6 RELATIONSHIP OF UNIFORMITY OF RATE OF HEAT RELEASE TO MAXIMUM FLUIDITY, IN DIAL DIVISIONS PER MINUTE, AS DETERMINED BY THE GIESELER TEST

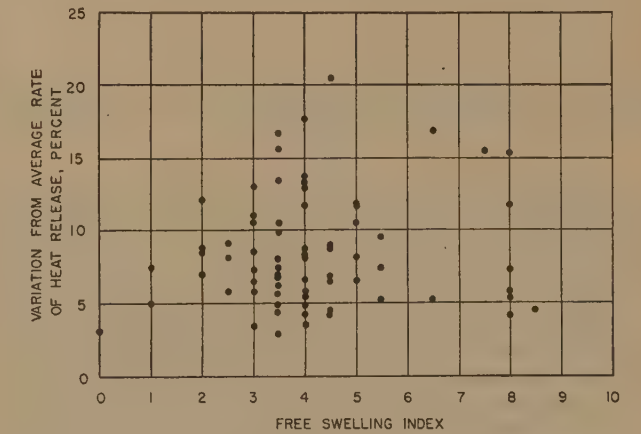


FIG. 7 RELATIONSHIP OF UNIFORMITY OF RATE OF HEAT RELEASE TO FREE-SWELLING INDEX

domestic stokers. One of the best indicators of coking tendency is thought to be the free-swelling index. However, Fig. 7 shows that the correlation is very low. Five coals with swelling indexes of 8 or greater burned quite uniformly.

The banded ingredients were not determined for all of the coals burned because of lack of suitably trained analysts. Fig. 8 shows the relationship between the uniformity of rate of heat release and vitrain for all coals analyzed. The general tendency seems to be for more uniform combustion with the higher-vitain coals. This may be a reflection of ash content, because the coals with more vitrain usually had lower percentages of ash.

As stated, the percentage of vitrain reported did not include

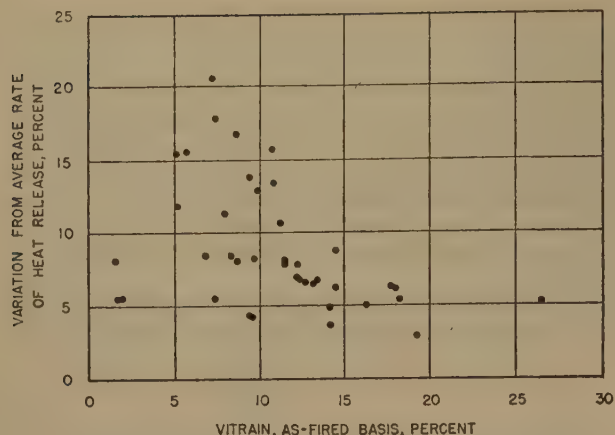


FIG. 8 RELATIONSHIP OF UNIFORMITY OF RATE OF HEAT RELEASE TO PERCENTAGE OF VITRAIN

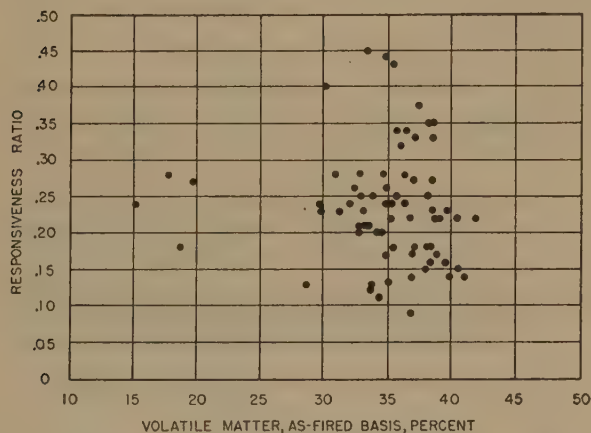


FIG. 9 RELATIONSHIP OF RESPONSIVENESS OF FIRE AFTER PROLONGED HOLD-FIRE PERIOD TO PERCENTAGE OF VOLATILE MATTER ON THE AS-FIRED BASIS

any bands less than 0.1 in. in thickness. The coals with "bright" clarain might have considerably more vitrain than reported, if no limit were placed upon the thickness of the bands. Therefore the correlations with the banded ingredients are considered to be incomplete. It is hoped that further investigations can be made.

Responsiveness of Fire to Heat Demands. No useful correlation was found between any of the chemical or petrographic tests and the responsiveness of the fire after a prolonged hold-fire period. The relationship of responsiveness and volatile matter is shown in Fig. 9. The widely scattered points were typical of all plots made.

The correlation of the responsiveness of the fire after a 45-min off period (called pickup) with any chemical or petrographic item was likewise poor. Fig. 10 shows a typical example.

The rate of heat release after the stoker shutoff did not exhibit a useful correlation with any of the chemical or petrographic tests made. A slight tendency for an increase in overrun to accompany an increase in the Gieseler softening temperature is shown in Fig. 11.

Clinker Characteristics. One of the primary requisites for a stoker coal is the ability to remove the ash in the form of clinker. This characteristic was indicated as the ratio of the weight of clinker removed during the tests and the total weight of ash formed. The ratio was always less than unity because the tests were started with no ash in the combustion chamber. A bed of

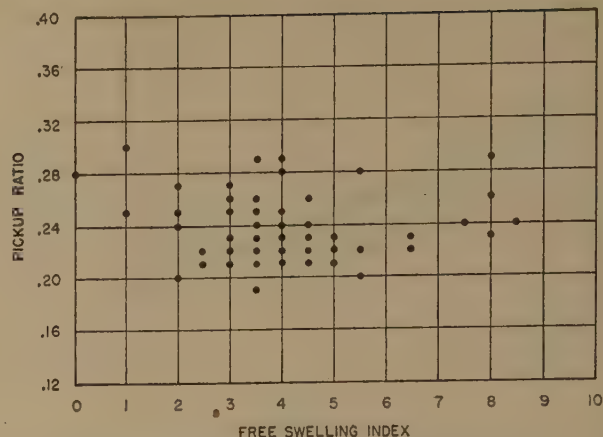


FIG. 10 RELATIONSHIP OF PICKUP RATE AFTER A 45-MIN "OFF" PERIOD TO FREE-SWELLING INDEX

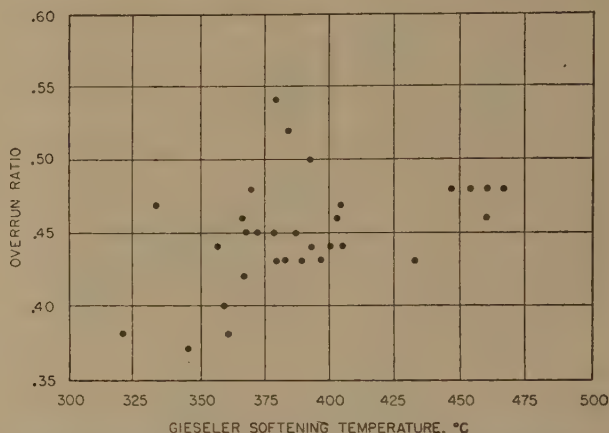


FIG. 11 RELATIONSHIP OF OVERRUN RATE AFTER 15-MIN "ON" PERIOD TO GIESELER SOFTENING TEMPERATURE

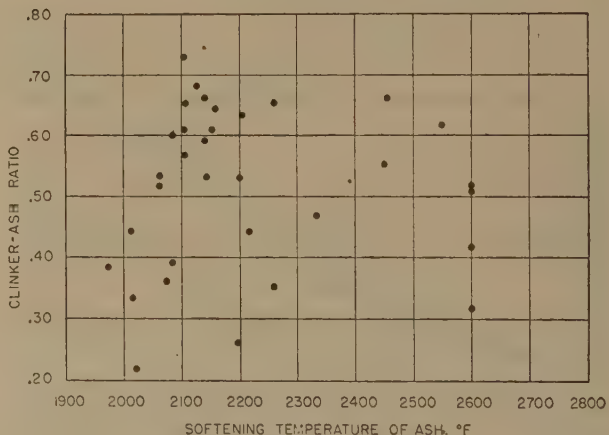


FIG. 12 RELATIONSHIP OF CLINKER-ASH RATIO TO SOFTENING TEMPERATURE OF ASH

loose ash had to be accumulated before clinker was formed. Less than unit values do not mean that all the ash could not be removed in the form of clinker with normal home operation.

Fig. 12 shows that the correlation of the softening temperature of the ash and the clinker-ash ratio is very poor. The clinker-ash ratio for several coals in the 2000 F range were lower than some in the 2600 F range.

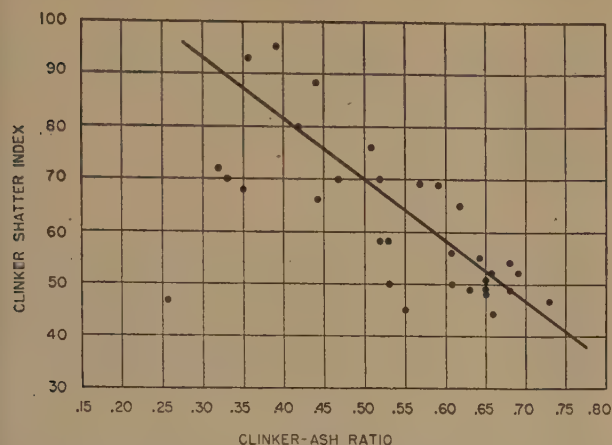


FIG. 13 RELATIONSHIP OF CLINKER SHATTER INDEX TO CLINKER-ASH RATIO

There appears to be a slight correlation between the clinker-ash ratio and the clinker-shatter index,⁵ Fig. 13.

CONCLUSIONS

The heat obtained from the coals tested exhibited a good correlation with several items given by chemical analyses. The most

⁵ The percentage of clinker by weight that broke into pieces weighing less than $\frac{1}{4}$ lb after two drops from a height of 6 ft.

useful relationship was with the heating value, on the as-fired basis.

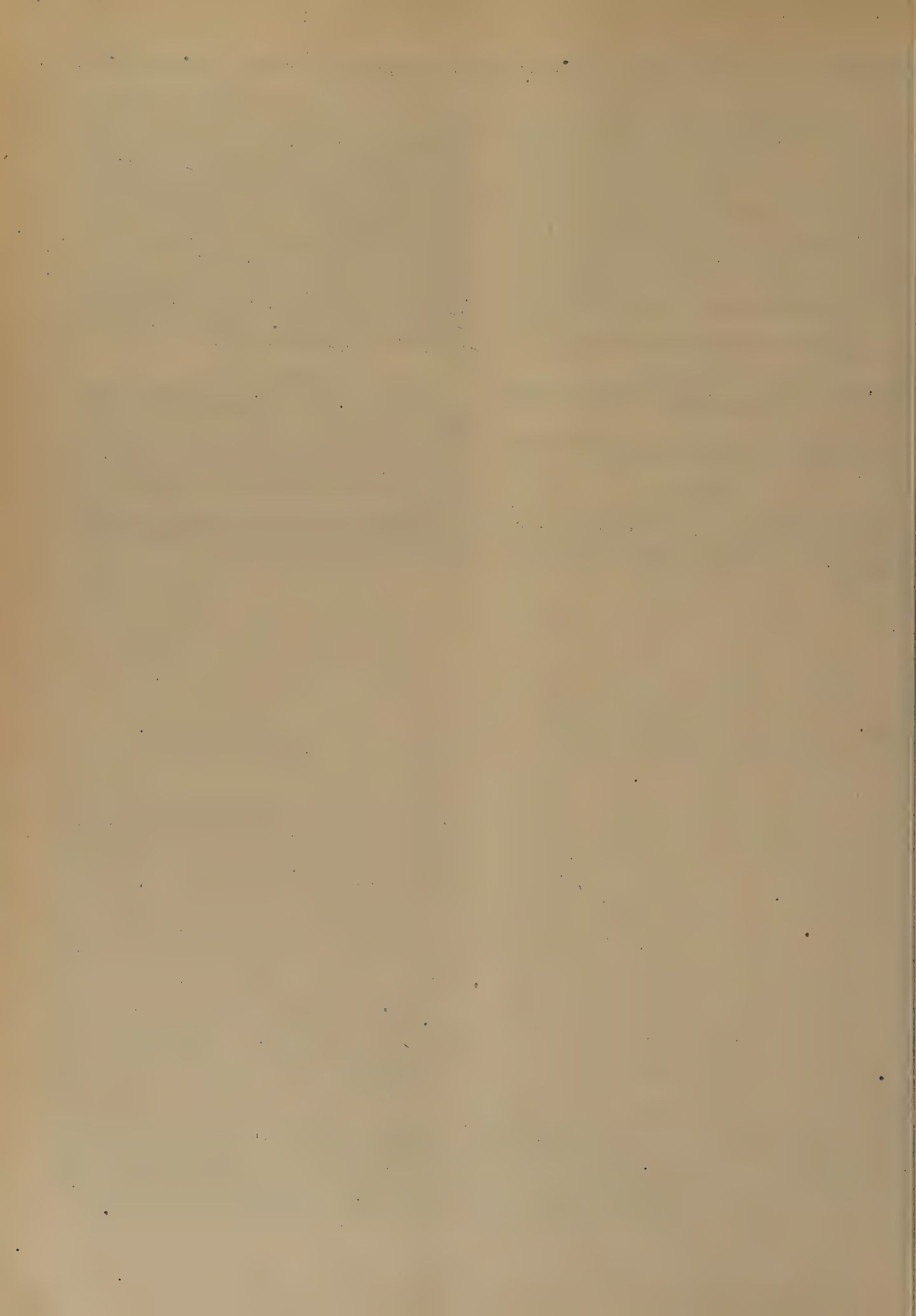
No single item, or combination of items, in the chemical or petrographic analyses gave good correlations with any of the other measured combustion characteristics. The percentage of ash, the fluidity of the coal as determined by the Gieseler test, and the percentage of vitrain exhibited a fair correlation with the uniformity of combustion.

The clinker-ash ratio appeared to have a fair correlation with the shattering characteristics of the clinker.

Sufficient space is not available to show additional graphs or to discuss the lack of correlation exhibited by more than one hundred combinations of chemical and combustion characteristics that were studied. Little or no useful correlation was exhibited by any except those mentioned in this paper.

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An Evaluation of the Performance of Thirty-Three Residential-Stoker Coals

By J. B. PURDY¹ AND H. W. NELSON,² COLUMBUS, OHIO

This paper is based upon experience gained from evaluation tests, sponsored by individual coal producers during the past 12 years, to determine the suitability of various coals for use in residential underfeed stokers of the clinkering type. The conclusions reached through the evaluation tests are in general agreement with the marketing experiences of the producers in the field. Comparisons are also made between the characteristics of the coals as determined by the evaluation tests and data determined by standard laboratory-test methods. Correlation in this regard was not good.

THE great majority of stokers used in residential heating installations are of the clinkering type. Because of inherent characteristics of the underfeed combustion process as it occurs in these small stokers, and because of the wide range of operating conditions encountered in residential heating schedules, not all bituminous coals can be used successfully in residential underfeed units. The problem of providing consumer satisfaction has been attacked by the joint efforts of the stoker manufacturers and the coal industry; the stoker manufacturers have been concerned with improving their product to permit the use of a wider range of coals, and coal operators have made every effort to market selected and better-prepared coals. In view of the wide range of physical characteristics and chemical properties of bituminous coals, the problems are real and are receiving increased attention.

Some 12 years ago a laboratory method was developed at Battelle Memorial Institute for the relative evaluation of coals for use in residential underfeed stokers. The method was developed in conjunction with work being done at that time for Bituminous Coal Research, Inc., and for individual coal producers, to obtain information on the characteristics of coal that determined the successful performance of the small stoker. Details of the equipment and test procedure were given by R. A. Sherman³ in a paper presented at the University of Illinois Fifth Short Course in Coal Utilization.

During the past several years the evaluation of over forty different coals has been determined by investigations sponsored at Battelle by coal operators in the eastern producing districts. The purpose of this paper is to compare the results and conclusions derived from the various tests with marketing experiences of the operators, and to discuss briefly the relationship of test results to standard laboratory determinations.

¹ Research Engineer, Fuels Division, Battelle Memorial Institute. Jun. ASME.

² Supervisor, Fuels Research, Battelle Memorial Institute.

³ "The Evaluation of Coal for Use in Domestic Stokers," by R. A. Sherman, Circular No. 39, University of Illinois, Engineering Experiment Station, 1939.

Presented at the ASME-AIME Joint Fuels Conference, White Sulphur Springs, West Va., November 3-4, 1948.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society. Paper No. 48-FU-1.

EQUIPMENT USED IN TESTS

Fig. 1 shows schematically the equipment used in the evaluation tests. The stoker is a commercial model of conventional design, having a nominal feed rate of 30 lb per hr. A special laboratory furnace is used, which is sealed tightly to prevent the entrance of secondary air. The complete stoker-furnace assembly is mounted on platform scales, which permits the measurement of combustible weight losses occurring during the test. The furnace stack, in which are located a thermocouple, flue-gas sampling tube, and smoke meter, is connected to the laboratory exhaust breeching through a frictionless connection.

Through use of an auxiliary blower system, provision is made for the measurement of the air delivered by the stoker fan to the wind box. A water seal in the ductwork eliminates interference with the process of weighing the stoker-furnace unit. A barometric damper is located in the furnace stack to assist in maintaining a constant draft. The damper is adjusted to give a draft of 0.05 in. of water during "off" periods of the stoker.

Smoke concentrations are measured by means of a smoke-density meter. The meter consists of a light source and photoelectric cell between which a sample of the flue gases is drawn at a constant rate. Variations in the absorption of light by smoke varies the output of the photoelectric cell, and the voltage is recorded continuously on a recording potentiometer.

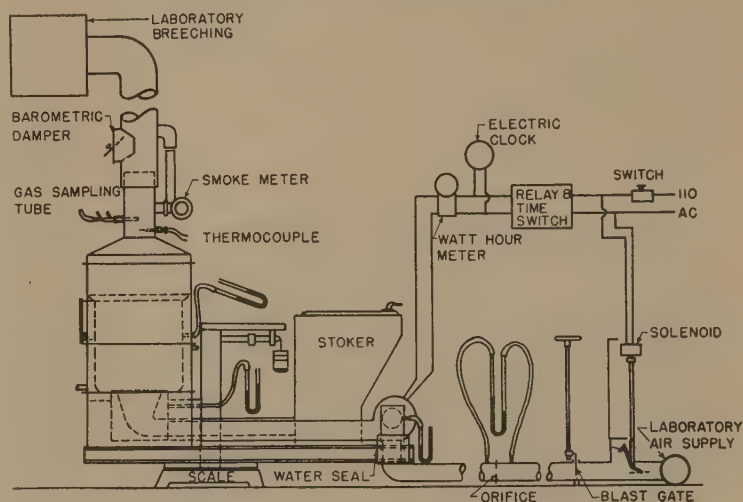


FIG. 1 DIAGRAM OF TEST EQUIPMENT

Appropriate instruments are provided for control of hold-fire operation, for recording stoker running time and power consumption, and for measurement of overfire and wind-box pressures. Flue-gas analyses are made at appropriate intervals throughout the test.

TEST PROCEDURE

At the start of the test the hopper is filled with the coal to be tested, and the stoker is run until the retort is filled and coal is level with the top of the tuyères. The feed of coal is then stopped, and 1 lb of charcoal is placed on top of the coal. Just prior

to lighting the fire, $1\frac{1}{2}$ pt of kerosene is poured over the charcoal. The charcoal is then ignited, and for a period of 5 minutes no coal is fed, but the stoker fan is permitted to run. At the end of 5 min the coal feed is turned on and allowed to run for the balance of the hour.

The air control on the stoker is set to furnish air at a rate 30 per cent in excess of that theoretically required by the rate at which the coal under test is being fed. Any correction necessary to obtain this rate of air delivery is made during the first hour by means of the stoker air control. No further adjustment of the air is made throughout the test.

The schedule of operation for an evaluation test is designed to simulate the operation of a stoker under conditions of varying heat demand, as would be expected during a typical heating season. The test is normally of 5 days' duration. During the first and second days, operation consists of alternate periods of continuous and hold-fire operation; alternate periods of intermittent and hold-fire operation occupy the third day, and the fourth day consists wholly of hold-fire operation. Intermittent operation is scheduled for the whole of the fifth day.

For the parts of the schedule indicating continuous operation, the stoker is started and stopped manually. For hold-fire operation, the stoker is started once each hour by a timer control and is stopped by a potentiometer controller when the temperature of the flue gases has reached 20 millivolts, or approximately 900 F. During intermittent operation, the stoker is started manually at intervals of 20 min, the duration of each period of operation being determined by the time required to burn one third of the quantity of coal necessary to release 187,500 Btu per hr. The stoker is stopped when the required weight loss is attained, and remains idle until the start of the next 20-min period.

As a test is always started with a completely clean hearth, the fuel bed is not disturbed until the evening of the fourth day, when a probe is made with clinker tongs to remove any clinker from the fuel bed. At the conclusion of the test the fire is quenched with water and the fuel bed is removed from the furnace. Pieces of clinker larger than 3 in. in size, and all coke retained on a 1-in. round-hole screen, are separated from the fuel bed and weighed.

BASES FOR EVALUATION

In evaluating a stoker coal, certain tangible factors must be selected on which a judgment of the coal may be based. These factors are chosen because of their importance in determining the degree of satisfaction that the consumer will obtain in using the coal, from the standpoint of manual attention, uniformity of heating, comfort, cleanliness, and cost. The main factors used as a basis for evaluation in this test are as follows: (1) Initial ignition; (2) clinker formation; (3) coke formation; (4) rate of fuel consumption during hold-fire operation; (5) resistance of fuel bed to flow of air; (6) uniformity of burning during intermittent operation; (7) smoke concentration.

COALS TESTED

Over forty tests have been conducted, and their results evaluated by the method described. A few of these were short-term tests, consisting of only the first and last days of operation. Of the 5-day tests, the results of thirty-three are included in the present study. With one exception, the coals were all high-volatile, and were largely from the eastern producing districts.

Table 1 presents the laboratory-determined characteristics of the coals used in these tests. In addition to the proximate analysis, the sulphur, calorific value, ash-softening temperature, and free-swelling index are given. The nominal size of the coal is indicated.

RESULTS OF TESTS

At the conclusion of each test, average values are determined for components of the flue-gas analysis, pressure measurements, smoke concentrations during "off" periods, and length of hold-fire running periods. These and other directly determined values are then subjected to analysis.

Table 2 lists values of many of these data for the coals tested. When reporting the results of tests, in addition to listing data shown in the table, plots of some groups of data are made for purposes of showing their degree of variability during the course of the test. Wide variations in the length of operating times during intermittent operation, for example, are indicative of excessive coke-tree formation, dirty fuel bed, or general lack of response to heat-demand operation, but average values alone do not indicate this variability. Supplementary data and information, including photographs and visual observation of the fuel bed, strength of the clinker formed, and the height and type of coke trees, are not subject to numerical evaluation but are useful in making an over-all interpretation of results.

The ratings of initial ignition shown in Table 2 represent approximate evaluations of the curve of flue-gas temperatures recorded during the first hour of the test. The word descriptions shown furnish an initial indication of the ignitibility and coking characteristics of the coal under test. The curves for initial ignition are discussed more fully in a later section of the paper.

For the coals tested, the range of values for the drop in pressure through the fuel bed was from 0.52 to 1.54 in. of water. The values are indicative of the general condition of the fuel bed, and any coal that has a fuel-bed resistance greater than 1.5 in. of water is considered to give difficulty in burning. Of the coals tested, only one showed values above this maximum.

As previously stated, during hold-fire operation the stoker is started once each hour and allowed to run until a stack temperature of 900 F is reached. Under this procedure the time of operation is governed by characteristics of the coal. A cutoff temperature of 900 F may be too high, as many coals will be capable of holding the fire at a lower temperature; however, time would not permit trial of minimum settings, and the results may be compared on a relative basis. For purposes of evaluation, coals having time values below 5 min per hr are considered satisfactory. The range of values shown in Table 2 was from 4.1 to 7.1 min per hr. Values of less than 5 min were determined for fourteen of the coals tested; and six required operating periods averaging over 6 min.

The values of smoke concentration were determined as the average density of smoke during the "off" periods of intermittent operation. Only coals showing average values above 20 on the logarithmic scale of densities would be apt to produce troublesome smoke during "off" periods, as combustion of volatile gases and dilution of smoke by air admitted over the fire would reduce substantially the smoke concentrations in a residential heating installation.

Values of the weight of coke remaining in the fuel bed at the end of a test give a numerical indication of the extent of coke formation and the depth of the fuel bed. Comparisons are significant only when all tests are run with closely similar percentages of excess air. The range of values found during the present tests was from 2.6 to 42.8 lb; eight of the coals produced 15 lb or more of coke.

The measure of clinker formation represented by data listed in Table 2 is the percentage of the total ash released during the test, which is formed into clinker. Percentage values of 20 per cent or more are considered to indicate satisfactory clinker formation.

TABLE 1 ANALYSES OF COALS TESTED

Test No.	Bed	State	Nominal size, in.	Proximate analysis, per cent					Sulphur, per cent	Calorific value, Btu/lb	Ash softening temp, F	Free-swelling index, number
				Moisture	Volatile matter	Fixed carbon	Ash	Total				
1	Wallins	Ky.	1-1/4X0	2.5	40.1	51.8	5.6	100.0	0.8	13,630	2660	—
2	Wallins	Ky.	1-1/4X1/4	2.8	40.0	52.4	4.8	100.0	0.8	13,700	2250	—
3	Upper Freeport	Pa.	3/4X1/4	1.1	33.8	56.0	9.1	100.0	1.6	13,800	2360	—
4	Pittsburgh	W. Va.	1X10 mesh	3.0	37.5	53.3	6.2	100.0	—	13,380	2380	4.5
5	No. 2 gas	W. Va.	1X10 mesh	0.9	34.6	61.2	3.3	100.0	0.8	14,760	2290	—
6	Taggart	Va.	3/4X0	—	—	—	3.1	—	—	—	2210	8.5
7	Taggart	Va.	3/4X0	—	—	—	4.5	—	—	—	2310	8.5
8	Chilton	W. Va.	3/4X3/8	1.1	31.9	58.5	8.5	100.0	—	—	2700	7.0
9	Sewell	W. Va.	3/4X1/4	0.2	23.2	72.6	4.0	100.0	—	15,140	2440	—
10	Pittsburgh	W. Va.	3/4X3/8	2.7	38.1	54.4	4.8	100.0	—	14,050	—	8.0
11	Elkhorn No. 3	Ky.	1X1/8	1.6	37.2	54.6	6.6	100.0	—	13,920	—	6.0
12	Elkhorn	Ky.	1X3/8	0.9	35.4	60.5	3.2	100.0	—	—	2630	6.0
13	Elkhorn	Ky.	3/8X1/8	1.6	32.9	59.8	5.7	100.0	—	—	2660	5.5
14	Elkhorn	Ky.	1X1/8	1.4	34.8	60.0	3.8	100.0	—	—	2630	6.0
15	Elkhorn	Ky.	1X50 mesh	1.3	34.1	59.5	5.1	100.0	—	—	2670	6.0
16	Mason	Ky.	1X1/4	0.6	37.8	56.4	5.2	100.0	1.6	14,160	2420	5.0
17	Mason	Ky.	1X1/4	1.7	38.3	58.3	3.7	100.0	—	14,089	2530	4.0
18	No. 11	Ky.	3/4X48 mesh	—	—	—	6.1	—	—	—	2200	4.0
19	High Splint	Ky.	1X5/16	0.7	39.0	52.5	7.8	100.0	1.0	13,400	2730	3.5
20	Blue Gem	Ky.	1-1/4X3/8	1.9	—	—	6.0	—	1.6	13,680	—	3.5
21	No. 2 gas	W. Va.	1X3/8	1.7	35.7	55.7	6.9	100.0	1.4	13,951	2680	8.5
22	(Unknown)	Ind.	3/4X3/8	7.5	41.0	44.0	7.5	100.0	3.0	12,397	2190	—
23	Hazard No. 4	Ky.	1X3/8	1.9	37.6	55.2	5.3	100.0	0.8	13,961	+2700	5.0
24	Hazard No. 4	Ky.	1X3/8	2.2	37.5	55.2	5.1	100.0	0.9	13,940	2410	4.5
25	Elkhorn No. 4	Ky.	1X3/8	1.1	38.5	55.8	4.8	100.0	1.0	14,970	2500	5.0
26	Harnshaw	W. Va.	1X3/8	0.8	32.3	63.8	3.3	100.0	0.7	14,690	2750	7.5
27	Rich Mountain	Tenn.	1-1/4X3/16	1.3	35.7	56.1	6.9	100.0	1.0	13,730	+2800	4.5
28	Shannon	Va.	1-1/4X1/4	1.9	33.6	53.4	11.1	100.0	1.1	12,283	2280	6.5
29	No. 2	Ky.	1X3/8	1.3	37.3	46.5	14.9	100.0	5.1	12,470	2310	3.5
30	No. 7	Ky.	1X1/4	6.0	38.0	51.4	4.6	100.0	1.8	12,780	2190	4.0
31	Powellton	W. Va.	1X3/8	1.8	35.5	57.1	5.6	100.0	1.1	14,330	2420	8.5
32	Hazard No. 4	Ky.	1X1/4	1.2	38.9	55.2	4.7	100.0	2.6	14,130	2340	5.0
33	Clintwood	Ky.	1X4 mesh	0.4	29.1	63.9	6.6	100.0	1.1	14,360	2700	8.0

TABLE 2 SUMMARY OF RESULTS FROM EVALUATION TESTS

Test No.	Bed	State	Initial ignition	Pressure drop through fuel bed, inches H ₂ O	Hold-fire time, minutes	Smoke concentration, average	Coke in fuel bed, end of test, lb	Clinker removed, per cent of ash released
1	Wallins	Ky.		1.16		11.8	4.1	16.8
2	Wallins	Ky.		0.86		13.6	2.6	27.2
3	Upper Freeport	Pa.		1.54	4.5	27.6	14.5	24.0
4	Pittsburgh	W. Va.		1.08	5.4	21.6	9.6	47.6
5	No. 2 Gas	W. Va.		0.84	4.1	15.6	16.5	27.9
6	Taggart	Va.	Satisfactory	0.76	4.3	12.0	4.3	13.4
7	Taggart	Va.	Very satisfactory	0.91	5.3	16.7	9.6	21.0
8	Chilton	W. Va.	Satisfactory	1.26	6.8	14.0	3.8	14.0
9	Sewell	W. Va.	Poor	1.19	4.2	7.5	16.1	25.3
10	Pittsburgh	W. Va.	Poor	0.79	5.4	33.4	4.9	16.3
11	Elkhorn No. 3	Ky.	Very satisfactory	1.01	5.8	14.7	10.8	28.4
12	Elkhorn	Ky.	Very satisfactory	0.52	4.6	14.7	8.4	12.5
13	Elkhorn	Ky.	Satisfactory	0.85	4.3	9.4	16.4	39.8
14	Elkhorn	Ky.	Satisfactory	0.65	4.2	10.9	7.8	26.2
15	Elkhorn	Ky.	Satisfactory	0.86	4.3	14.4	19.5	36.6
16	Mason	Ky.	Poor	1.05	5.3	15.8	9.1	37.1
17	Mason	Ky.	Very satisfactory	0.72	4.1	10.6	4.1	21.3
18	No. 11	Ky.	Satisfactory	0.97	4.3	17.9	8.4	31.0
19	High Splint	Ky.	Very satisfactory	0.77	4.8	5.0	3.6	17.8
20	Blue Gem	Ky.	Very satisfactory	0.77	4.9	12.8	13.7	53.3
21	No. 2 gas	W. Va.	Poor	0.81	6.0	19.0	19.0	16.2
22	(Unknown)	Ind.	Satisfactory	0.68	6.3	16.0	6.6	21.2
23	Hazard No. 4	Ky.	Satisfactory	1.09	5.4	12.0	7.2	23.5
24	Hazard No. 4	Ky.	Very satisfactory	0.97	4.5	16.0	7.4	14.9
25	Elkhorn No. 4	Ky.	Satisfactory	0.82	5.7	20.0	8.6	14.0
26	Harnshaw	W. Va.	Very satisfactory	0.78	4.6	11.0	5.4	11.6
27	Rich Mountain	Tenn.	Poor	1.26	6.3	21.0	11.9	17.5
28	Shannon	Va.	Poor	0.90	5.9	15.0	7.5	26.0
29	No. 2	Ky.	Poor	1.28	7.1	17.0	42.8	45.4
30	No. 7	Ky.	Very satisfactory	0.63	5.1	15.0	3.9	18.4
31	Powellton	W. Va.	Poor	0.93	5.7	22.0	17.5	36.0
32	Hazard No. 4	Ky.	Satisfactory	0.63	5.2	20.7	10.2	28.6
33	Clintwood	Ky.	Poor	1.04	6.2	18.6	15.5	30.5

RELATION OF CONCLUSIONS DRAWN FROM TEST RESULTS TO MARKETING EXPERIENCE OF OPERATORS

From consideration of the test results, conclusions are drawn with regard to the degree of satisfaction to be expected if the coal under test were to be marketed. Many of the coals tested could be classed immediately as satisfactory or unsatisfactory beyond

reasonable doubt; others were deficient in performance by one or more criteria, and were given an intermediate or limited classification.

Table 3 lists the coals tested, their classification based on test results, the principal reasons for the classification, if unsatisfactory or not completely satisfactory, and the marketing experience

TABLE 3 SUMMARY INFORMATION REGARDING CONCLUSIONS FROM EVALUATION TESTS AND MARKETING EXPERIENCE OF COAL OPERATORS

Test No.	Bed	State	Classification of laboratory test *	Reason for classification	Experience in marketing by Sponsor operator
1	Wallins	Ky.	S		No information
2	Wallins	Ky.	S		No information
3	Upper Freeport	Pa.	U	Strong coke-tree formation	Does not market residential stoker coal
4	Pittsburgh	W. Va.	S		No information
5	No. 2 gas	W. Va.	S		Marketed successfully
6	Taggart	Va.	S		Marketed successfully
7	Taggart	Va.	S		Marketed successfully
8	Chilton	W. Va.	U	Poor clinker formation and hold-fire characteristics	Not marketed for use in residential stokers
9	Sewell	W. Va.	S		No information
10	Pittsburgh	W. Va.	S-	Sluggish ignition; length of hold-fire periods above average; low clinker formation	No information
11	Elkhorn No. 3	Ky.	S		Marketed successfully
12	Elkhorn	Ky.	S-	Poor clinker formation	No information
13	Elkhorn	Ky.	S		No information
14	Elkhorn	Ky.	S		No information
15	Elkhorn	Ky.	S		No information
16	Mason	Ky.	S		Marketed successfully, principally in northern markets
17	Mason	Ky.	S-	Relatively poor clinker formation	Facilities not yet available for preparation of stoker coal
18	No. 11	Ky.	S		Marketed successfully
19	High Splint	Ky.	S-	Clinker formation questionable	Marketed successfully
20	Blue Gem	Ky.	S		Mine now closed; plan to prepare stoker coal when reopened
21	No. 2 gas	W. Va.	S-	Sluggish ignition, strong coke formations, clinker formation low	No information
22	(Unknown)	Ind.	S		New mine; upon completion of tipple will market stoker coal
23	Hazard No. 4	Ky.	S		Small mine; output not yet justifies preparation of stoker sizes
24	Hazard No. 4	Ky.	S-	Inadequate clinker formation	Has been marketed successfully
25	Elkhorn No. 4	Ky.	S-	Inadequate clinker formation	No stoker coal prepared for market
26	Hernshaw	Ky.	S-	Poor clinker formation	Preparation facilities lacking; no stoker sizes prepared
27	Rich Mountain	Tenn.	U	Sluggish ignition and hold-fire characteristics; high in coke formation; low in clinker formation	Only run-of-mine being marketed
28	Shannon	Va.	U	Sluggish ignition and hold-fire characteristics; high ash content	No stoker sizes prepared
29	No. 2	Ky.	U	Poor ignition and hold-fire characteristics; sluggish fuel bed, high ash content, heavy coke formation	Decision made not to prepare stoker coal
30	No. 7	Ky.	S		Stoker sizes not being prepared at present time
31	Powellton	W. Va.	S		Stoker sizes not being prepared at present time
32	Hazard No. 4	Ky.	S		New tipple under construction; stoker sizes will be prepared
33	Clintwood	Ky.	S		Equipment for preparation of stoker sizes not yet installed

* S Satisfactory S- Not completely satisfactory U Unsatisfactory

of the coal operators sponsoring the tests. Of the 33 coals tested, 20 were determined to be satisfactory for use in residential stokers, 5 were listed as being unsatisfactory, and 8 were given limited ratings.

The chief cause for limited ratings was inadequate clinker formation. Although satisfactory in other respects, use of coals in the restricted classification might be expected to result in complaints from consumers because of the accumulation of unfused ash in the furnace. Such coals might be marketed successfully in the northern regions, but might prove unsatisfactory in regions where climatic conditions require frequent and prolonged operation at low or hold-fire rates. Other factors affecting the evaluation of the coals adversely were excessive coke-tree formation, sluggish ignition and response during intermittent operation, high ash content, and high average smoke concentrations during "off" periods of stoker operation.

As shown in Table 3, there was general agreement between conclusions drawn from results of evaluation tests and the marketing experiences of the coal operators. Those coals classed as unsatisfactory have not been prepared by the respective operators in residential-stoker sizes. Most of the coals rated as being satisfactory are either being marketed successfully, or will be marketed when preparation facilities become available. Of those rated as not being completely satisfactory, three have been marketed successfully; the others are not now being prepared in residential-stoker sizes, or information regarding them is lacking.

The agreement between results of the evaluation tests and marketing information is reasonably good and appears to justify use of the method. Because it is a relatively short-time test, an appraisal of a coal can be accomplished much more quickly than if adequate field tests were attempted. Many of the requests for tests come in summer months when operators are interested in the market for the following heating season.

The method possesses limitations, particularly in regard to the evaluation of clinker formation with coals of intermediate ash-fusion characteristics. With coals of this class having low ash content, the fraction of the ash formed into clinker may be unduly low, although if the test were continued, the fraction would become greater. Admittedly, judgment of clinker formation is less susceptible to evaluation than any other factor involved in the test.

RELATION OF RESULTS OF TEST TO STANDARD LABORATORY DETERMINATIONS

Analysis of the data from the evaluation tests has furnished an opportunity for attempts to relate these results to data determined for the coals by standard laboratory determinations. Studies of this kind have been made by Shotts,⁴ on Alabama coals, and by Helfinstine and Boley,⁵ on Illinois coals. There is a natural interest in correlations of this kind, because of the possibility for simplification of an evaluation-test procedure.

Initial Ignition. The variation in the time required for the flue-gas temperature to reach an equilibrium value, as shown by the record of temperature during the first hour of the test, is considered an indication of the performance of the coal with respect to ignitibility and coking characteristics of the coal. Although not a measure of individual characteristics, it gives a preliminary indication of difficulties to be expected in maintaining ignition during operation under hold-fire conditions.

⁴ "The Relation of Free-Swelling Indexes to Other Characteristics of Some Alabama Domestic Stoker Coals," by R. Q. Shotts, AIME Technical Publication No. 2314, Feb., 1948.

⁵ "Correlation of Domestic Stoker Combustion With Laboratory Tests and Types of Fuels II. Combustion Tests and Preparation Studies of Representative Illinois Coals," by R. J. Helfinstine and C. C. Boley, Illinois State Geological Survey, Report of Investigations No. 120, 1946, p. 35.

Fig. 2 presents three examples of initial ignition curves from among the coals tested. These curves emphasize the difference that can exist between bituminous coals, in reaching an equilibrium flue-gas-temperature value during the initial ignition period. The top curve represents a coal having very good ignition characteristics; the maximum flue-gas temperature was reached early in the hour and declined only slightly during the balance of the hour. The middle curve, shown as a broken line, represents a coal having sluggish ignition characteristics; the flue-gas temperature declined sharply during the first 25 min and did not recover to its maximum value until the end of the first hour. The bottom curve represents unsatisfactory ignition, in that the fire had gone completely out at the end of 30 min of operation.

An attempt was made to relate data from curves of initial ignition to free-swelling indexes and to the time required for operation of the stoker during periods of hold-fire. This was done by plotting the variance of the flue-gas temperature during the initial ignition period against the free-swelling index, and against the average running time during hold-fire operation. No direct relationship was found to exist between these data, hence the temperature variation during ignition cannot be used alone as a criterion of coking or hold-fire characteristics.

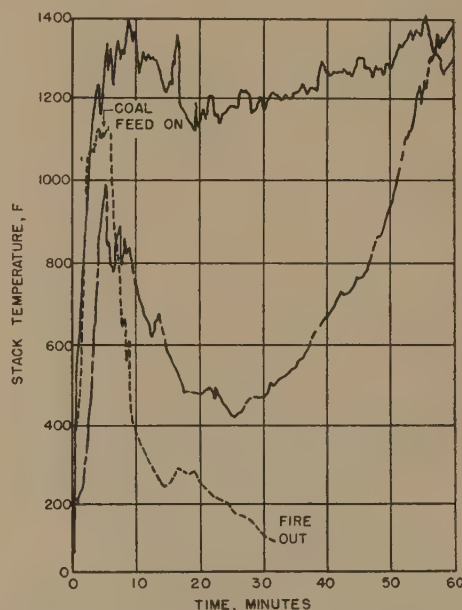


FIG. 2 FLUE-GAS TEMPERATURE RECORDS, INITIAL IGNITION PERIOD

Some years ago an investigation of ignitibility of bituminous coals was made at Battelle. In that study, ease of ignition in residential stokers was compared with ignition temperatures determined by the method developed by The Coal Research Laboratory at the Carnegie Institute of Technology. The correlation between these properties was not good. It is concluded that ease of ignition depends not only on ignition temperature, but also on other factors, chiefly coking characteristics and size variation.

Clinker Formation. Results of clinker formation in the evaluation test are expressed as the percentage by weight of the total ash released during the test which is formed into plus 3-in. clinker. If values for these percentages are 20 per cent or higher, the coals may be expected to give satisfactory service to the consumer in this respect; below this value, the clinkering characteristics of the coal are open to question.

Comparison of clinker values obtained in these tests and ash-

softening temperature show no correlation. Fig. 3 presents a graph of the relation between ash-softening temperature, as determined by the standard laboratory method, and the clinker removed from the fuel bed in the burning tests. Poor correlation is evident between these data. Similar results have been obtained by Helfinstine and Boley.⁵

The ash content of the coal, as determined by the proximate analysis, definitely predicts the degree of manual attention that will be required when burning the coal, due to the removal of clinker and unfused ash.

Fig. 4 presents the relation between the ash content of the coal and the clinker formation. The relationship here is little better than that shown in Fig. 3; however, no single laboratory-determined characteristic can predict accurately the performance of a stoker coal with respect to clinker formation.

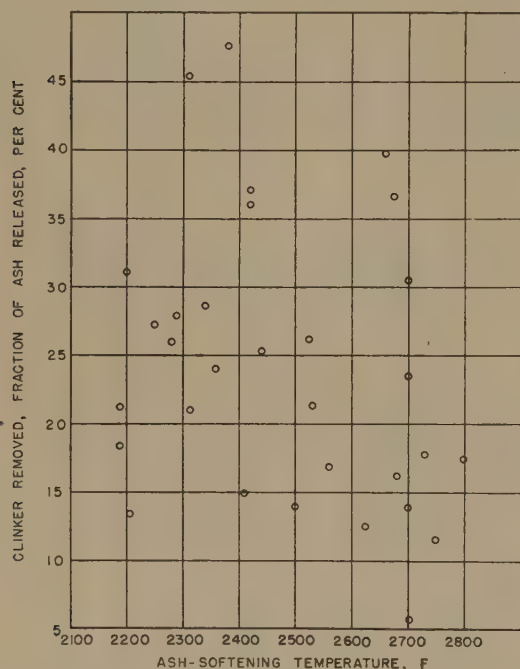


FIG. 3 RELATION BETWEEN CLINKER FORMATION AND ASH-SOFTENING TEMPERATURE

Coke Formation. In the evaluation test, a numerical measure of coke formation is obtained by the weight of plus 1-in. coke remaining in the fuel bed at the end of the test. The standard laboratory test used to obtain an indication of the coking tendencies of a coal is the free-swelling test as prescribed by ASTM designation D-720-46.

Fig. 5 shows the relation between these two methods. The correlation is not so close as might be expected, although the results are in agreement with those of Shotts.⁴ The reason for this lack of correlation is due largely to variations in the reactivity of the coke formed.

Rate of Fuel Consumption During Hold-Fire Operation. The stoker running time necessary to maintain the fuel bed in good condition is of importance to the consumer for reasons of economy and comfort, as it determines the amount of coal that will be burned during periods when there is no demand for heat.

An investigation was made of the relation between the time required to hold fire and laboratory-determined characteristics. Between time required to hold fire and the free-swelling index no relationship could be determined. A plot of the relation between hold-fire time and the ash content is shown in Fig. 6. These data

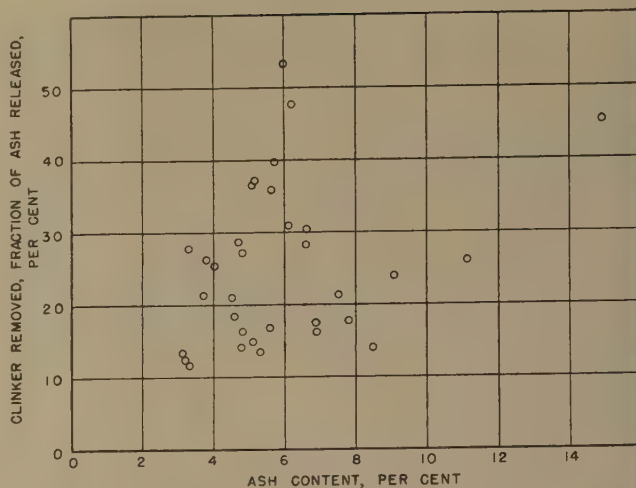


FIG. 4 RELATION OF ASH CONTENT TO CLINKER FORMATION

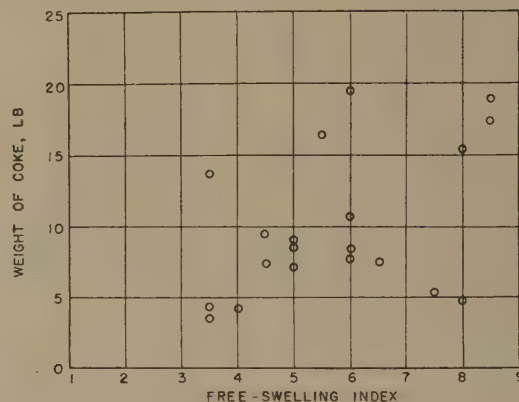


FIG. 5 RELATION BETWEEN COKE ACCUMULATION AND FREE-SWELLING INDEX

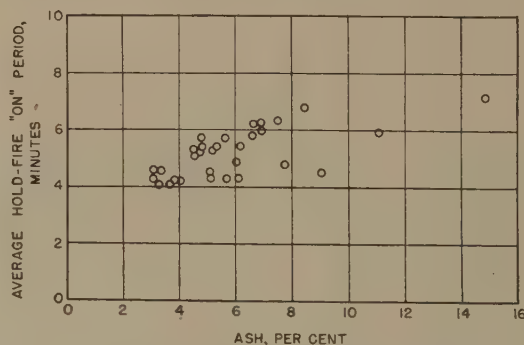


FIG. 6 RELATION BETWEEN ASH CONTENT AND AVERAGE TIME "ON" DURING HOLD-FIRE OPERATION

indicate that, in general, the hold-fire time is increased slightly with increase in the ash content of the coal.

Resistance of Fuel Bed to Flow of Air. The resistance of the fuel bed to the flow of air is of interest as it may reflect the general condition of the fuel bed and the ability of the stoker fan to furnish the necessary volume of air. The fan capacity of residential stokers is sufficient to overcome maximum fuel-bed resistances of 1.50 to 1.75 in. of water. With suitable automatic air controls the delivery of air will be maintained up to this pressure. The highest resistance found during the present study was 1.54, and

for most of the coals tested values were well below the limiting pressure.

The principal factors contributing to fuel-bed resistance are coking properties, size of the coal, and ash content. No correlation was found between free-swelling index, as indicative of coking properties and fuel-bed resistance. Fig. 7 presents a plot showing the relation of fuel-bed resistance to the ash content of the coals. The effect of increasing ash content upon the resistance offered by the fuel bed is evident in this graph.

Uniformity of Burning. The ability of a stoker coal to respond to abrupt demands for heat, and to burn uniformly during periods of heat demand are very important factors in providing satisfactory operation for the consumer. The uniformity of burning is judged mainly by the performance of the coal during the period of intermittent operation on the last day of the test. A measure of the performance during this period is obtained by the operating time necessary to liberate a given quantity of heat, and the uniformity of these values for successive operating intervals.

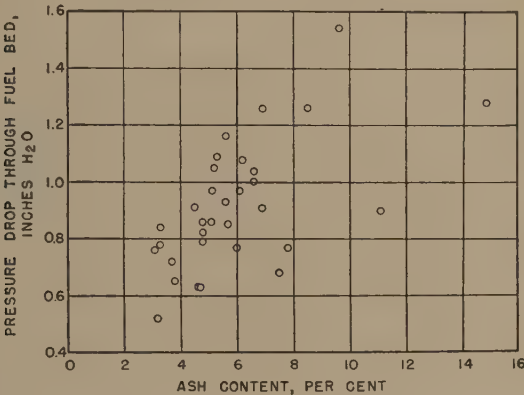


FIG. 7 RELATION BETWEEN PRESSURE DROP THROUGH FUEL BED AND ASH CONTENT, PER CENT

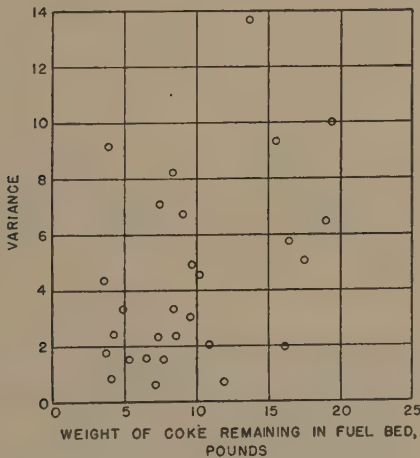


FIG. 8 RELATION BETWEEN VARIANCE IN TIME "ON" DURING INTERMITTENT OPERATION AND WEIGHT OF ACCUMULATED COKE

Fig. 8 shows the relation between the variance in time "on" during intermittent operation and the weight of coke remaining in the fuel bed at the end of the test. The trend of the data in this graph shows that increased quantities of coke are reflected in increased variations in the uniformity of burning. Fig. 9 presents a similar plot for clinker formation and variance. Here it is shown that the amount of clinker formed had much the same effect on the uniformity of burning.

A study of data relating the free-swelling index and the average "on" period during intermittent operation showed a complete lack of correlation.

Smoke Concentration. When the stoker is in operation during a test, under conditions of at least 30 per cent excess air as normally employed, smoke concentrations are of no consequence, even when burning high-volatile coals. Smoke emission is noticeable only for a brief period after the stoker shuts off.

Of the laboratory-determined characteristics, volatile matter would appear to be the most closely related to smoke concentration; the volatile-matter content usually is associated with the relative smokiness of a coal.

Fig. 10 shows in graphic form the relation of volatile matter and smoke concentration for the coals tested. In general, the correlation of the data for these two characteristics is poor, indicating that volatile matter alone cannot determine the amount of smoke that will result from burning a coal in residential stokers.

CONCLUSIONS

A study of data obtained during laboratory tests to determine the suitability of bituminous coals for use in residential underfeed stokers of the clinkering type has led to the following general conclusions:

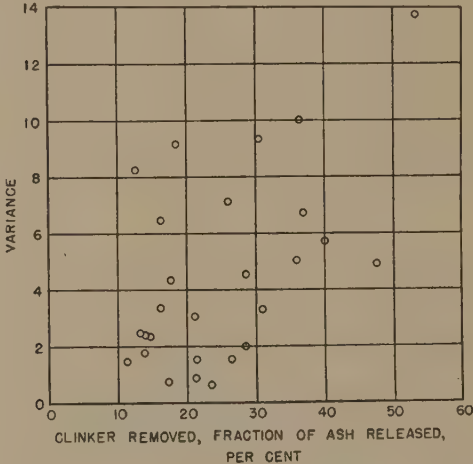


FIG. 9 RELATION BETWEEN CLINKER FORMATION AND VARIANCE

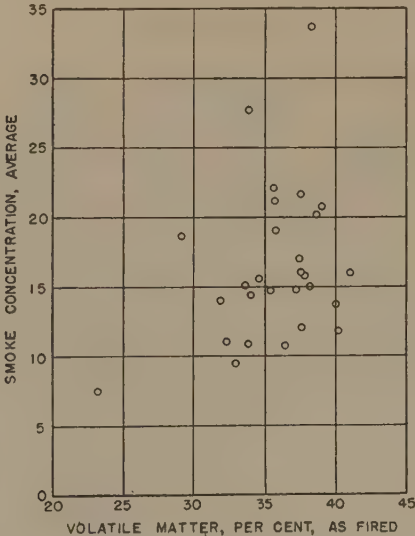


FIG. 10 RELATION BETWEEN SMOKE CONCENTRATION AND VOLATILE-MATTER CONTENT

1 Performance data obtained during the test furnish useful and practical information upon which to judge the performance of stoker coals.

2 Conclusions derived from the results of evaluation tests are in general agreement with use experience of the coals in the field. The method is of practical use in this regard.

3 Few direct relationships were found between data furnished by the evaluation test and determinations by standard laboratory tests. This is particularly true with regard to coke formation and clinker formation, two of the most important factors in determining the degree of satisfaction to be obtained in applications of the small underfeed stoker.

It has not been the intention in this paper to prescribe this evaluation test as the best method for the evaluation of coals for use in residential stokers. That there are other test methods having similar objectives serves to point out the necessity for full-scale laboratory tests.

At the present time, a standard test procedure for the evaluation of coals for use in residential stokers is not available. For the past several years a joint committee, appointed from the membership of the Residential Stoker Committee of Bituminous Coal Research, Inc., and from the Engineering and Research Committee of The Stoker Manufacturers Association, has been working to develop a standard procedure for testing and evaluating bituminous stoker coals. This procedure has been completed to the extent that it has been drawn up in tentative form. It is now planned to install the equipment in several laboratories and to run check tests, using portions of identical lots of coal, to determine the degree of reproducibility offered by the method. Completion of this standard test procedure should facilitate greatly the evaluation of coals for use in residential stokers.

ACKNOWLEDGMENTS

The co-operation of coal operators sponsoring the coal-evaluation tests, in granting permission to use data and results from their tests, is gratefully acknowledged. Acknowledgment is also made to Carroll F. Hardy, Chief Engineer, Appalachian Coals, Inc., for his interest and timely suggestions on the preparation of this paper. Special thanks are due to Ralph Sherman, Assistant Director, Battelle Memorial Institute, who was largely responsible for development of the evaluation procedure, for helpful suggestions and advice.

Discussion

C. F. HARDY.⁶ When a new mine is opened, there is always a question as to the suitability of the coal for various uses including domestic stokers. Until this service was offered by Battelle, it was customary to hand-screen a few hundred pounds of coal and distribute it among various engineers or other stoker users. Obviously, this is unsatisfactory and in no sense of the word standard procedure. There have been several instances where stoker plants were installed by this hit-or-miss system of evaluation, and the coal has proved unsatisfactory for use in domestic stokers after the plant was built. On the other hand, the Battelle standard evaluation tests, by showing that the coal was unsuitable or borderline, have prevented several companies from useless investment in stoker-screening facilities, and the cost of trying to promote the coal on the market.

This paper is particularly valuable in that it brings to the attention of the coal industry the fact that there is a method of testing available, which gives a reasonably accurate evaluation of a coal for domestic-stoker use before it is placed on the market.

⁶ Chief Engineer, Appalachian Coals, Incorporated, Cincinnati, Ohio. Mem. ASME.

This should stimulate further research which should tell why coals perform as they do in domestic stokers.

R. J. HELFINTINE.⁷ The correlation shown between the laboratory combustion tests and the public acceptance of the coal is of particular interest. This type of information is needed to bridge the gap between the laboratory and the consumer.

This paper furnishes more evidence that "armchair philosophy" is often used to condemn good stoker coals. Obviously, no coal should be branded as unsuitable for domestic stoker coal because of high (or low) ash-fusion temperature, or high free-swelling index. Actual combustion tests are required to establish the clinkering and coking characteristics of a coal, and judgment should be based upon performance and not upon appearance.

The tests described in this paper show that a longer hold-fire period was required with high-ash coals. This might be expected with the type of hold-fire control used for the tests. However, it is the writer's opinion that this relationship would not exist if the controls were of the time-interval type.

E. R. KAISER.⁸ As a former fuel engineer assigned to residential stokers at Battelle, the writer has read this paper with considerable interest. It is gratifying to note that earlier test techniques have since been developed and that criteria found important in the years 1935-1938 are still important in judging the performance of stoker coals for residential heating.

The authors have plotted a number of the primary coal data singly against performance data of the stoker in an effort to establish relationships. Unfortunately, wide scattering of points usually resulted, which did not illustrate what experience generally indicates to be the case. For example, Fig. 3 of the paper indicates almost no trend in the percentage of released ash converted to clinker with change in ash-softening temperature. Other factors, such as fuel-bed temperatures and zones of heat release must influence the results.

In the present state of our knowledge, we cannot explain all of the reasons why suitable stoker coals perform satisfactorily, nor why one satisfactory coal may be better than another. Therefore it would be better to approach the subject from the aspect of unsatisfactory coals whose properties might be more marked and therefore easier to evaluate. On examining the data for sixteen unsatisfactory coals tested at Battelle, not all of which were reported in the subject paper, it was concluded by one of the authors as follows: Unsatisfactory coals for residential stokers have one or more of the following general characteristics in the Battelle test:

- 1 Slow ignition with less than 20 lb of coal burned during the first hour.
- 2 Over 8 per cent ash content.
- 3 Over 2500 F ash-softening temperature.
- 4 Less than 20 per cent of the ash released converted to clinker.
- 5 A smoke reading of over 12 units average during the "off" period.
- 6 A wind-box pressure exceeding 1.25 in. water gage.
- 7 Weight of coke at end of test exceeding 10 lb.
- 8 Free-swelling index exceeding 8.
- 9 It is assumed that the top size does not exceed 1 1/4 in. and that the amount of minus 1/4-in. coal does not exceed that of normal resultant coals.

Returning to the question of correlating stoker performance

⁷ Mechanical Engineer, Illinois State Geological Survey, Urbana, Ill.

⁸ Assistant Director of Research, Bituminous Coal Research, Inc., Columbus, Ohio. Mem. ASME.

with basic laboratory tests, the stoker performance is so complicated that it is not reasonable to expect the performance factors to vary with single coal factors. At least two or three coal characteristics are at work at the same time in effecting coke formation and clinker formation. These factors are discussed under the same headings as used by the authors:

Ignition Properties. A coal with poor initial ignition shows symptoms of poor ignition later during "on" periods and hold-fire. Coking characteristics and size consist may be factors, but something inherent in the coal other than that measured by the Coal Research Laboratory method seems to be present. Perhaps it is liberation of CO_2 and water vapor from the coal, with absorption of heat, when the coal is subjected to ignition. Weakly coking coals from the Indiana-West Kentucky field sometimes exhibit poor ignition. Outcrop coals are notorious in this respect. Additional basic research is needed on this subject.

Clinker Formation. Clinkering of coal ash is evidently affected not only by the ash-softening temperatures but by the temperatures in the ash zone. A hot fire in the region beginning immediately above the tuyères and extending at least 5 in. upward will promote clinkering. A coal that ignites slowly and produces coke rather than heat is not likely to promote clinkering. A high ash content does not help rapid ignition nor rapid combustion. Furthermore, the ash-softening temperature has the limitation of not representing the ash in its segregated or heterogeneous state on being released from the coal.

Coke Formation. The poor correlation between free-swelling index and weight of coke in the fuel bed is probably due to the additional factors of slow ignition in the retort and reactivity of the coke formed.

Plasticity of the coal in the initial stage of coking is probably the dominant factor, with ignition second, and reactivity of the coke third. The plastic property obviously must be first in importance in forming the coke tree. Slow ignition causes green and unoxidized coal to move up into the heat zone where the temperatures are better for coking than for rapid combustion. Once coke is formed, air cannot penetrate the formation.

Because of hold-fire conditions and the cooling effect of the furnace walls, particularly in household furnaces, the coke loses ignition and often will not burn until it falls to the hearth and is reignited by the flames issuing from the annular space around the retort.

Hold-Fire Operation. The ignition property of the coal again must be a dominant factor in the length of time the stoker must be operated to brighten the fire. Therefore it is understandable why the free-swelling index did not alone prove to be the factor. A small fuel bed of hot coal with a low fuel maintenance is all that is necessary. A heavy fuel bed of smoldering coal results if the stoker feed screw continues to deliver coal that will not ignite at the rate fed.

Fuel-Bed Resistance to Air Flow. Coals which ignite slowly often cause high wind-box pressures. Green coal floods the tuyères rather than burning away promptly in the air jets. Low wind-box pressures result when the combustion is rapid and a free annular space is left around the coke tree and inside the clinker ring.

Any unclinkered ash that falls into the annular space contributes to the air resistance. The wind-box pressure also increases gradually with the amount of clinker in the bed simply by increasing the height of the annular space through which the gases must flow.

Uniformity of Burning. Coals which ignite promptly and do not coke too strongly burn with a small fuel bed. The uniformity of burning is good, neither too much nor too little, but approximately at the rate the coal is fed.

The combination of strong coking and slow ignition makes for large coke masses. While the coke is being formed, the rate

of burning is slow. After the coke mass breaks down and is reignited, the burning is fast. The excess air varies inversely with the rate of burning. The extreme case occurs during the "on" period following the removal of clinker from a hot fuel bed. The coke is broken down in the process and reactive surfaces are exposed. Long flames and appreciable carbon monoxide in the flue gas result for a few minutes following the resumption of full-rate combustion.

Smoke Concentration. Poor ignition also can contribute to "off" period smoke. Smoke increases with the amount of green coal and partially devolatilized fuel above the retort, particularly at the center of the fuel mass. Prompt ignition and intense burning near the tuyères reduce the coal left to cause smoke when the air flow is shut down for the "off" period.

Conclusion. The foregoing indicates strongly that the ignition property of bituminous coals should be investigated in a basic way to establish a laboratory method which would help not only in evaluating coals but in analyzing the performance of coals in combustion. Tests conducted at Battelle several years ago with fuel beds ignited from above and supplied with air from below showed promise of obtaining useful results.

Further study might well be given to the noncombustible gases liberated from coal on heating above room temperature. The heat required to raise the temperature of the coal, including that for liberating the gases, should also be measured. This heat is not important in relation to the total calorific value of the coal, but in underfeed ignition the amount of heat radiated in the ignition zone is small enough to make this factor highly important. If the tests could be conducted on the same coals used in the laboratory stoker tests it might be possible to develop a better correlation of all of the data obtained thus far, and to place a measured value on the ignition factor.

C. H. SAWYER.⁹ The work described in this paper represents the longest-standing program of such research with which we are familiar, and all of us who are interested in this subject have borrowed freely both in ideas and in method from it.

Both R. I. Bush, Director of our Pittsburgh Stoker Research Laboratory, and the writer had an opportunity to examine this paper in advance and he shares with the writer responsibility for the following comment:

We liked especially the authors' careful statement that values of fuel-bed resistance indicate "...the general condition of the fuel bed." Certainly, fuel-bed resistance cannot be ascribed to any one property of the fuel bed, such as thickness, as is so often claimed. We think this error has led to many false starts on air-control design and it certainly finds its way into much stoker advertising. We think the limit of 1.5 in. of water given as maximum trouble-free fuel-bed resistance exceeds the capabilities of several stoker fans we have tested and would be safer set at some lower value, say, of the order of 1.25 in. Fortunately, there is a trend toward higher-pressure fans among stoker manufacturers, and the point will assume less weight as time eliminates the old low-pressure jobs.

The authors have questioned the temperature selected for hold-fire pickups in their method. We use an entirely different method of evaluating hold-fire performance. In typical stoker operation out fires are most likely to occur at some time between 3 and 5 hr after the last prolonged operation of the stoker. Many coals, particularly the semifluid types represented by some of the popular low-volatile stoker coals, will pick up very sharply after this low-activity period and would thus show an average time of hold-fire operation little if any below the normal, even though their remaining alive may have been a touch-and-go proposition at the critical stage. We cannot, however, offer a completely

⁹ Eastern Gas and Fuel Associates, Pittsburgh, Pa.

satisfactory substitute for the method cited. We still use 3-min-per-hr timed operation as a hold-fire test. Most coals will hold fire at such a rate, and we think minimum activities, as indicated by stack-temperature records, give a closer indication of the likelihood of out fire than does the average reactivation time of the Battelle method. However, we pay for this policy with occasional out fires which means virtual inability to run a "standard" test on the coal in question since much other data of value are no longer reliable after rekindling such a fire. Possibly avoiding a hold-fire check until the end of a test schedule might overcome the objection to some such attempt to get a closer assay of hold-fire characteristics.

Mr. Bush has suggested that residual coke as determined is subject to too much accidental variation to allow significance by any means short of running several tests on each coal to obtain an average value. He does not feel that the size consist of residual coke is subject to as much variation as is the quantity, and suggests comparison of some figure such as average size of residual coke as an alternative attempt to index the likelihood of coke-tree trouble.

We find the lack of pattern in clinker data disappointing, as did the authors. It seems that no test of such short duration can answer satisfactorily the question of "clinkerability." However, in our own laboratory we have noted a slightly better correlation of clinker removed with ash-softening temperature than that indicated in the graph shown. The writer has wondered considerably about this difference since he knows how carefully this work is done at Battelle. One possible partial explanation for the difference is offered:

Our fires are undisturbed throughout the entire week. In the tests under discussion clinker is removed on the evening of the fourth day. It would seem only human nature for an operator to search a fuel bed showing little or no clinker more thoroughly than one containing some obvious pieces at this stage of the test. Such search would constitute cultivation which is always of help in clinkering difficult ash. Thus certain high-fusion or otherwise difficult-to-clinker coals might be unconsciously thrown out of line in this performance characteristic by even the most conscientious operator. As a matter of fact even an equivalent degree of cultivation (unavoidably entailed in any clinker search)

might favor high-fusion coals in this regard and tend to even the end quantities of clinker.

AUTHOR'S CLOSURE

Mr. Hardy has pointed out the economic advantages of a laboratory-conducted evaluation test. This of course is of prime importance. In addition, one of the strong points in favor of a test method of this type is that it permits the evaluation to be made regardless of the season or of fluctuations in outside temperature. It also permits making a relative evaluation of coals with comparative ease, since consideration of variations in equipment is reduced to a minimum.

Mr. Helfinstine's comment regarding the type of hold-fire control used is correct. However, as pointed out by the authors in the paper, the method used does provide a relative means of comparing the performance of coals during the periods of hold fire, even though it must be recognized that the period of operation during each hour may be longer than necessary. The method eliminates interruptions occasioned by attempts to find the lowest possible hold-fire setting for each coal under test, yet it furnishes a relative measure of performance.

Mr. Kaiser's knowledge of the background of these tests has provided a basis for a very interesting discussion. The limiting values generally found to be present among coals not considered suitable for use in residential underfeed stokers of the clinkering type, have been clearly summarized.

Of particular interest are the conclusions drawn by Mr. Kaiser in the need for additional basic research on the ignition properties of various coals and of the coke derived therefrom. It is hoped that additional work can be accomplished along these lines in the near future.

Mr. Sawyer's discussion of clinkering data is of special interest. Disturbing the fuel bed certainly has an effect on the subsequent formation of clinker in spite of all the care that is taken in handling the clinker tongs. However, when burning a high-ash coal at a feed rate of 30 lb per hr, as employed in these tests, sufficient ash has been deposited by the end of the fourth day to make it often necessary to remove the clinker at that time, thus insuring a relatively clean fuel bed for the intermittent heat-demand run of the last day of the test.

Marine Propulsion-Gear Testing at the Naval Boiler and Turbine Laboratory

By IVAN MONK,¹ PHILADELPHIA, PA.

This paper describes the testing phases of the propulsion-gear development program of the United States Navy. Full-sized reduction-gear units are tested to destruction under controlled conditions. The facilities and procedures employed in these tests are described, together with some of the problems encountered. Results are presented from tests in which four destroyer-escort reduction-gear units were able to transmit more than three times their designed full power before failure occurred.

IF ONE could devise a means of reconciling, in a practical manner, the necessary high speed of revolution of the turbine with the comparatively low rate of revolution required by an efficient propeller, the problem would be solved, and the turbine would practically wipe out the reciprocating engine for the propulsion of ships. The solution of this problem would be a stroke of great genius."

These words were written in May, 1904, by Rear Admiral George W. Melville and Mr. John H. Macalpine after a thorough investigation of the then-existing status of the steam turbine as applied to the propulsion of ships. That the problem was solved is evidenced by the preponderance of geared-turbine-driven ships in the naval and merchant fleets of the world.

The first ship in the United States to be equipped with reduction gears for main propulsion was the U.S.S. *Neptune*, a fleet collier commissioned in 1911. In 1945 the combined capacity of propulsion gears in United States naval vessels was more than 50,000,000 hp. This figure makes the Navy by far the world's largest single user of marine propulsion gears.

Above all, the propulsion machinery of a combatant ship must be reliable. Experience has shown that naval reduction gears have possessed the desired degree of reliability, but there is no doubt that the Navy, in specifying designs that would insure reliability, has sacrificed other desirable features. One of the most tempting methods of obtaining reliability is to specify conservative designs, with large factors of safety. This of course results in larger gears, consuming additional weight and space which otherwise could be used for fuel. The greater the fuel capacity of a ship, and the more efficient its machinery, the greater will be its steaming radius. The importance of the steaming radius of a ship was demonstrated time and again during the past war, when our fleet units were required to remain at sea for prolonged periods, frequently in enemy waters.

The Navy had long felt the need of facilities for evaluating the performance of reduction gears during full-scale operation before approving them for installation in a large number of ships. It was to fill this need that the Bureau of Ships, in 1941, started to build gear and turbine testing facilities at the Naval Boiler Laboratory, now known as the Naval Boiler and Turbine Laboratory.

¹ Commander, U.S.N., Assistant Director, Naval Boiler and Turbine Laboratory, Philadelphia Naval Shipyard.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society. Paper No. 48-A-50.

DESCRIPTION OF LABORATORY

The Naval Boiler and Turbine Laboratory, located in the Philadelphia Naval Shipyard, had its beginning in 1909 when the Bureau of Steam Engineering established it as the Fuel Oil Testing Plant. Through the years the missions and facilities of the laboratory have grown, and at present it is the sole government agency in which full-sized marine boilers, turbines, and gears are tested under simulated service conditions with principal variables accurately controlled. A detailed description of the laboratory facilities has been given in a paper² by Capt. E. Kranzfelder, U.S.N.

All gear tests are conducted in a building specially designed for testing large prime movers and reduction gears. Testing space is such that three complete sets of propulsion machinery, including boilers, turbines, and gears, for large combatant vessels can be set up for testing simultaneously. The building is 60 ft high, 255 ft long, 165 ft wide, and is divided into three test bays, each of which is equipped with an overhead traveling crane of 50 tons capacity.

To insure a solid foundation for a large variety of test installations, special consideration was given to the design of the floors in the test areas. On top of closely spaced piling was constructed a concrete mat, approximately 5 ft thick. The mat was then covered with heavy cast-iron slabs, 8 in. thick, which were grouted in place and anchored with the holding-down bolts embedded in the mat, thus forming a level test floor of great rigidity. The cast-iron slabs are cross-hatched with T-slots which form a convenient means for firmly securing test installations to the floor.

The laboratory has a large assortment of steam turbines for use in testing gears. These have been obtained from surplus Navy equipment and have capacities ranging up to 25,000 hp per turbine. Turbine speeds up to 9000 rpm are available; in those cases where higher speeds are required, a special step-up gear is installed between the turbine and test gear.

Three boilers with a combined steam capacity of more than 600,000 lb per hr supply steam to the turbines. Circulating water for the condensers is supplied from the shipyard basin by three pumps having a total capacity of 150,000 gpm.

PURPOSE OF TESTING

Gears are tested at the laboratory for many purposes but these may be divided into the following general categories:

- (a) Contract acceptance tests.
- (b) Development tests.

In contract acceptance tests the gears are operated under simulated shipboard service conditions. The object of these tests is to determine whether the gears meet the contract specifications and are suitable for the purpose for which they were designed and built.

During a naval shipbuilding program, emphasis usually is placed on the mass production of units of standardized designs. There is an understandable reluctance to approve new and un-

² "The Naval Boiler and Turbine Laboratory," by Capt. E. Kranzfelder, U.S.N., Trans., the Society of Naval Architects and Marine Engineers, vol. 55, 1947, pp. 341-358.

tried designs for mass production before their reliability has been demonstrated. Consequently the reduction gears for each class of ships represent the best designs of known reliability existing at the time the building program started.

The best time to build and try experimental propulsion machinery of advanced designs is before the shipbuilding program starts. Future standardized designs can then safely include new and progressive ideas which otherwise might be considered too radical. This is the keynote of the laboratory's development test program. The general objective is to assist industry in producing better reduction gears for the Navy. Some of the factors which contribute to better reduction gears are the following:

- (a) Greater reliability.
- (b) Lighter weight.
- (c) Smaller size.
- (d) Higher efficiency.
- (e) Quieter operation.

METHODS OF TESTING

Two methods of testing gears are employed at the laboratory:

- (a) Dynamometer testing.
- (b) Torque testing.

Dynamometer-Test Arrangement. The dynamometer method is illustrated in Fig. 1. The gear is driven by a steam turbine, and the transmitted power is absorbed either by a water brake or by an electric dynamometer. Except for the roll and pitch of the ship, this test setup reproduces actual shipboard operating conditions very realistically.

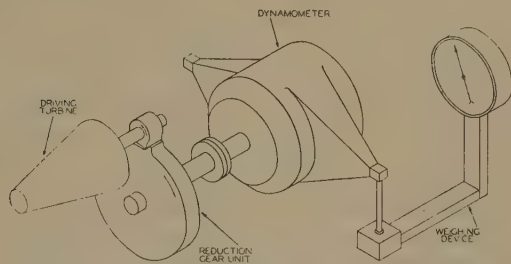


FIG. 1 DYNAMOMETER-TEST ARRANGEMENT

Water brakes have been used as load-absorbing devices in most of the gear tests to date. The laboratory has eight large water brakes with capacities ranging from 25,000 to 40,000 hp, rated at various speeds up to 6000 rpm; also, numerous smaller brakes of various sizes are available so that practically any type of propulsion gear, from battleship to motorboat, can be tested.

The water brake is an ideal load-absorbing device for simulating shipboard operating conditions since the power absorbed by both the ship's propeller and the water brake is proportional to the cube of their speed. To prevent unstable operation, however, it is essential that the water supply to the brake be maintained as steady as possible. A uniform water pressure is obtained at the laboratory through the use of large constant-level supply tanks mounted in the roof structure of the building.

The accuracy of a water brake is limited only by the accuracy of the weighing device used to measure the reaction of the stator arm. The laboratory formerly used platform scales for this purpose. With properly calibrated scales, the over-all water-brake error should not exceed 0.2 per cent. During one of the larger tests in which a 35,000-hp water brake was used, numerous scale casualties occurred, due to water-brake vibration. In some instances the scales were wrecked after 2 or 3 hr of operation. In an effort to find a more rugged substitute the laboratory tried several different types of weighing devices which appeared to be suitable. After exhaustive tests two of them were found to

possess the required durability and accuracy. One of these devices consists essentially of a hydraulic pressure-indicating system. The reaction of the brake stator arm is applied to a piston which is held in place by oil pressure. Since the oil pressure is proportional to the applied load, the latter can be determined by measuring the oil pressure with an accurate pressure gage. The pressure gage is calibrated to indicate the load directly in pounds, and is compensated for changes in temperature.

The other weighing device used by the laboratory is similar in some respects to the one just described, except that air is used as the working fluid. In the pneumatic device the load is applied to a diaphragm which is held in position by air pressure. A pilot valve, actuated by movement of the diaphragm, regulates the air-chamber pressure to keep the diaphragm in equilibrium. As the applied load decreases, air is bled from the chamber; as the load increases, compressed air is admitted from an external source. Load is indicated by an accurate pressure gage connected to the air chamber.

Both the hydraulic and pneumatic weighing devices are frequently checked for accuracy by means of proving rings which have been calibrated by the National Bureau of Standards. The over-all accuracy of a water brake and its weighing device may be tested statically by applying known torques to the water-brake stator. In this method heavy test weights are suspended from one end of a long lever, the other end of which is secured to the stator.

Water-Brake Vibration. In addition to replacing platform scales with sturdier weighing devices, the laboratory modified the 35,000-hp water brake to reduce its vibration. It was found that the frequency was 15 times the shaft speed. The brake, which was of the Froude type, had 15 rotor vanes and 16 stator vanes on each end. This combination was very satisfactory from the standpoint of torsional excitation, but was creating a bad lateral excitation. It was decided that the vibration was being caused by unbalanced forces produced as the rotor vanes passed the stator vanes. The forces were unbalanced because the stator had an even number of vanes and the rotor had an odd number of vanes. Consequently the impulses occurred one at a time, without an equal and opposite impulse on a vane 180 deg away, and the unbalanced forces were transmitted to the foundations. Had both the rotor and stator been provided with even numbers of vanes the impulses would have occurred in pairs, diametrically opposite, thus balancing each other as far as lateral excitation was concerned.

The foregoing analysis proved to be correct. The 15 original rotor vanes were replaced with 18 new vanes, equally spaced on each end. The resulting combination of 18 rotor vanes and 16 stator vanes eliminated the vibration troubles. It might be added that the welded-plate construction employed in this brake was susceptible to erosion and corrosion, particularly at the welds. In the vane modification just described stainless steel (25-20) welding rod was used and the welds have proved to be much more durable than the original carbon-steel welds.

Although the water-brake-dynamometer method of testing gears closely simulates shipboard operating conditions, it has the disadvantage of requiring large prime movers capable of developing all the power transmitted by the gears. The cost of boiler fuel alone is usually one half the total operating cost of the test. Another disadvantage is the difficulty of determining gear efficiencies. To date, the laboratory has been unable to find a torsion meter capable of measuring the power input to the gear with the requisite degree of accuracy. The combined errors of the torsion meter and absorption dynamometer are usually of the same order as the gear losses. Attempts to determine gear losses by measuring the heat carried away by the lubricating oil have been only partially successful.

Torque-Test Arrangements. Many of the disadvantages inherent in dynamometer testing may be avoided by using the torque-test setup. In this method two gear units are tested simultaneously, with one unit loaded against the other. Several loading arrangements are possible, depending on the type of gear units being tested. The two arrangements used by the laboratory are the "front-to-front" and the "back-to-back."

The front-to-front arrangement is shown in Fig. 2. Two gear units, one port and one starboard, are erected end to end so that the corresponding first-reduction pinions may be connected together with torque bars. The torque bar connecting the pinions on the low-pressure-turbine side has a flange-type coupling at its mid-length to permit the application of torque to the gear units. Wrenches are used to twist the torque-bar flanges in opposite directions until the desired torque has been applied to the gear trains. The flanges are then bolted together and the gear units brought up to the desired operating speed. The turbine used to drive the gear units is usually coupled to one of the quill shafts which connect the first-reduction gears with the second-reduction pinions. A typical front-to-front torque test is shown in Fig. 3.

The back-to-back test setup is similar to that just described, except that the gear units are arranged so that the bull gears of the port and starboard units may be coupled together as shown

in Figs. 4 and 5. Both torque bars are provided with adjustable couplings and the high-pressure and low-pressure turbine branches are loaded separately. This feature permits the load distribution between the two branches to be controlled; in the front-to-front arrangement both branches must transmit the same power. Another advantage of the back-to-back method is that the bull-gear teeth can be loaded in the same direction by the pinions, thus simulating shipboard operating conditions. In the front-to-front method the pinions are loaded against the bull gear in

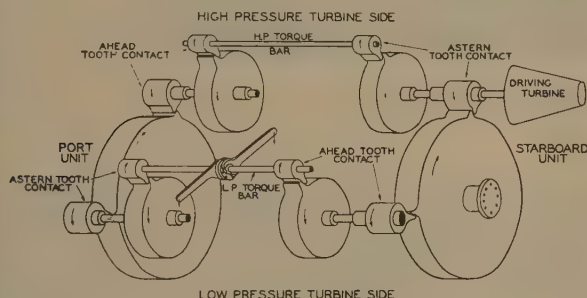


FIG. 2 FRONT-TO-FRONT TORQUE-TEST ARRANGEMENT
(Both units rotate in ahead directions.)

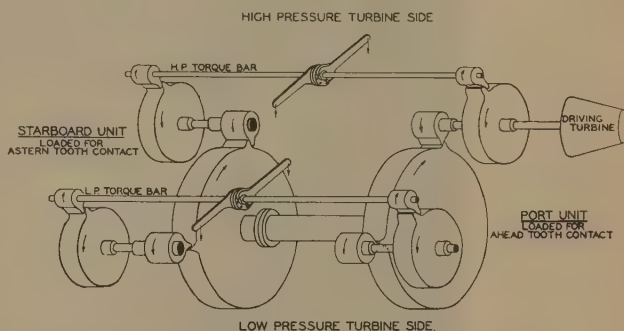


FIG. 4 BACK-TO-BACK TORQUE-TEST ARRANGEMENT
(Both units rotate in ahead directions.)

opposite directions, causing the bull-gear teeth to be subjected to reversed bending.

One disadvantage of the back-to-back method is the difficulty of connecting torque bars to the first-reduction pinions. Since the pinion couplings are on opposite ends of the two units, the pinions must be hollow to permit the torque bars to pass through. The gear cases also must be modified for the same reason. Gears usually can be set up for a front-to-front test without these difficulties.

It should be noted that the expressions front-to-front and back-to-back have been used arbitrarily to specify the manner

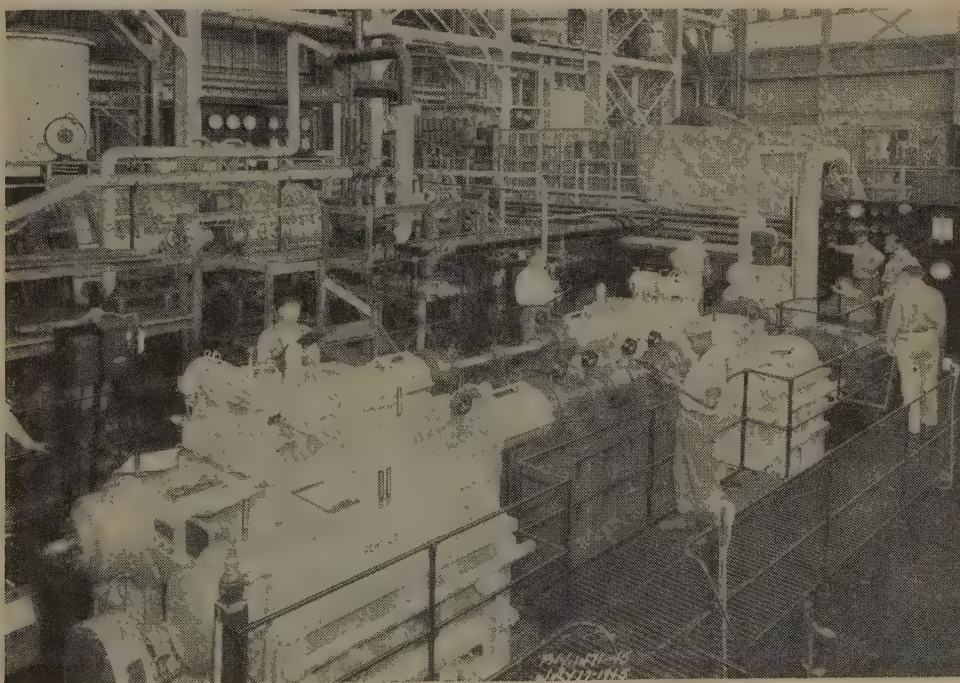


FIG. 3 FRONT-TO-FRONT TORQUE TEST OF DESTROYER-ESCORT REDUCTION GEARS

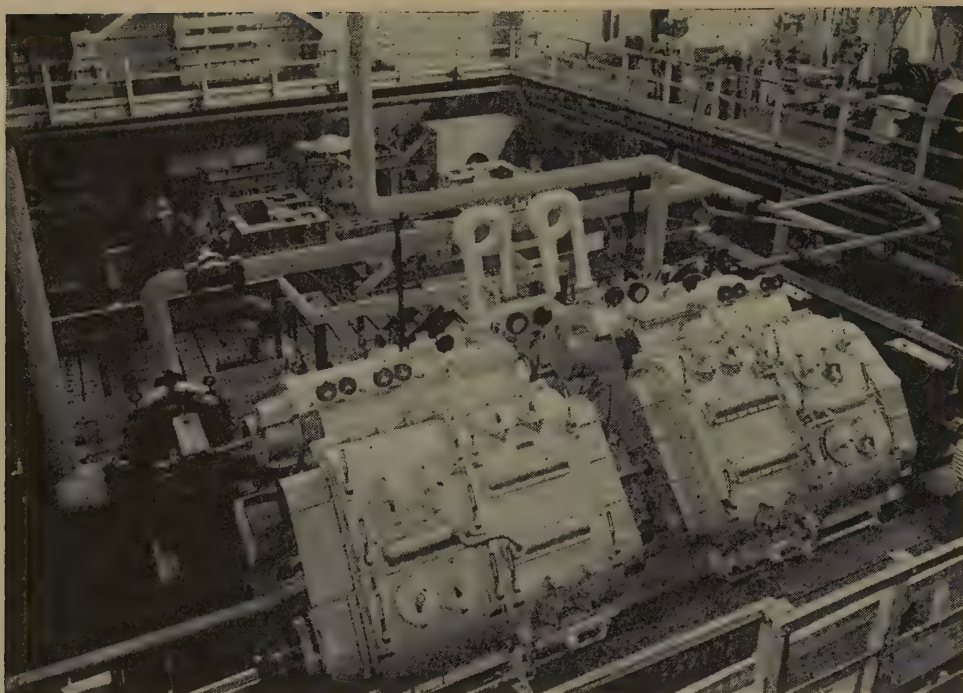


FIG. 5(a) BACK-TO-BACK TORQUE TEST OF DESTROYER REDUCTION GEARS
(Driving turbine shaft is at lower left; other turbine and two motors are for starting.)

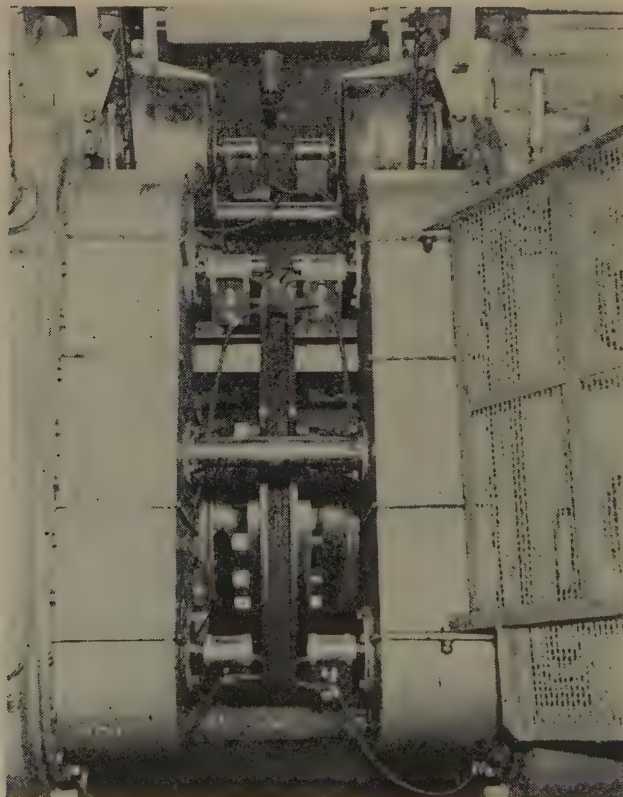


FIG. 5(b) BACK-TO-BACK TORQUE TEST OF DESTROYER REDUCTION GEARS
(Showing torque bars, starting sheave and torsion-meter pickups.)

in which the rotating elements of two gear units are connected together for testing. Actually, these expressions merely indicate the relative positions of the units, and it is quite possible that the bull gears could be connected together in either arrangement; or even in a "front-to-back" arrangement. For all torque tests conducted at the laboratory to date it has been more convenient to use the back-to-back arrangement when the bull gears were to be connected together, and the front-to-front arrangement when they were not; hence the association of loading arrangement with relative position of the units.

Both of the torque-testing arrangements just described permit accurate determination of gear efficiencies. The output of the driving turbine is measured by means of a torsion meter. Since this output represents only the gear losses, a small torsion-meter error of 1 or 2 per cent will not seriously affect the accuracy of the efficiency determination. In the case of the water-brake method of test, the turbine output represents the total power transmitted by the gears, and the same small torsion-meter error may be greater than the actual gear losses. Another obvious advantage of the torque-test setup is the comparatively small amount of power (and fuel) required to drive large gears at full power.

Before the power transmitted by gears can be determined, the applied torque must be known accurately. It was found that the calculated torque (torque wrench length multiplied by applied force) gave questionable results, due primarily to the static friction in the bearings and gear elements. Usually, the torque required to twist one half of the coupling was several times as great as that required to hold the other half in place; also, the average of the two changed after the gears were run.



FIG. 6 CALIBRATION OF STRAIN-GAGE TORSION METER

Torsion Meters. To avoid these difficulties the laboratory uses torsion meters for measuring locked-in torque. One type that has given good results is the wire-resistance strain gage. This device can be installed on one or more of the quill shafts and requires only minor modifications to the gear elements. The shaft and strain-gage assembly is carefully calibrated as shown in Fig. 6. The strain-gage torsion meter has been particularly useful for measuring load distribution among the various trains of locked-train gear units.

Another torsion meter used by the laboratory is of the Amsler type. It consists of an elastic torque shaft and concentric sleeve, fastened together at one end and free at the other. Relative torsional displacement of the two elements at the free end is proportional to the applied torque. This device is installed as part of the regular torque bar, and a stroboscope is used to take readings while the gears are running. Both the strain gage and

Amsler torsion meters indicate that locked-in torque is a linear function of torque flange displacement. No such correlation could be established between torque flange displacement and calculated wrench torque.

Torque Applier. One of the difficulties encountered in torque-testing large gear units is getting them started after the torque has been locked in. A tremendous external driving torque is required to overcome the static friction of the bearings and gear elements. A typical starting arrangement is shown in Fig. 5(a). Two large motors and an auxiliary turbine are geared to three of the quill shafts through clutches. Fig. 5(b) shows a starting sheave between the low-speed-shaft flanges. Once the gears are rotating and adequate lubrication has been established, the regular driving turbine can carry the load.

More serious than the difficulty of getting the gears started are the abnormal friction loads imposed during starting. Some of the laboratory's gear tests have required the locking-in of torques greater than 300 per cent of the designed full-power torque. These severe starting conditions have caused a considerable amount of uncertainty in evaluating damage sustained by bearings and gear elements. One could not be sure whether the damage occurred during running or while starting.

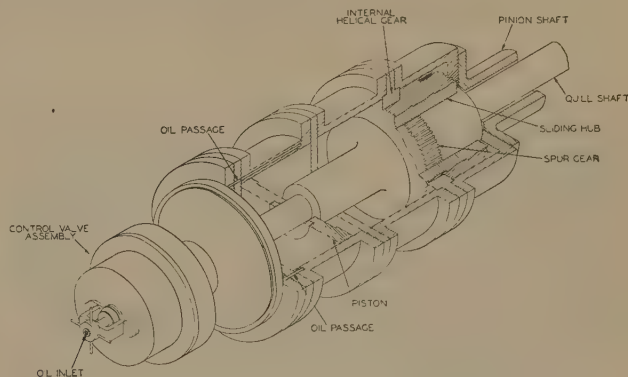


FIG. 7 HYDRAULIC TORQUE APPLIER

The ideal situation would be to bring the gears up to speed under no load and then to apply the desired torque. This problem, like many others, was solved by the Navy Gear Industry Committee, whose Gear Testing Sub-Committee designed a hydraulic torque applier which accomplished the desired result. This device, shown in Fig. 7, consists of an oil-operated piston whose axial movement causes relative torsional displacement between a pinion and its shaft. The torque-applier unit rotates with the pinion and shaft; actuating oil is supplied under pressure through a rotary shaft seal. This unit has performed very satisfactorily and similar units will be used on future torque tests.

TEST PROCEDURES

When the present gear-testing program was being planned, it was generally agreed that one of the best methods of determining the weak points of a gear was to operate it at progressively increasing loads until failure occurred. As a result, most of the gears tested at the laboratory ultimately have been run to destruction.

During the erection and operation of a gear test, every effort is made to eliminate all variables except those under investigation. This policy was adopted after much discussion concerning the practicability of attempting to reproduce and evaluate in the laboratory all the variables associated with actual shipboard operating conditions. It was reasoned that if gear performance could be evaluated under ideal conditions, the results would

provide valuable criteria for comparing and predicting performance under shipboard service conditions. The problem of achieving an ideal installation aboard ship would be greatly facilitated if accurate information were available concerning the optimum performance under ideal laboratory conditions, and the effects of various departures therefrom, such as foundation distortion when a ship is in a rough sea.

In the laboratory the gear units are secured to the iron-slab test floor as rigidly as possible to prevent foundation distortion. Alignment receives special attention; first the gear cases and bearings are carefully aligned with the rotating elements removed. After the gears have been assembled, all meshing elements are checked for proper tooth contact. Prior to operation all elements are inspected, photographed, and measured. Measurements include the thickness and surface finish of designated teeth so that wear and damage may be evaluated during future inspections.

Lubricating-Oil System. Experience has shown that it is desirable for each gear unit to have an independent lubricating-oil system. While this is impracticable aboard ship, the laboratory installs separate systems for the gears, driving turbine, and dynamometer. These systems are in the form of semiportable units, consisting of two pumps (one steam and one electric), magnetic strainers, spiral bag filters and a cooler, all mounted on a large, rectangular tank. The electric stand-by pump is set to cut in automatically in case the steam pump fails. After each period of operation, the lubricating oil is pumped to a settling tank, heated, and renovated by means of centrifugal purifiers. Except for a few designs requiring special lubricants, all gears have been tested with standard Navy turbine oil, Symbol 2190T.

Instrumentation. The instrumentation for most gear tests is quite elaborate, with particular emphasis being placed on temperature measurement. Temperatures of oil to and from bearings and meshing elements are measured by calibrated mercury-in-glass thermometers, resistance thermometers, or thermocouples, depending upon the accuracy desired. Recorders employed are both single and multiple-point of the direct-current potentiometer type for thermocouples and of the Wheatstone-bridge type for resistance thermometers. Thermocouples are particularly useful for measuring temperatures in inaccessible places. The laboratory has not been able to develop a completely satisfactory method for measuring the temperatures of rotating surfaces, such as the planet-gear bearings of planetary-type gear units. Various combinations of slip rings and thermocouples have been tried and found unsatisfactory. A substitute method which has given good results consists of measuring the temperature of the oil as it is thrown from the moving bearings. A small stationary cup containing a thermocouple is mounted inside the gear case so that oil samples are entrapped and measured each time a bearing passes by.

The quantity of oil to each gear unit is measured by positive-displacement-type meters equipped with self-venting air eliminators. The flow to individual bearings and sprays is usually measured by rotameters similar to that shown in Fig. 8. Since the rotameter case is made of glass, the oil passing through may be inspected for such things as air bubbles, cleanliness, and color.

Safety Devices. Testing large machinery units to destruction involves a certain amount of risk to personnel as well as to material. Needless to say, one of the laboratory's primary considerations in any test is safety, and many devices and procedures have been developed for this purpose. Since most gears fail while being driven at or above their full power rating, it is imperative that the driving turbine be prevented from running away when its load is suddenly removed. Some of the laboratory's smaller turbines, such as those designed for turboelectric-drive ships, are equipped with governors which, if maintained

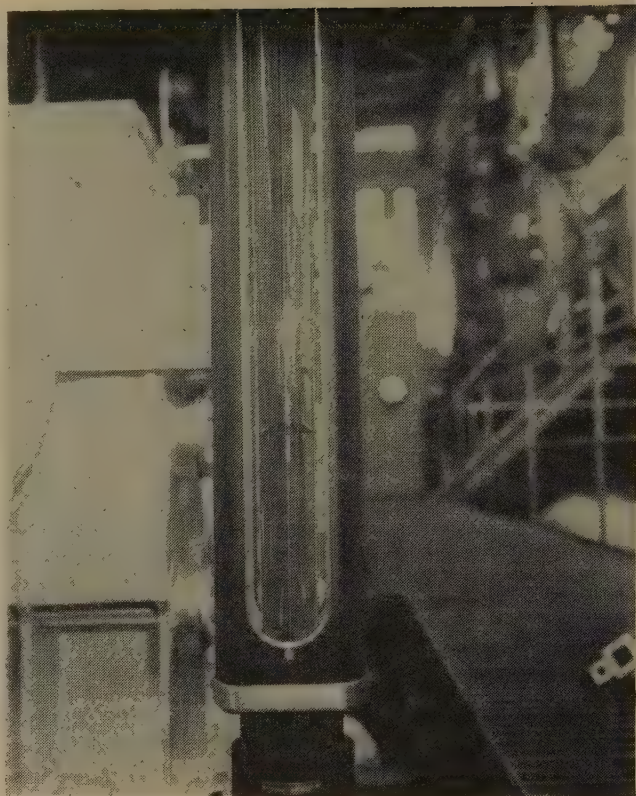


FIG. 8 ROTAMETER

properly, are adequate. Most of the larger turbines have been designed for manual control, and require the addition of overspeed protective devices before they can be used safely for test purposes.

After experimenting with several types of overspeed trips, the laboratory adopted an electrical system which has proved to be dependable, simple to operate, and adjustable over a wide range of speeds. In the steam line ahead of the turbine throttle valve is installed a solenoid trip valve, actuated by a small, 3-phase, alternating-current generator connected to the turbine shaft. Between the generator and solenoid valve is an adjustable relay whose operation depends upon the frequency of the generator output and is not affected by variations in voltage. This system not only provides reliable overspeed protection but is useful for other purposes. Turbine speeds are measured accurately by connecting tachometers and revolution counters to the generator output. Another application for this generator was found during a clutch test, where a friction-type clutch was installed between two turbines, and it was desired to determine whether the clutch was slipping while transmitting power at 3000 rpm and, if so, whether the slippage was intermittent or continuous. To each turbine shaft was attached a generator, one phase of which was connected to a cathode-ray oscilloscope. The vertical sweep of the oscilloscope was connected to one generator and the horizontal sweep to the other. The resulting pattern was such that slippage of only a fraction of a revolution could be detected, regardless of the clutch speed.

One of the problems associated with analyzing the causes of gear failures is that of stopping the gear as soon as failure commences, before the damage spreads to other parts of the unit. Most gear and bearing failures at the laboratory have occurred with little or no advance warning. In the case of bearing failure, the lubricating oil from the element concerned usually shows a slight temperature rise a few seconds before a casualty occurs,

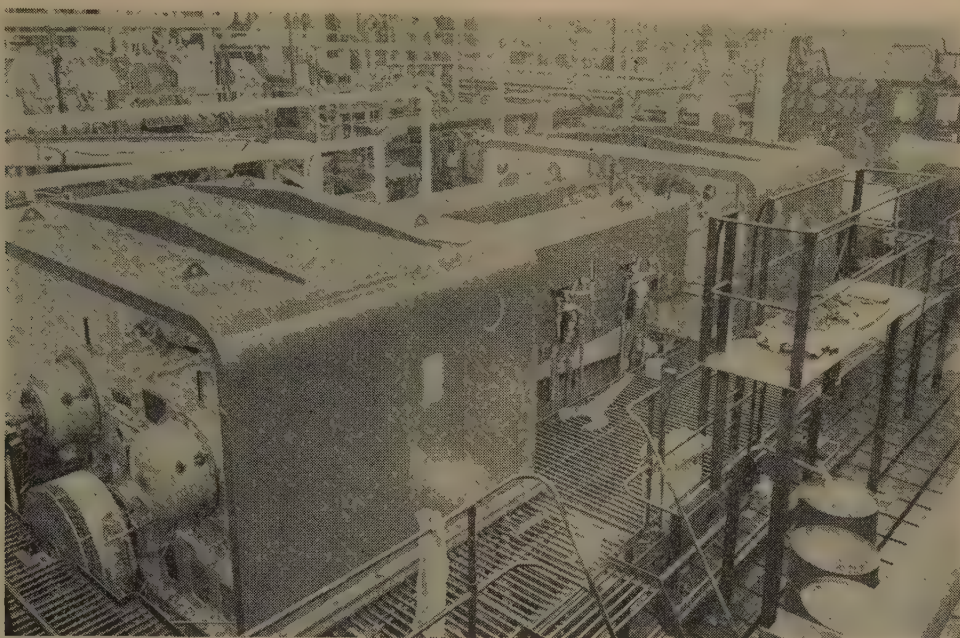


FIG. 9 PROTECTIVE ARMOR AROUND DESTROYER-ESCORT GEAR UNITS

followed by a large temperature rise as the damage spreads. Formerly, the laboratory installed alarm systems in the lubricating-oil return lines from bearings and meshing elements, so that the operators could be warned to stop the test. This proved inadequate, and several gears were completely wrecked before their turbine throttle valves could be closed. A solution for the problem was then found in an automatic system which required no action on the part of the operators. Thermocouples, installed in strategic locations throughout the lubricating-oil system, are connected through relays to the solenoid valve ahead of the driving turbine. If adjusted properly, the thermocouples actuate the solenoid valve and stop the turbine when normal operating temperatures are exceeded by more than a few degrees. In this arrangement the solenoid valve can be tripped by either the thermocouple relays or the overspeed relay, and neither system interferes with the other.

Other laboratory safety precautions include the erection of protective armor plating around highly loaded gear units, as shown in Fig. 9. Its only value to date has been psychological, since all damage has been confined by the gear cases.

Inspection. Normal test procedure consists of operating the gears continuously at constant torque and speed for some predetermined period, usually from 24 to 100 hr. The load is then increased for each subsequent period until failure occurs. At the end of each operating period the gears are dismantled and inspected carefully for signs of distress. Inspection includes measurements for wear, photographs, and Faxfilm impressions of selected surfaces. The latter procedure has been especially useful in following the progress of tooth-pitting.

Faxfilm consists essentially of a small plastic film upon which is reproduced the surface being investigated. One side of the film is softened by use of a solvent so that when it is pressed against a hard surface an impression of the surface will be made on the film. The film may then be placed in a projector and magnified to approximately 100 diam. A typical Faxfilm enlargement is shown in Fig. 10.

TEST RESULTS

Due to the large diversity of designs and characteristics of the

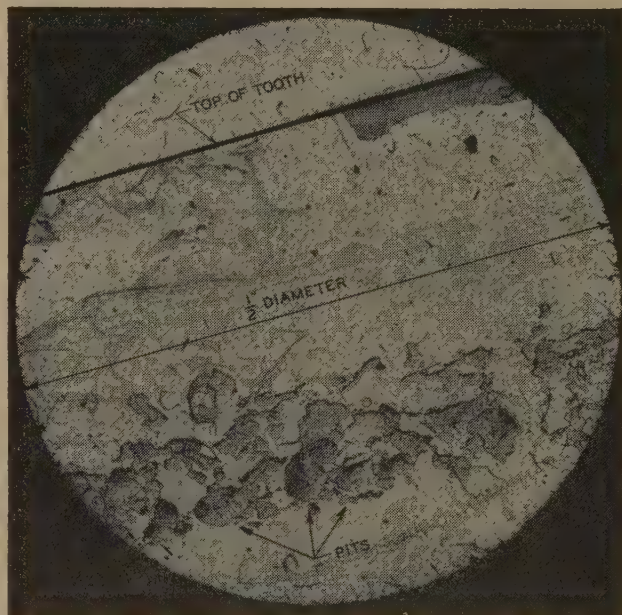


FIG. 10 FAXFILM ENLARGEMENT OF PITTED TOOTH SURFACE (Destroyer-escort hobbled gear.)

gears tested to date the results have varied between wide limits. However, each test has furnished valuable information which may be applied to future designs. In general, it may be stated that the tooth loadings permitted by Navy specifications during World War II were conservative and can be increased without compromising reliability. This statement can be illustrated best by describing a typical gear test.

Destroyer-Escort Gear Tests. Commencing in July, 1945, the laboratory conducted a series of torque tests on two standard destroyer-escort reduction-gear units. The initial purpose of the tests was to determine the influence of material and manufacturing processes on the load-carrying capacities of representative naval reduction gears.

Both gear units, obtained from Navy surplus equipment, had been manufactured by normal production methods from the same general designs, so that units built by different manufacturers were interchangeable. The gear teeth of the starboard unit had been cut by the hobbing process, with no further finishing. The teeth of the port unit had been produced by a shaping process in which the teeth are generated by a reciprocating gear-shaped cutter. The surfaces of the shaped teeth had then been finished by lapping, with the result that they were somewhat smoother than those of the plain hobbed teeth.

Each double-reduction unit contained two gear trains, one for the high-pressure turbine, and one for the low-pressure turbine, with a common low-speed gear, and had been designed in accordance with Navy specifications to transmit 6000 hp at a propeller speed of 400 rpm.

The units were coupled together for torque testing in a front-to-front arrangement shown in Figs. 2 and 3. A strain-gage torsion meter was installed on the port high-pressure quill shaft to determine locked-in torque, another on the driving-turbine shaft to evaluate power losses. Torque was applied by means of wrenches.



FIG. 11 HOBBED BULL GEAR AFTER 150 PER CENT RUN

Hobbed Gear Loaded Against Shaped and Lapped Gear. The gears were operated in 100-hr runs with the locked-in torque being increased for successive runs. All overload-capacity runs were conducted at the nominal full-power speed, i.e., a bull-gear speed of 400 rpm. At the end of the 150 per cent full-power run, tooth-pitting appeared on the hobbed bull gear, see Fig. 11. With the load at 150 per cent full power, three additional 100-hr runs were made; the pitting increased during the first two but appeared to have stabilized during the third. The shaped bull gear showed only slight pitting.

During subsequent runs the torque was increased in 15 per cent increments until 225 per cent full power was reached; thereafter 25 per cent increments were used. At the end of the 275 per cent full-power run the appearances of the shaped and hobbed bull gears were as shown in Figs. 12 and 13, respectively. The shaped teeth were still in excellent condition while the hobbed teeth were badly pitted.

Three runs were made at 300 per cent full power, followed by another at 325 per cent. At the end of the 325 per cent run, the shaped bull-gear teeth appeared to be in good condition although many of them were discolored by a brown stain, later found to be caused by fretting corrosion originating in fatigue cracks in the root fillets, see Fig. 14. The hobbed bull-gear teeth were still pitting, as indicated in Fig. 15.



FIG. 12 SHAPED BULL GEAR AFTER 275 PER CENT RUN



FIG. 13 HOBBED BULL GEAR AFTER 275 PER CENT RUN

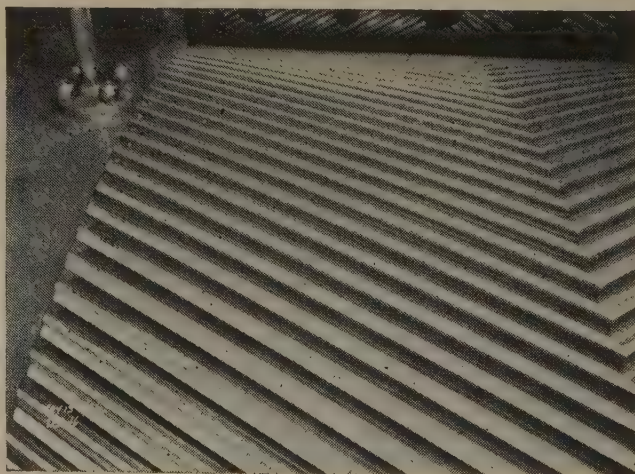


FIG. 14 SHAPED BULL GEAR AFTER 325 PER CENT RUN



FIG. 15 HOBBED BULL GEAR AFTER 325 PER CENT RUN

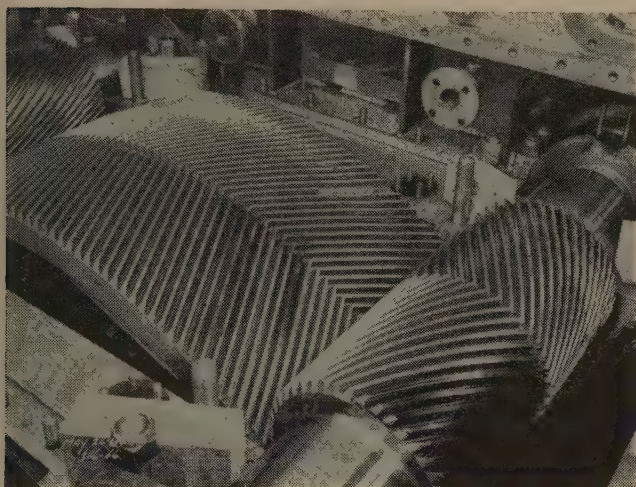


FIG. 16 SHAPED BULL GEAR AND PINIONS AFTER FAILURE

A second 325 per cent run was begun but the shaped bull gear failed after $77\frac{1}{2}$ hr due to tooth breakage. Fig. 16 shows the shaped bull-gear and second-reduction pinions after the casualty. The surfaces of these teeth were still in good condition but fatigue cracks appeared in most of the fillets, as evidenced by fretting-corrosion marks. It was found that fretting-corrosion marks indicate tooth cracks more effectively than either magnaflux or zygló equipment, and are positive signs of incipient failure.

Although both bull gears had been subjected to the same tooth loads for the same number of reversed bending cycles (about 38,000,000), the hobbled teeth showed no signs of impending breakage. The fillet radius of the hobbled teeth was $\frac{1}{8}$ in., while that of the shaped teeth was only $\frac{3}{64}$ in.; also, the shaped teeth contained well-defined toolmarks at the roots. It was believed that both the toolmarks and small fillet radius contributed to fatigue failure.

Hobbled Gear Loaded Against New Shaped and Lapped Gear. A new shaped (and lapped) bull gear and second-reduction pinions were installed in the port unit for further testing. Since all first-reduction elements were still in excellent condition, and the hobbled bull gear was still serviceable, no other changes were made. To investigate the effects of toolmarks on gear-tooth performance, the tool marks along the tooth fillets were partially removed by hand-grinding; lack of facilities prevented complete removal.

Since there had been considerable discussion concerning the effects of work hardening on tooth surfaces, it was decided to omit the customary low-power runs and to compare the pitting resistance of the new bull gear at high loads with that of its predecessor. Accordingly, the first run was made at 263 per cent full power. An inspection then failed to disclose any change in the tooth surfaces, indicating that the usual lower rate runs had little effect on the pitting resistance of shaped and lapped gears. Had the teeth pitted badly, the value of work-hardening runs at lower powers would have been demonstrated. Unfortunately, the test was not conclusive because the gear had been subjected to running-in operation at the factory.



FIG. 17 NEW SHAPED BULL GEAR AFTER 322 PER CENT RUN

At the end of the 295 per cent run a few small pits appeared on the shaped bull-gear teeth; the original hobbled bull-gear teeth were getting progressively worse but were still serviceable. The next run, conducted at 322 per cent full power, resulted in the appearance of fretting corrosion marks at the roots of the shaped bull-gear teeth, see Fig. 17. Because tooth breakage was imminent, the damaged gear was removed from test. The original hobbled gear was still capable of sustained operation.

Hobbled Gear Loaded Against Hobbled and Shaved Gear. By now it appeared that a gear possessing the beam strength of the hobbled teeth and the pitting resistance of the shaped and lapped teeth would be very successful indeed. Since no such combination was immediately available, it was decided to investigate the performance of a hobbled gear whose tooth surface finish had been improved by shaving. In the shaving process the gear is run in mesh with a small gear-like tool whose teeth are serrated with numerous small cutting edges; the cutting edges are set at an angle so that they will "shave" a small amount of material from the surfaces of the mating gear teeth as the teeth of the gear and shaving cutter slide in and out of contact. This finishing process was applied to a hobbled bull gear and second-reduction pinions which happened to be available, and they were then installed in the port unit. The port first-reduction elements and all star-board elements were left intact.

During the first 100 hr the load was increased progressively to 150 per cent full power. Subsequent runs were conducted for



FIG. 18 SHAVED BULL GEAR AFTER 200 PER CENT RUN



FIG. 20 SHAVED BULL GEAR AFTER 341 PER CENT RUN

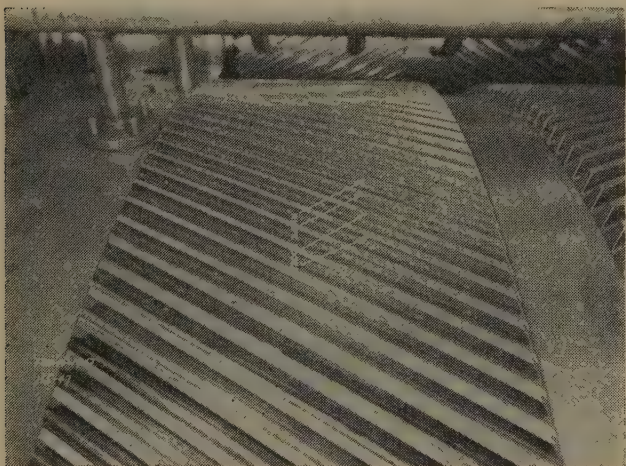


FIG. 19 SHAVED BULL GEAR AFTER 325 PER CENT RUN



FIG. 21 ORIGINAL HOBBED BULL GEAR AFTER 341 PER CENT RUN

100 hr each with the load being increased in increments of 25 per cent full power. Slight pitting appeared on the shaved bull-gear teeth at the end of the 150 per cent full-power run and slowly increased as the load was raised. Fig. 18 shows the teeth at the end of the 200 per cent full-power run. The appearance of the same teeth after the 325 per cent full-power run is shown in Fig. 19. In addition to progressive pitting, there were galling and rolling, with knife-edges on the tops of most of the teeth. The tooth surfaces of the hobbled bull gear were badly deteriorated, but still serviceable.

At this point a hydraulic torque applier was installed, thereby eliminating the difficulty of starting the units with locked-in torques greater than 325 per cent full power. The units were then brought up to speed and operated for 100 hr at 341 per cent full power. Inspection at the end of this run revealed that the shaved gear, like its predecessors, had succumbed to the pace set by the original hobbled gear of the starboard unit. The familiar brown radial stains, peculiar to fretting corrosion, emanated from cracks in every shaved tooth. At the ends of some teeth the cracks extended completely through the teeth, as shown in Fig. 20.

Although the original hobbled bull gear had succeeded in destroying three running mates, its last victory had not been an easy one. As shown in Fig. 21, a piece approximately $2\frac{1}{4}$ in. long had broken from one tooth and fatigue cracks were found

in three others. This gear had operated for 2953 hr at various loads up to 341 per cent full power, and was still capable of running at full power. However, it was decided to replace the second-reduction elements of both the port and starboard units with new elements of special designs, manufactured by experimental processes. These tests are in progress at the present time.

The purpose of the foregoing review of a typical gear test was to show the severe treatment that a conventional naval reduction gear can withstand. For this reason no attempt was made to present such details as the evaluation of test results, a discussion of the first-reduction elements, the influence of load and other factors on efficiency, and the performance of couplings and bearings. It might be of interest to point out that all runs were made without a bearing failure, notwithstanding the fact that they were subjected to heavy overloads. (There were two instances in which minor wiping of locating thrust bearings occurred—probably while starting under load.) No sign of tooth distress appeared on either hobbled or shaped first-reduction gears and pinions.

CONCLUSION

Much important information is being obtained from the laboratory's gear-testing program. This information would have only limited value, however, unless it were placed in the hands of those who design and build the Navy's reduction gears. Also, the in-

formation would be practically useless unless it were new and served to advance existing knowledge of the art.

In setting up a program to improve its reduction gears, the Navy sought help from those best qualified to give it—members of the gear industry. Their enthusiastic response led to the establishment of the Navy Gear Industry Committee, whose members represent the leading marine propulsion-gear manufacturers of the United States. This committee, through its Gear Testing Sub-Committee, has advised and guided the laboratory through every major gear test performed to date. At this time the author would like to pay special tribute to those members of the sub-committee who started in 1944, and are still working on the laboratory's testing program; they are: Mr. L. J. Collins, of the General Electric Company; Mr. H. Engvall, of the De Laval Steam Turbine Company; Mr. E. Gasser (later succeeded by Mr. N. L. Shaw), of the Farrel-Birmingham Company; and Mr. H. W. Semar, of the Westinghouse Electric Corporation.

Members of the sub-committee meet once a month to discuss and evaluate current test results, review agenda for new tests, and make plans and recommendations concerning the objectives and procedures for future tests. In addition to the regular members, every third sub-committee meeting is attended by approximately 40 representatives from various activities having interests in gear problems. A typical meeting will include representatives from the fields of metallurgy, lubrication, automotive gearing, aircraft gearing, and marine engineering, as well as from educational institutions and laboratories. The Navy is extremely fortunate in having men like these contribute their knowledge and services to the solution of its problems.

Today the Navy is engaged in a research and development program embracing many phases of science and engineering. Whatever the advances made in ships' propulsion machinery, prime movers of the future probably will rotate at speeds even higher than those currently employed. Thus there will still be a need for means of efficiently reconciling high prime-mover speeds with low propeller speeds. The propulsion-gear development program, now in progress at the Naval Boiler and Turbine Laboratory, is designed to fill this need.

Discussion

HARRY ENGVALL.³ The gear-testing program undertaken by the Navy, as discussed in the author's paper, is an outstanding contribution to the art of gear building. The tests are planned and executed in a scientific manner with adequate instrumentation to insure that all possible information is obtained from each test. The scale on which this program is carried out is of such magnitude that no private organization could hope to undertake it. The information already obtained corresponds to that resulting from many years of normal development. With all information made available to the gear industry as a whole, we can look forward to better gearing much sooner than would otherwise be possible.

The author has very ably described the testing facilities available at the Naval Boiler and Turbine Laboratory and no comments on this part of the paper seem called for, except to note that the list of equipment is indeed impressive.

The safety devices employed by the laboratory during the testing of gears have in their final development proved themselves to be very effective. Their proper functioning has at various times resulted in confining the damage resulting from minor failures to the parts involved and prevented extensive damage or complete wreckage from occurring.

In the description of the destroyer-escort gear test, the author

compares hobbled gears with shaped and lapped gears. The method used to produce the gears is not what really counts, but rather, the tooth form and surface finish produced determine the performance of a gear. In the tests described, the hobbled gear had generous fillets at the root of the teeth and therefore the teeth were appreciably stronger than the teeth in the shaped gear which had very small fillets at the roots of the teeth. The surface finish of the teeth in the hobbled gear was exactly as produced by the gear hobber; whereas, the tooth surfaces of the shaped gear had been finished very carefully after the teeth had been cut and therefore would withstand higher loadings before pitting or other surface failures would occur.

When running overload tests on gears, it must be understood that the surface stresses and bending stresses do not increase in the same proportion, since the contact stresses which cause pitting increase in proportion to the square root of the load (the contact area increases with load), and the bending stresses increase directly with load. Therefore in the tests run at 325 per cent load, the contact stress was only 187 per cent of normal full-load stress, while the bending stress was 325 per cent of normal.

It should be noted that, although the hobbled and shaped gears were subjected to the same total tooth loading, the load per inch of face was higher in the hobbled gear than in the shaped gear because of the wide gap between the two helices.

The effect of tooth form on bending stresses is considerable, and using established formulas as given in AGMA Standard 220.01 and stress-concentration factors based upon a paper by T. J. Dolan and E. L. Broghamer,⁴ the comparison of the stresses in the two types of fillets shows the following:

- (a) The mean bending stress of the small fillet tooth was 135 per cent of the stress in the full fillet tooth.
- (b) The peak bending stress for the small fillet tooth was 140 per cent of the stress in the full fillet tooth.

The foregoing comparison is based on the same quality of finish in the fillets of both tooth forms. With definite toolmarks present in the smaller fillets, an even greater difference in resistance to breakage, than just mentioned, is to be expected between the two tooth forms.

It is to be hoped that the Navy will continue its gear-testing program at least until a number of fundamental questions are answered. For example: What size tooth is best for a given application taking into account strength, wear resistance, noise, etc.? What relationship exists between load-carrying capacity and speed? How does the load-carrying capacity of a speed reducer compare with that of a speed increaser? There is no doubt that the interest and ingenuity shown by the personnel of the Naval Boiler and Turbine Laboratory will result in further rapid progress toward solving the many problems that are encountered in connection with design of modern gearing.

H. C. E. MEYER.⁵ As one who has had the opportunity of firsthand knowledge of the full-scale testing of marine propulsion gears at the Naval Boiler and Turbine Laboratory, it is a pleasure to commend the author on an able presentation of this important phase of investigations, conducted under the auspices of the Bureau of Ships, Navy Department. The contributions to the advancement of marine propulsion-plant design by the Naval Boiler and Turbine Laboratory are of great value.

The development of the hydraulic torque applier described in the paper is particularly interesting. As a point of information, do the data developed up to now indicate that the torque-testing

³ Chief Engineer, Helical Gear Department, De Laval Steam Turbine Company, Trenton, N. J. Mem. ASME.

⁴ "A Photoelastic Study of Stresses in Gear Tooth Fillets," by T. J. Dolan and E. L. Broghamer, Bulletin No. 31, University of Illinois, Urbana, Ill., March 24, 1942.

⁵ Gibbs & Cox, Inc., New York, N. Y. Mem. ASME.

method may ultimately be superior to the dynamometer-testing method, when the techniques of the former method have been perfected? The small amount of power and fuel required for driving, and the possibility of accurate determination of gear efficiencies seem to present decided advantages.

H. W. SEMAR.⁶ The real need for testing on the scale carried out at the Naval Boiler and Turbine Laboratory was long recognized by the gear builders as well as by the Navy. Hence we were not only willing but eager to co-operate and to assist in every way possible, so that the tests would yield a maximum of information.

On gears of this size the manufacturer is limited to nondestructive testing. He can carry out acceptance tests to rated power and some overload but cannot afford to risk a test which might result in the loss of a complete train of gears.

Another reason for the full-size test is that the tooth-stress limits which can be readily determined on smaller gears cannot be extrapolated to the larger dimensions, greater loads, and higher peripheral speeds of main propulsion gears. For instance, in the destroyer-escort test described in the paper, some pitting took place on second-reduction gears at 150 per cent load, while the first-reduction gears, just as highly stressed at normal power and with much higher velocity, carry 341 per cent load without surface distress. This result simply could not have been predicted on the basis of experience or test with smaller units.

From our experience with large gears, we know that there is a decidedly greater tendency to pitting on the lower-speed second reduction than on the higher-speed first reduction. In this case, a phenomenon which has been difficult to understand has been brought under laboratory scrutiny.

The author reports the 341 per cent load carried by the destroyer-escort gears as an illustration that tooth loadings can be increased. This overload should not be taken as a measure of the amount to which the rated load can be increased. The degree that power can be increased can be determined only by evaluating all of the variables associated with actual shipboard operating conditions. This point is made early in the paper, but it is one that the writer believes deserves special emphasis.

Much can be learned about some factors which influence gear performance by relatively inexpensive means. For instance, tooth materials can be investigated by roller tests to determine their relative resistance to wear and pitting. Other factors may be studied analytically. But these means yield limited information.

The final proof of a gear unit must be its performance in actual service, for there alone can it be subjected to every combination of stress which it must withstand. However, there are practical limitations to studying every variable under service conditions.

Full-size laboratory tests likewise have limitations. But when these limitations are understood and the tests are planned to supplement other sources, then only will valuable basic information be gained.

L. J. COLLINS.⁷ The author has presented in concise form the outstanding accomplishments of the subject tests. The writer feels it can be stated without reservations that the information gained by these tests has advanced the design of gears many years ahead of what could have been expected had these tests not been made.

It should be appreciated by the readers of the subject paper that further analysis must be made. For instance, since all

first-reduction elements were without distress, we must determine the reasons why this was so.

At the beginning of these tests we had anticipated distress on bearings and flexible couplings. However, these elements performed their functions without incident.

The author points out that these tests were laboratory tests on full-size units. Since this is true, then we know how units will perform under ideal conditions. If in actual service, similar units do not duplicate these results, then we know that variables are present, and we are in a better position to evaluate the cause and effect.

As a native born citizen of the United States, the writer is appreciative of the service the Navy rendered in World War II, and for the farsighted programs similar to that described by the author, which have been undertaken by the Navy for the mutual benefit of all.

H. M. KEYES.⁸ The writer feels that those who build gears of the size described, should be quite thankful that there are facilities in this country for full-scale testing. In general, there are few, if any, companies which could afford to make tests of such magnitude at their own plants. The cost of running gears for hundreds of hours would be almost prohibitive for a private concern, particularly the straight-through tests where the turbine supplies the rated gear horsepower. It is true that the Navy has a special purpose in its research—it needs heavily loaded gears. It is likewise true that taxes on the people and industry provide these facilities. Nevertheless, from the results of these tests the manufacturer obtains information which adds to his stockpile and eventually benefits him on other than Navy gears.

To give some conception of the amount of work and care with which the tests are carried out, reference is made to one test in which torque-meter readings were taken over an 81-hr period at 6-min intervals. This test was made at full speed of the gear unit and a torque load of 300 per cent of designed full torque. To show the consistency of the strain-gage meter, an average was struck through the readings, and the deviation above and below this average was found to be 0.44 per cent, i.e., less than one half of one per cent.

In order to give a rough idea of the relative loading during the 38,000,000 cycles to which the author refers, it should be noted that some 35,000,000 of these loadings were above 100 per cent torque and, of these, 20,000,000 were above 200 per cent torque.

Further, with reference to the gear-unit loading, it seems in order to mention the first reduction elements and how they stood up under the tests. The *K*-factor on the first reductions was essentially the same as that on the second reductions, while the pitch-line speed was some $2\frac{1}{2}$ times that of the second reductions. These elements did not show any distress, even though the number of cycles of loading were considerably greater than that on the second reductions. More explicitly, at the time of breakage, the first-reduction gears had sustained approximately 207,000,000 repetitions of load with some 187,000,000 of these at torques above 100 per cent, and out of these 110,000,000 loadings were above 200 per cent torque. For the high-speed pinions, the number of cycles was approximately 4 times the figures just quoted or about 800,000,000 cycles.

The absence of distress possibly might be explained by dynamic loading. The actual dynamic load may reach several times the calculated load as a result of deformation of the teeth and error in tooth form or spacing. Both reductions are of course subject to dynamic load. However, the time interval between successive contacts is considerably shorter in the case of the first reductions.

⁸ Turbine Division, Elliott Company, Jeannette, Pa. Mem. ASME.

⁶ Westinghouse Electric Corporation, South Philadelphia Works, Steam Division.

⁷ Division Engineer, Gear Engineering Division, General Electric Company, River Works, Lynn, Mass. Mem. ASME.

Mr. Buckingham has pointed out in one of his papers that if there is sufficient time interval between successive contacts, there are two peak loads for every tooth contact. As the speeds increase so that the time interval between contacts is decreased, there will not be time for the double load cycle. As the time interval becomes less, the intensity of the single-impulse load becomes less, which may be why our first-reduction element performed so well.

Finally, it is desirable to mention the increased efficiency obtained by loading the gearing somewhat closer to its strength limitation. At 300 per cent torque, the per cent power loss decreased by 2.7 per cent over the per cent power loss at 100 per cent torque.

AUTHOR'S CLOSURE

The author wishes to express his appreciation to the discussers for their thoughtful and well-prepared contributions. Their views are particularly pertinent to this subject because all of them are prominent designers of marine gearing and propulsion machinery.

Mr. Engvall has pointed out correctly that the performance of gear is determined by the characteristics of its teeth rather than by the method in which they were produced. None of the gear tests conducted to date at the Naval Boiler and Turbine Laboratory has indicated that there is any single best method of producing gears. The destroyer-escort gear test described in the paper is only one of many tests conducted with gears of various designs and produced by several different methods. The surface finish, fillet radius, and other characteristics of the destroyer-escort gears are not necessarily inherent in the methods by which they were produced.

All of the development tests at the laboratory are directed toward determining the optimum characteristics of marine propulsion gears. So far, the data have been useful for all methods of producing gears, and it is hoped that this will continue. Mr. Engvall mentioned a few of the fundamental questions for which the laboratory is seeking answers. Although most of the answers are not yet in sight, it is felt that progress is being made on many of them. Regarding the relative performance of speed reducers and speed increasers, the tests at the laboratory have indicated that at high overloads the tendency for the speed increasers to show distress first is accentuated.

Mr. Meyer's question concerning the relative merits of torque-testing and dynamometer-testing is one that has received much study and discussion. There is no doubt that the two setups represent two different dynamic systems, both of which are

different from the mass-elastic system of a shipboard installation. The dynamometer method approaches shipboard operating conditions more closely than the torque method. For example, in the latter arrangement it usually is necessary to load one unit ahead and the other astern, while some of the trains act as increasers and others as reducers. From a practical viewpoint, torque-testing requires two, two-branch gear units of opposite hand but identical in all other respects. Otherwise, a special gear train would have to be provided to complete the "four-square" loading arrangement, and probably could not be used for testing other gears of different characteristics. It thus lacks the versatility of the dynamometer.

In comparing the two methods of testing gears, the author feels that it is not a question of which reproduces shipboard operating condition the more realistically, but rather, how valuable is each as a criterion for evaluating gear performance in the laboratory, in terms of shipboard-service requirements? Based upon the performance of the same gear when tested by both methods, and upon machinery performance records from the fleet, the author is of the opinion that both the torque and dynamometer tests yield valuable data and will continue to be used. The choice of method for a particular gear test will depend upon several considerations, such as the purpose of the test, type of gear to be tested, availability of test facilities, probable duration of test, and the availability of funds.

All of the foregoing comments, and indeed the fundamental concept of testing gears, are summarized very succinctly in the last two paragraphs of Mr. Semar's discussion. Although full-size laboratory tests do not reproduce all the variables encountered in shipboard service, these test results, when evaluated properly and judiciously applied, provide a powerful tool for the gear designer.

The author is indebted to Mr. Keyes for presenting in his discussion the interesting data regarding the relative performance of the first- and second-reduction elements. As a naval officer during the war, Mr. Keyes played an active part in organizing and building up the Navy's gear development and testing program.

At the risk of oversimplification, the development testing of gears at the Naval Boiler and Turbine Laboratory may be described as exchanging dollars for data. Although many dollars have been spent to date, the data obtained have pointed the way toward saving many times their cost in future propulsion machinery. Thus, in addition to the military value of these developments—to which no dollar mark can be assigned—it is to the Navy's economic advantage to spend some of its increasingly limited funds for useful technical data.

The Parallel Development of Heavy Self-Contained Hydraulic Presses in the United States and Great Britain

By F. H. TOWLER,¹ RODLEY, NEAR LEEDS, ENGLAND

The object of this paper is to survey the parallel development of heavy self-contained hydraulic presses in the United States and Great Britain in the hope that an exchange of ideas may be helpful to future development in both countries. The self-contained press has been made possible by the development of a compact high-speed pump using oil as the hydraulic medium; and the construction of the pump has a considerable influence on the design of the press. In the United States the tendency has been to use pumps of the rotary-valve type operating at a maximum pressure of about 3000 psi, with a comparatively thick oil of around 300 Saybolt at 100 F. In Great Britain there is a growing tendency to use pumps of the seated-valve type capable of a working pressure of 7000 psi, and using a thin oil of 60 Saybolt at 100 F. The relative advantages and disadvantages are discussed.

IT is the object of this paper to survey the parallel development of heavy self-contained hydraulic presses in the United States and Great Britain, for there seems to be no doubt that our two countries have been first and foremost in the development of that type of press. It is a development which at the outset involved a change from the water-hydraulic system to the oil-hydraulic system, and consequently the shortage of oil immediately prior to and during the war constrained Germany to retain the water-hydraulic system. At any rate there is no evidence of a compara-

ble development in Germany in so far as heavy hydraulic presses are concerned, although they did of course employ oil-hydraulic transmission on aircraft, and it was developed to a very high degree of efficiency. The continent of Europe has been for so long a part of Germany, or under German domination, that we may regard it as having been similarly constrained in the development of the self-contained press.

UNITED STATES PRESSES AND THE HELE-SHAW TYPE RADIAL PUMP

The United States was first and foremost in the development of the heavy self-contained hydraulic press, which occurred around 1926, and among the pioneers we may name Howard F. MacMillin and Walter Ernst of The Hydraulic Press Manufacturing Company, Mount Gilead, Ohio. The development of a self-contained press entailed the employment of a high-speed hydraulic pump with a shaft speed of 400 to 1800 rpm, capable of reasonably high pressure, and sufficiently compact to be mounted in or on the press frame. The only pump then available was a variable-stroke radial pump invented by Dr. Hele-Shaw in the period 1914-1918, and manufactured in the United States by The American Engineering Company of Philadelphia. The construction of the Hele-Shaw pump is shown in Fig. 1 which appeared in a paper by Dr. Hele-Shaw in 1921.² At the outset, The Hydraulic Press Manufacturing Company employed the Hele-Shaw pump; later it manufactured a pump of similar

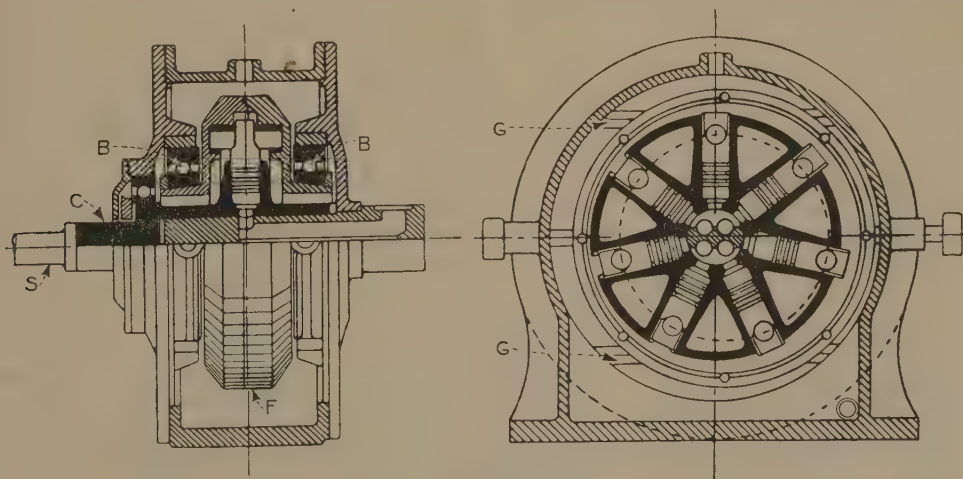


FIG. 1 HELE-SHAW PUMP

¹ Director, Towler Bros. (Patents), Ltd., and Electraulic Presses Ltd., Rodley, near Leeds, England.

Contributed by the Hydraulic Division and presented at the Annual Meeting, New York, N. Y., November 28-December 3, 1948, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society. Paper No. 48-A-33.

construction. Today, a number of firms manufacture the Hele-Shaw type of pump, and it may be said that the majority of the large self-contained hydraulic presses in the United States and Canada have been built around this type of pump. Of course a

² "Power Transmission by Oil," by Dr. H. S. Hele-Shaw, Proceedings of The Institution of Mechanical Engineers, vol. 2, 1921, p. 849.

similar development did occur in Great Britain, but not on any large scale and, in so far as the author is aware, the Hele-Shaw pump was only applied, prior to 1926 anyway, to comparatively small presses and in particular to testing machines.

ADVANTAGES OF THE HELE-SHAW TYPE PUMP

The Hele-Shaw pump, having a pintle-type rotary-sleeve valve, can be operated either as a pump or as a rotary motor. It was designed primarily as the component of a steplessly variable-speed gear, that is to say, by coupling a pump and motor of this construction together, and varying the stroke or displacement of one of them, it was possible to obtain infinitely variable speed of rotation at the motor shaft. Another application of the Hele-Shaw type, reversible-flow, variable-stroke radial pump, which has become almost universal, is to the operation of ships' steering gear.

LIMITATIONS OF THE PINTLE-TYPE ROTARY-SLEEVE VALVE

Valve Clearance and Pump Slip or Leakage. The pintle-type rotary-sleeve valve imposes certain limitations on the Hele-Shaw type of pump, such that it is the author's contention that this pump is not ideal for the operation of large hydraulic presses. The periphery of the pintle throughout 180 deg, and of a length sufficient to cover the valve ports with a suitable lap, is subject to the maximum hydraulic pressure engendered by the pump. This area of the pintle is supported between two bearings upon which the pump body rotates, and therefore the pintle must deflect under the bending moment imposed by this hydraulic load. Consequently the clearance between the pintle and valve sleeve must be sufficiently large to prevent the two coming into metal-to-metal contact due to this deflection. If the clearance is large there will be considerable valve leakage or slip; if it is too small the parts will seize. There is therefore a limitation of pressure to around 2000 to 3000 psi and, in order to maintain a reasonable clearance without serious leakage, it is necessary to use a comparatively thick oil as the hydraulic medium, an oil having a viscosity around 300 Saybolt at 100 F. Even so, after a pump of this type has been in operation for some time, the slip is in the region of 20 per cent, or in other words, the volumetric efficiency is down to 80 per cent. It must be remembered that as the slip or leakage increases, it heats up the oil, which therefore becomes thinner and increases the leakage. A pump of 100 hp may be converting 20 hp into heating up the oil in the hydraulic system, and it is for this reason that presses equipped with this type of pump are usually fitted with cooling coils to dissipate this heat. It is like a slipping clutch.

Thick Oil; Variations in Viscosity; Cavitation. One disadvantage of using a thick oil is that it has a wide range of viscosity, Fig. 2. Thus one oil which is commonly used has a Saybolt viscosity of 920 at 70 F and 120 at 140 F; therefore at low temperatures the oil may become so thick and sluggish that it will not flow into the pump chambers with sufficient rapidity and cavitation will occur. Consequently it is frequently necessary to provide both heating and cooling coils, and thermostatic control, to maintain the oil at the desired viscosity.

The use of a thick oil also imposes some limitation on the speed at which the press ram can be operated. Most self-contained hydraulic presses are of the downstroking type in which the press ram comes down to meet the work, the ram falling by gravity and the press cylinder being filled with slack oil from an overhead supply tank. The oil flows into the press cylinder through a large prefill or nonreturn valve which closes when the press ram meets the work, and then the pump supplies the pressure fluid to effect the working stroke. On completion of the stroke the prefill valve is opened to allow the oil in the press cylinder to return to the overhead tank during the rapid return

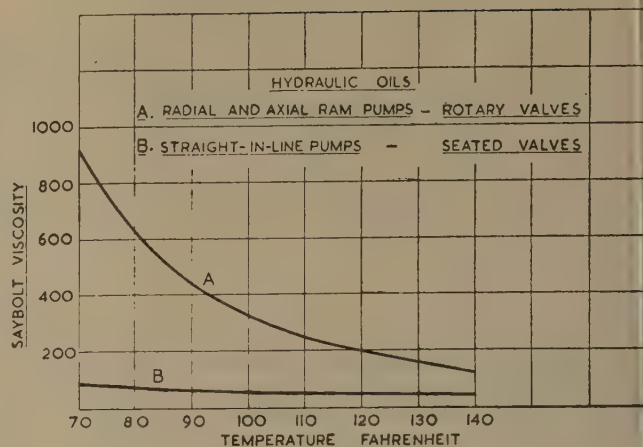


FIG. 2 HYDRAULIC OIL

stroke of the press ram, which is effected by means of push-back rams or a return annulus to which the pump is then connected. If the oil flows freely into the press cylinder during the rapid approach stroke of the press ram, the cylinder will be completely filled when the ram meets the work, the prefill valve will be closed, and the pump will immediately build up pressure in the press cylinder and perform the working stroke at the required speed. On the other hand, if the prefill valve is too small or the oil is cold and sluggish, the press cylinder will not be filled completely when the ram meets the work; consequently, after the prefill valve is closed, there will be some delay while the pump fills the press cylinder prior to building up pressure and performing the working stroke. This may be referred to as "cavitation-dwell." There is also some danger of air being drawn into the cylinder during cavitation and, if this takes place, the air will be compressed during the working stroke and the extra resilience of the liquid will impede the opening of the prefill valve prior to the return stroke. This may be referred to as "compression-dwell."

Large Prefill Valves and Pressurized Prefill Tanks. In order to avoid cavitation when using a thick oil of around 300 Saybolt at 100 F, it is usual to make the prefill valve of such a size that the velocity through the valve does not exceed 5 fps at the maximum approach speed of the press ram, assuming that the oil flows freely by gravity from the overhead tank. If the approach speed is 1 fps, this means that the prefill valve must be at least one fifth of the area of the press ram or equivalent to a pipe bore of about one half the ram diameter, which is a very big valve. Therefore to increase the permissible oil velocity through the prefill valve without cavitation, it is common practice to superimpose air pressure on the overhead supply tank. Thus additional energy has to be expended to force a thick oil at high velocity through the prefill valve on the rapid approach stroke of the press ram; and similar additional energy must be expended in forcing the oil back through the prefill valve on the rapid return stroke of the press ram. This additional energy is converted into heating up the oil in the hydraulic system, over and above that caused by pump slip. Thus we have what might be described as an "oil-boiling system."

Speed Variation and Sustained Pressure. Nevertheless, this system has certain advantages which, up to now, have seemed to outweigh the disadvantages. The Hele-Shaw type pump is reversible and capable of infinitely variable capacity by varying the stroke of the pump rams, so that it is possible to vary the speed of the working stroke during the pressing operation and to vary the speed of reversal and return stroke. It is also possible to maintain pressure at a given figure during the curing period of a

Let us put it another way, suppose the pump at this near-zero stroke is operating a press through a working stroke with a maximum pressure of 1500 psi, then it will deliver 10 gpm to the press cylinder at 1500 psi; but if the pressure required to do the job falls to 1000 psi during the working stroke, then the pump will deliver 13.3 gpm to the press cylinder, and the speed of the pressing stroke will be increased by 33 per cent, which is not so good. On certain pressing operations, such as lead extrusion or cable sheathing, it is at the slow speeds that one requires the most accurate speed control; it is when extruding small sections with a large lead ram that the speed control must be most delicate, and a variation of 33 per cent might cause rupture of the sheath or section.

³ Illustration from Janney's British Patent Specification No. 7478/08.

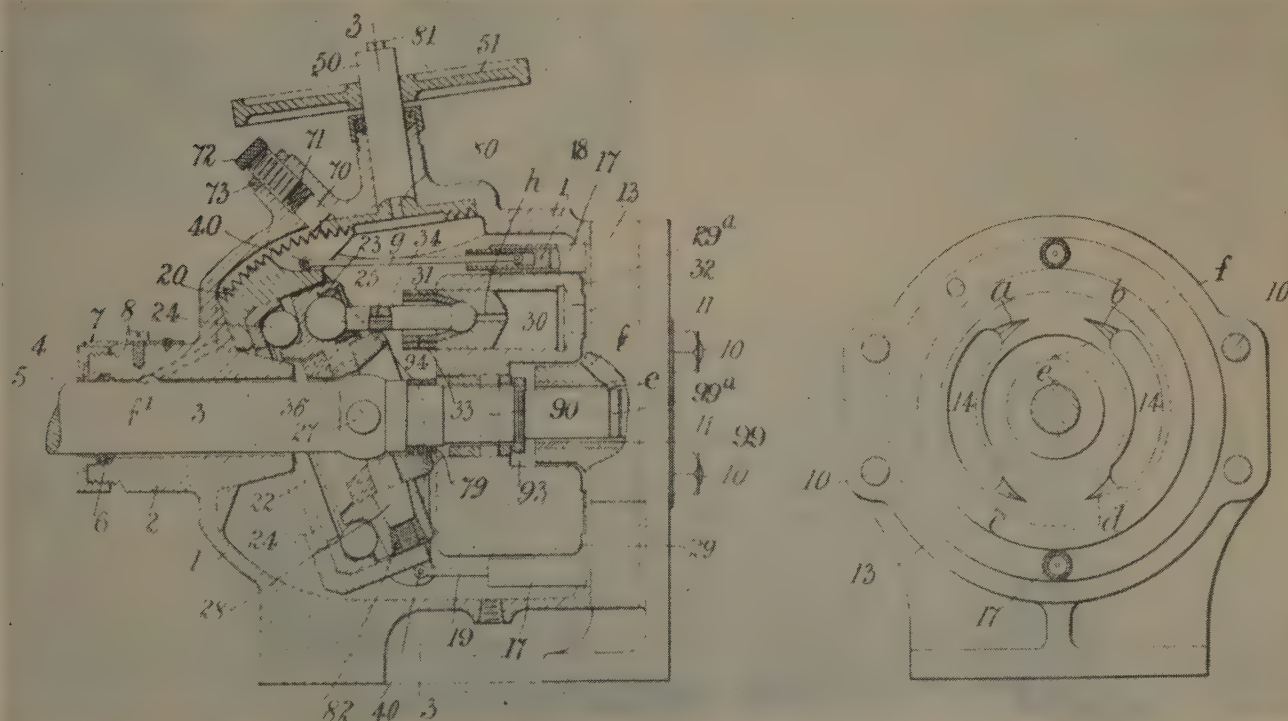


FIG. 3 JANNEY'S PUMP

Nevertheless, this lag of 8 years behind the parallel development in the United States, had its advantages because it allowed time for the development of high-speed pumps capable of much higher pressure, 6000 to 7000 psi. One pump, the Paul pump⁴ which came from the Continent, is of the radial type, having seated valves. It has been widely adopted, but its construction seems to necessitate the use of a thick oil similar to the Hele-Shaw pump, and it has a limiting shaft speed of around 750 to 1000 rpm. Also, aircocks are fitted in the end of each pump chamber to eliminate air lock, which seems to indicate that the pump is not self-priming.

STRAIGHT-IN-LINE PUMP WITH SEATED VALVES

Another pump which is manufactured by the author and his brother, and which may account for any slight bias creeping into this paper, is a straight-in-line pump with seated valves, capable of a normal working pressure of 7000 psi with a shaft speed of 1500 rpm. It is made in sizes up to 150 hp in one unit. It has a volumetric efficiency of 97 per cent at 1500 psi; the volumetric efficiency is somewhat diminished at higher pressures, owing to the compression of the oil; but even at 14,000 psi the volumetric efficiency is in excess of 90 per cent. Pumps of this construction have been operated regularly at a momentary pressure of 12,000 psi for testing gas containers for aircraft. The mechanical efficiency is 96 per cent at 7000 psi or maximum pump-ram load. Fig. 4 shows the general construction of the pump, and Fig. 5 is an enlarged section through the pump chamber.

Advantages and Limitations of Straight-In-Line Pump. This pump has four limitations: It is expensive to manufacture; the stroke is not variable without considerable complication; the flow is not reversible; and it will only operate satisfactorily with a very thin oil having a viscosity around 60 Saybolt at 100 F.

Cost of Manufacture. It is expensive to manufacture because the three-throw pump employs two roller-bearing journals and three roller-bearing eccentrics; and the five-throw pump, in addition to the journals, employs five roller-bearing eccentrics; the eccentric bearings are "specials," because in the large pumps the ram load on each eccentric is over 3 tons. To offset this, the pump has a life of 3 to 6 years without overhaul under heavy working conditions; it will stand heavy momentary overloads of 50 to 100 per cent on occasion.

The life of the pump is determined by the roller-bearings; in other words, an overhaul means a new shaft and bearings. Strangely enough, the suction and delivery valves and seats

do not show much sign of wear after 3 to 6 years, provided that a thick oil is not used. This may be accounted for by the fact that at 1500 rpm the valves have to make a complete cycle, open-close, in $1/50$ sec; and the valves are not seated under pressure for more than $1/50$ sec. Hence there is always an oil film between valve and seat, i.e., there is insufficient time to squeeze it out. On the other hand, the use of a thick oil would cause cavitation and valve hammer, with disastrous results.

Variation of Stroke Not Practicable. Another limitation is that the capacity of the pump is not readily variable. However, this is not so serious as it might seem. There are many pressing operations in which it is of no advantage to vary the speed of the pressing stroke; full speed is required right through. On the other hand, with up-stroking presses and horizontal presses, a

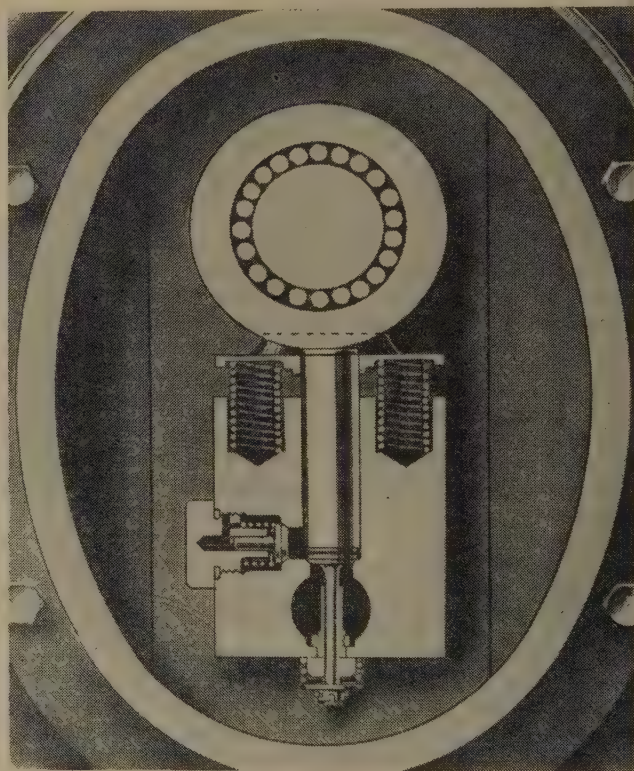


FIG. 5 ENLARGED SECTION THROUGH PUMP CHAMBER

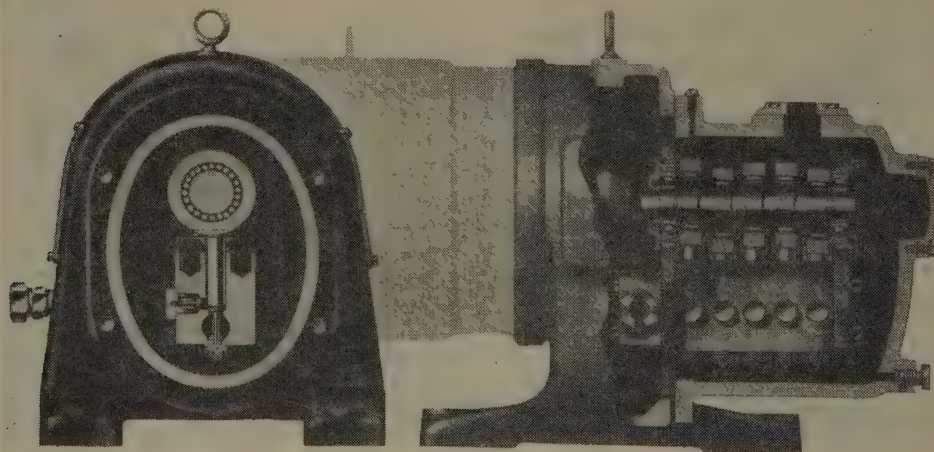


FIG. 4 STRAIGHT-IN-LINE PUMP

⁴ British Patent No. 341,667. 1929.

ELECTRAULIC PATENT DOUBLE ACTING
CONTROL AND UNLOADING VALVE
1" NOMINAL BORE

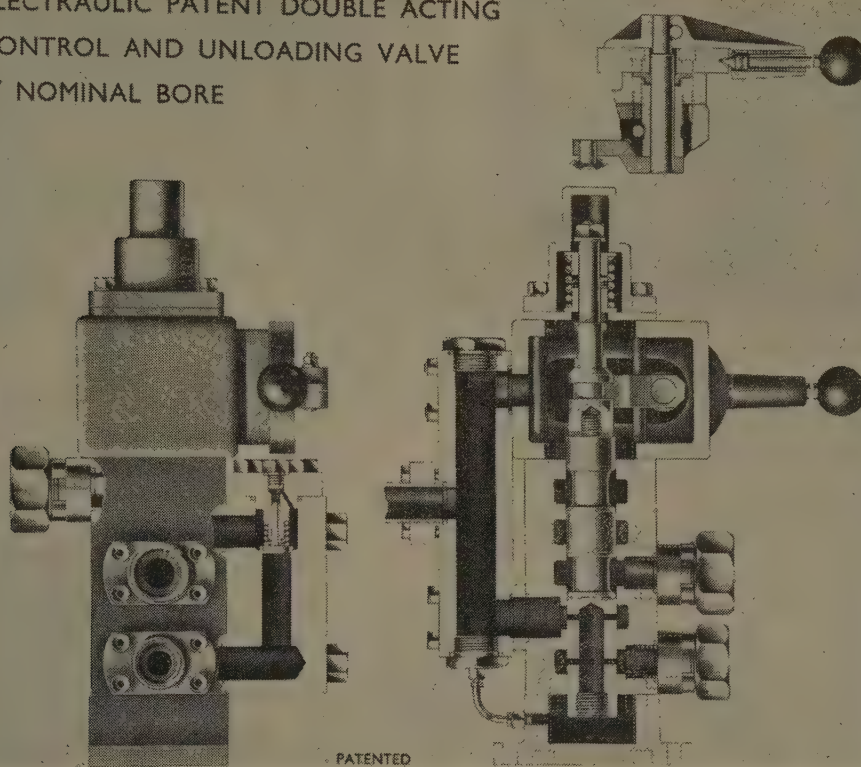


FIG. 6 DOUBLE-ACTING CONTROL VALVE

two-stage pump is most suitable. An auxiliary low-pressure gear pump or vane pump is used to effect the rapid approach stroke, and a high-pressure ram pump to effect the working stroke of the press ram.

Reversing Flow by Control Valve. In pumps of the seated-valve type, the flow through the pump is not reversible, and therefore it is necessary to use a double-acting control valve, such as that shown in Fig. 6, to control the forward and return movement of the press ram. This is not necessarily a disadvantage, because a valve spindle usually can be moved much more easily and quickly than the reversing gear of a radial or axial pump.

Controlled Steplessly Variable Capacity Without Stroke Variation. In those pressing operations in which it is necessary to vary the speed, it is seldom essential that the speed should be steplessly variable. With a combination of three constant-delivery pumps, which may be driven by one shaft, and having capacities of 1, 2, 4 gpm, it is possible to obtain seven speeds corresponding to 1, 2, 3, 4, 5, 6, 7 gpm. If steplessly variable speed is essential, it is possible to bridge the gap between speeds by using a metering valve to by-pass up to 1 gpm. This will give a steplessly variable-speed control which is practically unaffected by variations of pressure, because a pump, having a volumetric efficiency of 97 per cent, has no appreciable slip; whereas already it has been shown that a variable-stroke pump, having a rotary valve, does not control the speed at near-zero stroke; the speed is then a variable dependent upon slip and pressure. Controlled speed is usually more important than an uncontrollably variable speed.

Sustained Pressure. This still leaves us with near-zero delivery at full pressure, such as is required to sustain pressure during the curing period of a compression-molding press, or the clamping pressure for a die-casting or injection-molding machine.

It has been shown previously that this near-zero delivery

with the Hele-Shaw type of pump is really a quite substantial delivery, because the pump is in continuous operation, pumping its own slip in addition to making up leakage in the system. This sort of operation can be performed much more satisfactorily by a constant-delivery straight-in-line pump provided with a pressure-actuated unloading valve which by-passes the pump to tank when maximum pressure is reached, and reloads the pump or closes the by-pass when the pressure has fallen a given amount. In this way the pump is running idle most of the time, circulating oil at tank pressure, and it is loaded momentarily at intervals to make up leakage. Furthermore, the full capacity of the pump is immediately available when required, without any stroke-changing lag. With this system, the pump is not an "oil boiler" and therefore no cooling coils are required.

Advantage of Using a Thin Oil. The straight-in-line pump will only operate satisfactorily with a very thin oil. It will be obvious to anyone who has had the misfortune to read thus far, that this is an almost unmitigated advantage. For what is good for the pump is good for the press. A thin oil is essential to this type of pump in order to insure that the oil will flow freely through the spring-loaded suction valve, with the assistance of atmospheric pressure and only a few feet of head, and completely fill the pump chamber in $1/60$ sec. That is exactly what we want for the press. The oil must flow freely through a spring-loaded prefill valve into the press cylinder with the greatest rapidity under almost identical conditions; and that is what it does.

It has been proved by experiment⁵ that an oil at a viscosity of 165 Saybolt will flow through a prefill valve at 33 fps by gravity and with the assistance of atmospheric pressure alone, without cavitation in the press cylinder. Thus it is quite safe to allow for

⁵ "The Modern Direct-Hydraulic System," by F. H. Towler, Proceedings of The Institution of Mechanical Engineers, vol. 154, 1946; discussion by W. F. Cully, p. 190.

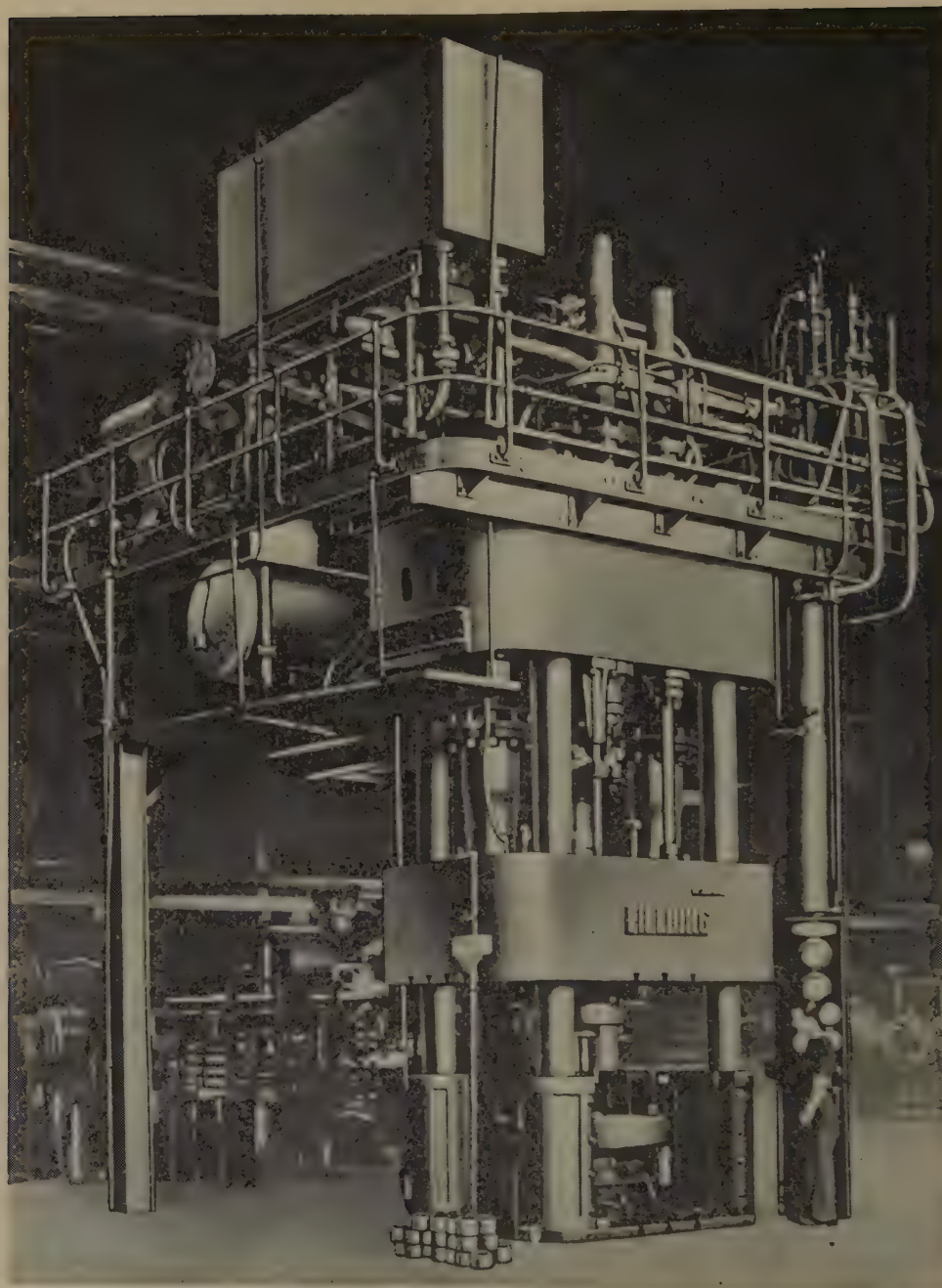


FIG. 7 1700-TON FORGING PRESS, OPERATED BY THREE 150-HP PUMPS, 4500 PSI

a prefill velocity of 20 fps, with an oil of 60 Saybolt, at maximum approach speed. Even when the oil is cold and old its viscosity will not exceed 165 Saybolt—and it must be admitted that oils do get thicker after much use, due to oxidation and the assimilation of foreign matter of colloidal dimensions. Here, by way of qualification, it should be said that this type of pump is equally capable of accepting oil of a viscosity up to 165 Saybolt. It will be appreciated that one of the greatest advantages of a thin oil is the fact that it has such a flat viscosity curve, as shown in Fig. 2, so that the performance of a hydraulic press does not vary with the temperature of the oil, and there is no need for thermostatic control.

Possible Disadvantages of a Thin Oil. A possible disadvantage of using a thin oil is that it requires very high precision in the

manufacture of the packingless-piston-type control valves for the press, Fig. 6.

They must be a fine lap fit, with a clearance not exceeding 1 micron per in. of diam, in order to hold pressure satisfactorily at 7000 psi. As to press glands, there does not appear to be any greater difficulty in holding a thin oil than a thick one; furthermore, if a gland packing is good for 3000 psi, it would seem to be equally good for 7000 psi. The disadvantages of a thin oil are probably more apparent than real.

Fig. 8 illustrates an 850-ton press having a 20-in-diam piston fitted with five plain cast-iron piston rings, and operating at 6000 psi. Several of these presses have been in continuous operation for the past 5 years, and there is no sign yet of undue cylinder wear or excessive leakage. Fig. 7 shows a 1700-ton forging

press,* operated by three 150-hp pumps, 4500 psi. Fig. 9 gives some idea of the size of these pumps.

Compressibility of Oil at High Pressures. Thin oil is slightly less compressible than thick oil, and it may be less liable to aeration, but compressibility does become more of a problem at higher pressures. As a rough approximation, oil compresses $\frac{1}{2}$ per

* U. S. ton of 2000 lb.

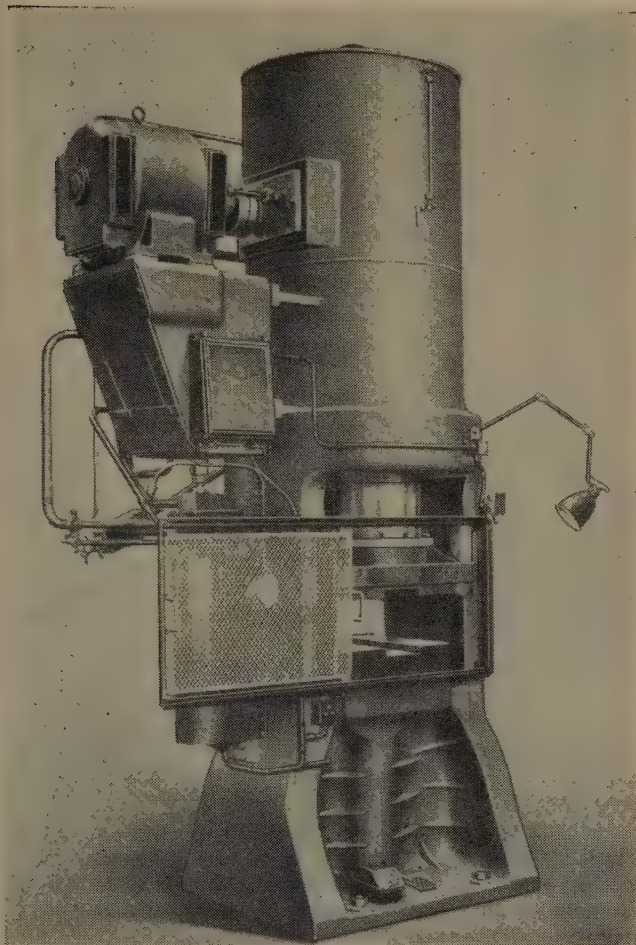


FIG. 8 850-TON SELF-CONTAINED PRESS, 6000 PSI

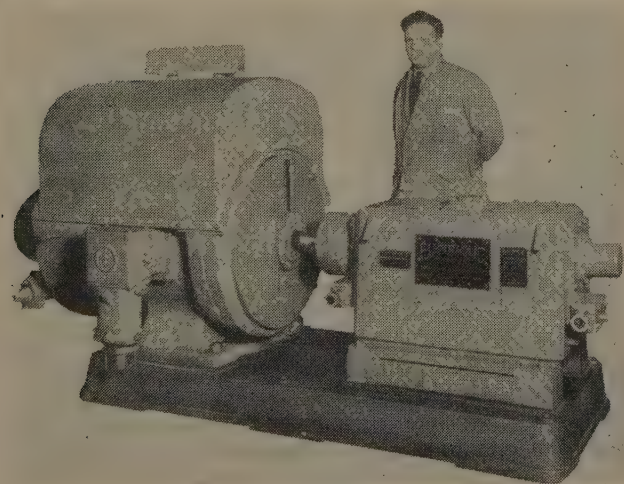
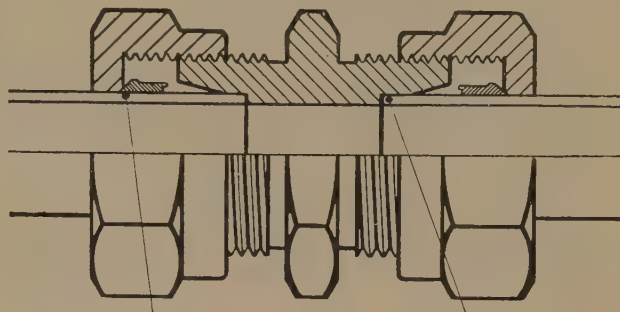


FIG. 9 150-HP STRAIGHT-IN-LINE PUMP, 4500 PSI

cent per 1000 psi and therefore the energy stored in the oil at 6000 psi is twice that which would be stored in a press of the same power operating at 3000 psi. However, this is not a serious difficulty, provided proper provision is made to release this energy at the end of the pressing operation before the press ram is reversed. It should be emphasized that the actual loss of energy is usually more than offset by the saving in energy required to move one half the volume of oil throughout the operating stroke of the press.

Pipe Fittings for 7000 Psi. As for pipe fittings, it will be admitted freely that the taper-screw thread, which is common

BEFORE ASSEMBLY.



AFTER ASSEMBLY.

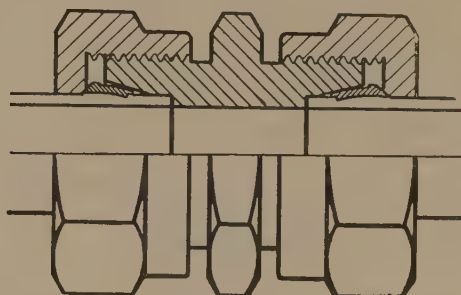


FIG. 10 ERMETO COUPLINGS

practice in the United States for low pressure, is not good enough for 3000 psi and it is certainly not good enough for 7000 psi. The author's firm has used "Ermeto" couplings, Fig. 10, on pipes up to 1 in. bore at pressures of 7000 psi for the past 10 years, and they have proved highly satisfactory. Similar couplings, "Ferulok" and Ermeto, are now available in the United States. These couplings have the advantage that the pipe is not weakened by screwing, and therefore it is not necessary to use heavy-gage tubing for 7000 psi. A pipe, which when screwed would only be good for 3000 psi, would be quite satisfactory for 7000 psi in an Ermeto coupling. However, for pipes larger than 1 in. bore, flanges are to be recommended.

CONCLUSION

The author suggests that for heavy self-contained hydraulic presses there is considerable advantage to be gained in using a pump of the seated-valve type, capable of operating satisfactorily at working pressures up to 7000 psi with a thin oil of 60 Saybolt at 100 F. Compared with present practice in the United States, the following advantages are claimed:

- 1 Operating at twice the pressure, the press rams and cylinders are proportionately smaller.
- 2 Only half the volume of oil has to be moved.
- 3 The oil can be moved at 4 times the speed; 20 fps instead of 5 fps, through the prefill valve.
- 4 It is possible to use smaller pipes and valves for the same pressing speed, or to operate at up to 8 times the speed with the same-sized pipes and valves.
- 5 The oil has a flat viscosity curve, so that there is little variation of viscosity with variations of working temperature. There is no need for thermostatic control.
- 6 There is no need for air pressure to force the oil through the prefill valve into the press cylinder.
- 7 With the absence of pump "slip," there should be no need for cooling coils in a properly designed hydraulic system.
- 8 For the same reason (no slip) it is possible to control the pressing speed more accurately.
- 9 Pumps of this type have a longer working life, can stand serious overload, and they are not so easily damaged by the presence of grit and scale in the hydraulic oil.

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Discussion

T. E. BEACHAM.⁷ The writer was associated with the late Dr. Hele-Shaw, from 1909 onwards. English press manufacturers took considerable interest in the Hele-Shaw pump during the early stages of its development but they insisted on pressures of 2 tons per sq in. and upward to suit existing press designs whereas it would appear that American manufacturers were prepared to make large presses for lower operating pressures.

The practical limit for the early Hele-Shaw pumps was in the neighborhood of 1 ton per sq in. on account of risk of seizure on the central valve. A two-stage pump gave satisfactory results on test up to 6000 psi, but this design was never used in practice, and Hele-Shaw pumps for press operation were usually installed with intensifiers, as a rule with an automatic valve which changed the connections as soon as a pressure of 1 ton per sq in. was reached.

Pumps with seated valves have not the pressure limitation imposed by the rotary valve, and the author and his brother have made an important contribution to hydraulic-machinery design by their work in developing pumps of this type up to speeds of 1500 rpm.

High-pressure pumps with seated valves, almost of necessity,

⁷ Consulting Engineer, London, England.

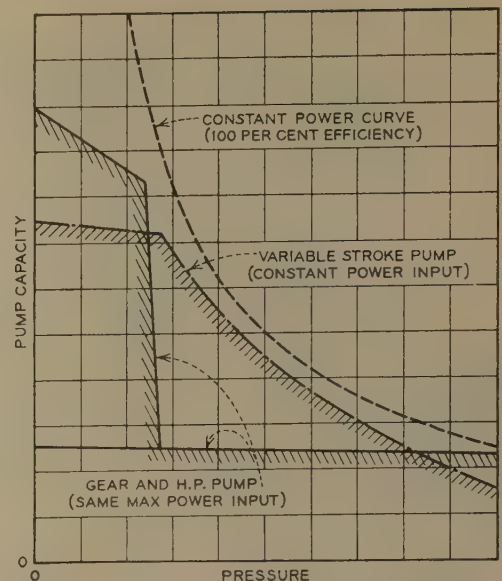


FIG. 11 COMPARATIVE PERFORMANCE OF VARIABLE-STROKE AND TWO FIXED-CAPACITY PUMPS UNDER VARIABLE PRESSURE AND CONSTANT POWER CONDITIONS

have fixed cylinder blocks and rotating cranks. While a very large number of mechanisms have been evolved for obtaining variable stroke from rotating cranks, these have never come into general use except for such purposes as steam-engine valve gear.

The more widely used types of variable-stroke pumps have nonrotating cranks where the adjustable element can be directly attached to the control mechanism. The nonrotating crank involves a rotating cylinder block which in turn is ideal for use with a rotary valve but most unsuitable for seated valves.

In general, therefore, the variable-stroke pump suffers the limitations imposed on it by the rotary valve, and on the other hand the seated-valve type of pump has the limitation of fixed capacity.

The author has pointed out that variable output can be obtained by using two or more fixed-capacity pumps. Where a constant power characteristic is required, the combination of a gear pump with a fixed-capacity high-pressure pump will give approximately the same operating time as a variable-stroke pump with constant power control. The point is illustrated by Fig. 11 of this discussion, in which the chain dotted line shows the capacity/pressure relationship for a variable-stroke pump with a 3/1 range, and the full line the same relationship for a gear pump in combination with a high-pressure pump of the seated-valve type, the relative capacities of the combination being 3 to 1.

When direct control of speed is required, the use of two fixed pumps will give 3 alternative capacities, three pumps, 7 capacities and 4 pumps, 15 capacities, and so on. For radial and swash-plate types of pump, two pumps can be arranged conveniently in the same casing.

It is the opinion of the writer that for the great majority of press applications, the balance of advantages is in favor of the high-speed seated-valve type of pump.

G. I. CHINN.⁸ The author has presented an able story for the constant-displacement pump in press applications. However, we wonder whether he has experienced any difficulty with his double-acting control valve because of binding of the piston in the body. It seems to us that with clearances of the order of one micron per

⁸ Chief Engineer, Gerotor Pump Division, Gerotor May Corporation, Baltimore, Md. Mem. ASME.

inch of diameter, which, incidentally, is difficult for us to visualize from a production standpoint, changes in oil temperature or deformation of the valve body under high pressure would take up this clearance with sufficient binding to preclude actuation by the lever means shown. Further, operation of balanced piston valves under high pressures for long intervals usually results in a hydraulic bind due to the building up of a leakage path on one side of the plunger at the expense of the other, thereby forcing the plunger against the body. So-called equalizing grooves, balancing slots, and mounting the plunger in bearings not exposed to the hydraulic pressure are methods usually employed to overcome this defect, but none is completely successful. What has the author done in this connection?

WALTER ERNST.⁹ The author presents his case for a seated-valve-type high-speed pump for the operation of self-contained hydraulic presses. Use of this pump, according to him, results in the following characteristics and advantages:

- 1 High volumetric efficiency of pump, no slip, less heating.
- 2 Low-viscosity oil, flat viscosity curve; advantages operational:
 - (a) No thermostatic control of oil necessary, due to flat viscosity curve.
 - (b) No cooling of oil required, due to high efficiency.
 - (c) High prefill efficiency and gravity prefill, due to light oil.
 - (d) Accurate control of pressing speed, due to high volumetric efficiency.
 - (e) Rugged construction, due to absence of pintle.
- 3 High working pressure; advantages: first cost, small cylinders, pipe, and valves.

The author has made a good case for his selection of type of pump and operating system. A closer examination of some of these points calls for comments which may modify the final conclusion.

The writer does not subscribe to the categorical statement that "after a pump of this type has been in operation for some time, the slip is in the region of 20 per cent." Volumetric efficiencies of pintle-type pumps range from 85 to 95 per cent, depending on make, pressure, type of design, and capacity. Mechanical efficiencies range from 90 to 95 per cent. These efficiencies may or may not deteriorate, depending on service conditions, maintenance, cleanliness, and so on. The writer does not consider efficiencies of pumps too important for hydraulic-press service, and in his experience the volumetric losses have only a second-order effect in high-speed press operations, due to the fact that the pumps are under pressure only a fraction of the time and at peak pressure only a fraction of that fraction (this refers to high-speed metalworking or forming presses, not to extruders or upsetters). The writer has found that mechanical losses in pumps and presses, turbulence and eddy losses in the system, and auxiliary heat-generating devices bear the greatest responsibility for temperature rise of the oil.

Radial and axial pintle and flat-plate-valve pumps do require oils of higher viscosity to maintain pressure-sealing oil films. From an extreme of 1500 SSU used in some early Hele-Shaw pump installations, most manufacturers have gradually worked down to the 300 SSU which the author mentions. A few die-hards still use viscosities upward of 500 SSU for little more than historical reasons.

Radial-pump installations are equipped with coolers, not so much for the reason that they create more heat than other systems, but because they stand less heat. This is due to the temperature-viscosity relationship pointed out by the author. It is

the writer's opinion that here there is much room for research and improvement on radial-pump systems. The oil companies must continue research to flatten the viscosity curves of their oils and produce oils which withstand higher temperatures, and information has been received that they are doing this. The new synthetic oils may be a step in that direction.

Pump and press manufacturers must improve their designs, so that oils of still lower viscosity may be used, and the units will work at higher temperatures. Ambient temperatures have a great deal to do with necessity for cooling coils. A production shop in Detroit in midsummer is a lot hotter than one in the Midlands. The writer believes that ultimately we will see elimination of cooling coils, but before this happens, the press companies must make material changes in such heat-producing devices as die cushions, blankholders, and the like, which waste a disproportionate amount of power and create excessive heat. So much for points (a) and (b).

As to point (c), the writer's experience has been that within the commercially used range the viscosity of the oil has no bearing on the efficiency of prefill. Speeds of not 5 but 10 fps are regularly used by one of the large hydraulic-press manufacturers, with oils as high as 1000 SSU, and ram speeds of 200 fpm and more are common. The writer has operated valves at 20 fps experimentally without cavitation. With a little more fairing and streamlining, speeds of upward of 20 fps with oils in the 500-sec viscosity range are entirely feasible and practical without traces of cavitation. The reason for this is that most prefill valves operate above the critical range of Reynolds numbers, from 3000 on up, where the viscosity has little or no influence on the flow resistance. Air pressure prefill, used in the early days of development, has been completely abandoned and to the writer's knowledge there is no press manufacturer in the country at this time using air pressure prefill.

As to point (d), much more accurate control of linear pressing speeds may be had with an infinitely variable delivery pump than with any other means. Controls, ranging from simple slip compensators to servomechanisms with feedback from the press platens to the control to continually correct deviation from the ideal speed by stroke adjustment, have been developed and are available at reasonable cost.

Concerning point (e), performance records of radial variable-delivery installations are available equaling or bettering the figures mentioned by the author and under conditions of incredible neglect.

The writer should like to add two outstanding points which have not been covered in the paper. These are the utilization of the reversible-discharge and variable-delivery characteristics for the attainment of certain specific purposes.

He contends, and realizes that he is starting a controversy, by the use of the reversing type of pump, radial or axial, a heavy press, machine tool, or other hydraulic device may be operated at high speed with a smoothness of acceleration and deceleration, decompression of oil, and freedom from hydraulic shock not attainable by any other means. To decelerate, stop and maintain in suspension a weight of several tons operating at a speed of 200 fpm, is easily accomplished by a hookup to the stroke-reducing control of the pump. To do the same thing with a large-capacity constant-delivery pump would seem quite an engineering accomplishment to the writer.

In the application to a compression-molding press during the curing process, the variable-delivery pump exhibits a characteristic called "follow-up." This application is only superficially a sustained pressure application. In reality the material during the curing process gives and yields, and the pump must follow on the heels of this yielding and keep up the pressure. It acts very much like an accumulator which has earned this type of control

⁹ Vice-President for Engineering, The Commonwealth Engineering Company, Dayton, Ohio. Mem. ASME.

the name "accumulator-type control." A constant-delivery on-off control cannot duplicate this action, and the writer ventures to say that delicate parts, such as Amphenol plugs, could never be produced on a press thus equipped.

As to point (3), a high working pressure is not an unmixed blessing. Let us make a few calculations to see what happens to the speed of a press at higher working pressure. To this end we shall compare two presses of equal tonnage and horsepower, one operating at 3000 and one at 6000 psi.

Dow and Fink¹⁰ have shown that the bulk modulus of elasticity of oil is

$$C = \frac{A - 2Bp}{1 + Ap - Bp^2}$$

where $A = 4.38 \times 10^{-6}$ and $B = 5.65 \times 10^{-11}$ (at 100 F).

Thus

$$\begin{array}{ll} \text{At } p = 3000 & C = 4.10 \times 10^{-6} \\ \text{At } p = 6000 & C = 3.65 \times 10^{-6} \end{array}$$

If we assume now that the low-pressure press has a cylinder volume of 10,000 cu in., and the high-pressure press one of 5000 cu in., the additional oil required to produce full tonnage will be for the low-pressure press

$$V_L = 10^4 \times 3 \times 10^3 \times 4.10 \times 10^{-6} = 120 \text{ cu in.}$$

and for the high-pressure press

$$V_H = 5 \times 10^3 \times 6 \times 10^3 \times 3.65 \times 10^{-6} = 110 \text{ cu in.}$$

If we assume now that the 3000-psi pump puts out, say, 240 cu in. per sec, then the time to compress the oil will be $\frac{1}{2}$ sec.

If the 6000-psi pump, having the same horsepower, puts out 120 cu in. per sec, then the time to compress the oil will be 0.91 sec. That this is a serious disadvantage for a production press cannot be denied. The American owner of the press is not interested whether this energy loss can be recovered by saving in energy required to move $\frac{1}{2}$ of the volume of oil. All he is interested in is how many pieces he can get off the press per unit time.

Now, what does all this add up to? The writer believes that the author's system possesses outstanding advantages in the application to heavy extruders of aluminum, magnesium, and brass shapes and tubing, for coating presses for welding electrodes and lead sheathing, for die casting and injection machines, upset and extrusion forging machines, wherever heavy sustained pressure and volumetric output are required. He questions whether it will displace present reversible-discharge radial- and axial-pump systems on high-speed medium and light production presses for metalworking, forming, drawing, and straightening.

Compression molding presses had best be operated by variable-delivery pumps or accumulators, and in that connection the Greer hydraulic accumulator seems to hold promise.

J. G. FRITH.¹¹ The writer's company has used the author's pumps on several occasions, not only in conjunction with presses, but also in combination with air-loaded accumulators to serve the roll-balance cylinders on rolling mills, and also to operate the hydraulic roll-feeding mechanism for a railway wheel rolling mill.

In all cases we have had no trouble whatever and have found them a very reliable type of pump.

C. J. LAMB.¹² The author is to be congratulated upon the sub-

¹⁰ "Some Properties of Lubricating Oils at High Pressure Density," by R. B. Dow and C. E. Fink, *Journal of Applied Physics*, vol. 11, 1940, p. 353.

¹¹ Davy and United Engineering Company Limited, Sheffield 4, Yorks, England.

¹² Consulting Engineer, The Hydraulic Press Manufacturing Company, Mount Gilead, Ohio. Mem. ASME.

ject matter of this paper. His company can take pride in the development of the pumps described, working at such pressures and efficiencies, with such extremely small clearances. In view of his reference to our company and its part in pioneering the development of the heavy self-contained hydraulic press, and since our radial pump is also mentioned, we feel that comment by us is in order.

Like the author, we also may have a slightly biased viewpoint. We do desire to point out, however, that when, after building high-pressure water hydraulic presses and pumps (up to 15,000 psig) for approximately 50 years, we went into the development of the heavy self-contained oil hydraulic press, we gave full consideration to the oil pressures to be employed and to the types of oil pumps available, both here and abroad. After shop and field testing of various types of pumps, we were forced to the conclusion that in order to maintain our known reputation for reliability and freedom from trouble, it would be necessary to develop and build our own pumps. It should be emphasized here that while we adopted the principle of the variable-stroke radial pump invented by Dr. Hele-Shaw, our construction is not at all similar and varies appreciably from that design primarily to overcome the limitations referred to by the author. As may be noted in the illustration of our pump, Fig. 12 herewith, we have incor-

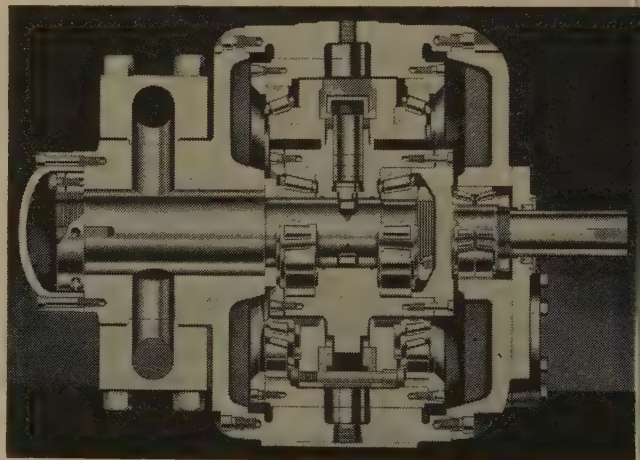


FIG. 12 VARIABLE-STROKE HYDRAULIC PUMP

porated into our design patented features which practically eliminate the pintle deflection referred to as a limitation of the Hele-Shaw pump. The unique feature, whereby we maintain the close clearance between the rotor and the pintle, lies in the use of adjustable tapered roller bearings, mounted between the rotor and pintle, adjustable, moreover, to micrometer accuracy in the field by the operating personnel without opening up or dismantling the pump.

With respect to the question of oil viscosities, we do use higher oil viscosities than employed in the author's pumps. While it is true that the friction losses in pumps result in oil-temperature rises, it is neither difficult nor expensive to maintain the oil at the temperature range of 90 to 120 F for which we design. The heavier oil is commonly available throughout the world, and our pumps are frequently used in remote places. Oil at high pressure and low viscosity is harder to contain, and the high-pressure system presents a greater fire hazard due to lower flash and fire points. Thus our successful use of heavier oil at lower pressures over a long period of years has been well justified. As might be expected, tests have demonstrated that the viscosity of hydraulic oils under pressure is appreciably higher than the same oils at atmospheric pressure.

With respect to the points raised regarding ram speed, uncontrolled speed variations, and the like, we have not encountered such difficulties. We are convinced that the practical and efficient development of the wide variety of hydraulic processes we have pioneered has resulted because the press, pump, and valves which make up the complete hydraulic system were designed as a unit to accomplish a specific result, rather than being assembled of existing components. Some of our presses operate at speeds of 60 to 100 cycles per minute, for example. Many of our systems require reversible flow and variable volume in either direction of flow, each cycle.

The author has frankly admitted that his pump has four limitations. He has omitted mention of a fifth one, namely, that of greater power consumption, required by the straight-in-line pump. From our viewpoint, these limitations are of such magnitude that they would have prevented the development of the majority of modern press applications at economically justified costs.

While we cannot speak on behalf of the various builders of Hele-Shaw-type pumps, the records of thousands of installations throughout the world for an extended period of time demonstrate that our pump efficiencies are maintained at approximately 90 per cent, and that the pump life, before overhaul is necessary, ranges from 6 to 12 years, depending upon the service. For example, our first pump installed for the operation of a marine steering gear in the Mississippi River automobile ferry motor vessel *Westside*, has operated 11 years (this is a single pump installation) without repairs or overhaul, and there are many other similar installations which might be mentioned.

AUTHOR'S CLOSURE

Mr. T. E. Beacham's contribution to the discussion is particularly valuable because he designed the original Hele-Shaw pump. He has had a very wide experience in the development of the oil-hydraulic system from its early beginnings to the present day, and among his most recent achievements is the invention of the variable-pitch air screw, in recognition of which he was awarded the James Clayton Prize by The Institution of Mechanical Engineers in 1948. It will be noted that Mr. Beacham is of the opinion that, for the great majority of press applications, the balance of advantages is in favor of the high-speed seated-valve type of pump.

Hydraulic Lock. The reference to hydraulic lock by Mr. G. I. Chinn is of considerable interest to all makers and users of hydraulic packingless piston valves. The author's firm has only had two isolated instances of hydraulic lock, and therefore he cannot claim to be an authority on the subject. Hydraulic lock might be defined as the condition in which a valve spindle becomes locked after pressure has been applied for a given length of time, and then, after the pressure has been released, in due course the valve spindle becomes free again. This latter condition distinguishes it from the more common condition in which the spindle binds due to foreign matter becoming lodged between the spindle and the bore. True hydraulic lock is conditioned by time and pressure, i.e., up to a certain pressure the valve spindle does not lock and then above that pressure the valve spindle locks if it is subject to pressure for a given length of time.

The author does not think that very fine clearances have much influence on the incidence of hydraulic lock, but he thinks that the main cause of this phenomenon is probably due to the deformation of the valve body due to the unsymmetrical application of hydraulic pressure at the valve ports. If the valve body is of light construction, the deformation may be several thousandths of an inch, and therefore increasing the clearance between the spindle and the bore will not make very much difference; at least that is the author's experience. On the other

hand, the increasing of the clearance does increase the danger of foreign matter becoming lodged between the spindle and the bore.

Valve Clearance. Mr. Chinn is quite correct in regarding a clearance of 1 micron per in. of diameter as being too fine for commercial production. In the author's firm the commercial limits are between 1 micron and 4 microns per in. of diameter for its standard packingless piston valves which operate up to 7000 psi working pressure. A micron is equivalent to approximately 40 microinches, or say one-fiftieth of the thickness of a human hair. The author is of the opinion that there is considerable room for research into the sealing mechanism of the packingless piston valve, and into the effect of grooving the spindle or the bore.

Light-Production Presses. In spite of his predilection for the Hele-Shaw type of pump, the views of Mr. Walter Ernst command respect, because he is one of the outstanding pioneers in the design of high-speed hydraulic presses in the United States. In the penultimate paragraph of his contribution he appears to support the author in the view that the high-pressure seated-valve type of pump "possesses outstanding advantages" when applied to heavy self-contained presses, or "wherever heavy sustained pressure and volumetric output are required." On the other hand, he questions "whether it will displace present reversible discharge radial- and axial-pump systems on high-speed medium and light production presses for metalworking, forming, drawing, and straightening."

The author was careful to use the term "heavy self-contained hydraulic presses" in the title of this paper because the advantages of high pressure are most marked in connection with large hydraulic presses. Also, it seems reasonable to expect that the mechanical press will remain supreme in the field of "light production presses for metalworking," forming, and blanking. On the other hand, the hydraulic press has considerable advantages for deep-drawing and heavy forging work.

Deep-Drawing Press: Operating Cycle. In defending the low volumetric efficiency of the Hele-Shaw type of pumps, Mr. Ernst uses the argument that the pumps are under pressure only a fraction of the time and at peak pressure only a fraction of that fraction, but he is careful to qualify it by saying that this refers only to high-speed metalworking or forming presses. If one were to apply this argument to a deep-drawing press, which is an application for which the hydraulic press is particularly suitable, it would be seen that the pumps are under pressure for quite a large proportion of the time during which the press is in operation. The following is a typical operating cycle:

500 tons	
22-in.-diam ram by 25-in. stroke	
Pump 95 gpm (U. S.), 22,000 cu in. per min	
15 in. per sec approach	
1 in per sec full load	
	Sec
15-in. approach.....	1
10-in. draw (80 per cent load).....	10
Total build-up and dwell (100 per cent load).....	$1\frac{1}{2}$
Servoshift to reverse pump (100 per cent load).....	$\frac{3}{4}$
25-in. return at 15 in. per sec.....	$1\frac{3}{4}$
Decompression, slowdown, pickup speed, etc.....	1
Total cycle.....	15

Pipe Losses. The author adheres to the view that low volumetric efficiency of the rotary-valve type of pump is the principal cause of heating up the oil in the hydraulic system, thereby necessitating the use of cooling coils, but he would agree that turbulence and eddy losses do add to the amount of heat generated and consequently increase the rate of temperature rise. These losses will obviously be greater when using an oil of higher viscosity.

Let us consider the relative influence of pipe friction on two

presses of the same power and ram speed, press A having a maximum working pressure of 3000 psi, operated by a rotary-valve type of pump using a relatively heavy oil, curve A Fig. 2 of the paper; and press B having a maximum working pressure of 6000 psi, operated by a seated-valve type of pump using a light oil, curve B, Fig. 2 of the paper.

Obviously, if the presses operate at the same ram speed, press A will have to move twice the quantity of oil moved by press B. In so far as the feed pipes are concerned, from the supply tank to the pump and to the press cylinder, the conditions for both presses are identical; the size of the feed pipes will be governed by the gravity head of oil which is available. Thus the feed pipes of press A have to pass twice the quantity of oil for the same pipe loss, as compared with press B. At 70 F oil A, Fig. 2, has a kinematic viscosity of 200 centistokes, and that of oil B is 20 centistokes.

Table 1 of this closure is based upon the friction loss through straight, smooth-bore pipes (the flow condition for oil B is turbulent in all cases, but in the case of the smaller bore pipes the flow condition is laminar for oil A). It will be clear by reference to Table 1 that the bore of the feed pipes for press A must be at least $1\frac{1}{2}$ times larger than for press B to meet the conditions just stated.

Now let us consider the friction loss in the pressure pipes; in this case we are not limited by gravity head, we are limited by the amount of pipe loss we are prepared to accept. If the total pipe loss is, say, 150 psi, then this will represent a loss 5 per cent of the total power of press A (i.e., on 3000 psi), whereas it will represent a loss of only $2\frac{1}{2}$ per cent for press B (i.e., on 6000 psi). Thus the pressure pipes of press A have to pass twice the quantity of oil at one half the pipe loss of press B, if both presses are to operate with the same mechanical advantage.

Table 2 of this closure is based upon these conditions, and it is clear from this that the pressure pipes of press A are required to be nearly twice the bore of those for press B. On the other hand, if we decide to accept a greater pipe loss on press A, then the effective power of the press will be reduced, and the additional friction loss will contribute to heating up the oil in the press system.

Tables 1 and 2 are based upon well-established formulas for

the calculation of friction loss in straight, smooth-bore pipes (see Lewis F. Moody¹³).

The formulas are as follows

$$R = 3159 \cdot \frac{Q}{u d} \dots \dots \dots [1]$$

$$f = \frac{0.3164}{R^{0.25}} \text{ (turbulent flow)} \dots \dots \dots [2]$$

$$f = \frac{64}{R} \text{ (laminar flow)} \dots \dots \dots [3]$$

where

Q = gpm (U. S. gal.)

R = Reynolds number

u = kinematic viscosity, centistokes

d = bore, in.

f = friction factor

Intersection of curves [1] and [2] at $R = 1187$

(Note: In long, straight pipes the change from laminar to turbulent flow may occur around $R = 2000$.)

$$\text{Oil A, 200 centistokes; laminar flow when } \frac{Q}{d} < 75 \dots \dots [4]$$

$$\text{Oil B, 20 centistokes; laminar flow when } \frac{Q}{d} < 7.5 \dots \dots [5]$$

$$P = 0.01348 \frac{\phi \cdot f \cdot Q^2}{d^5} \dots \dots \dots [6]$$

where

P = pipe loss, psi per ft

ϕ = specific gravity

Oil A; $u = 200$ centistokes; $\phi = 0.90$

¹³ "Friction Factors for Pipe Flow," by L. F. Moody, Trans. ASME, vol. 66, 1944, pp. 671-684.

TABLE 1 COMPARATIVE PIPE BORES : FLOW A = 2 X FLOW B : SAME PIPE LOSS (PL)

Oil B: Kinematic viscosity = 20 centistokes (95 SSU) specific gravity = 0.86

Oil A: Kinematic viscosity = 200 centistokes (950 SSU) specific gravity = 0.90

Oil	Bore, in.	Flow, ^a gpm	PL, psi/ft	Bore, in.	Flow, gpm	PL, psi/ft	Bore, in.	Flow, gpm	PL, psi/ft	
B	$\frac{1}{2}$	6		$\frac{1}{2}$	12		$\frac{1}{2}$	18		B Turbulent
A	0.98	12	0.640	0.86	24	2.15	0.797	36	4.38	A Laminar
B	1	24		1	48		1	72		
A	1.71	48	0.271	1.51	96	0.906	1.47	144	1.84	B Turbulent
B	2	96		2	192		2	288		A Turbulent
A	3.02	192	0.113	2.94	384	0.381	2.94	576	0.775	
B	4	384		4	768		4	1152		
A	5.9	768	0.0476	5.9	1536	0.160	5.9	2304	0.325	

^a Flow in U. S. gallons of 231 cu in.

TABLE 2 COMPARATIVE PIPE BORES : FLOW A = 2 X FLOW B : PIPE LOSS (PL) A = $\frac{1}{2}$ PL B

Oil B: Kinematic viscosity = 20 centistokes (95 SSU) specific gravity = 0.86

Oil A: Kinematic viscosity = 200 centistokes (950 SSU) specific gravity = 0.90

Oil	Bore, in.	Flow, ^a gpm	PL, psi/ft	Bore, in.	Flow, gpm	PL, psi/ft	Bore, in.	Flow, gpm	PL, psi/ft	
B	$\frac{1}{2}$	6	0.640	$\frac{1}{2}$	12	2.15	$\frac{1}{2}$	18	4.38	B Turbulent
A	1.17	12	0.320	1.02	24	1.08	0.948	36	2.19	A Laminar
B	1	24	0.271	1	48	0.906	1	72	1.84	
A	2.05	48	0.136	1.80	96	0.453	1.70	144	0.92	
B	2	96	0.113	2	192	0.381	2	288	0.775	B Turbulent
A	3.59	192	0.056	3.4	384	0.191	3.4	576	0.388	A Turbulent
B	4	384	0.0476	4	768	0.160	4	1152	0.325	
A	6.8	768	0.0238	6.8	1536	0.080	6.8	2304	0.163	

^a Flow in U. S. gallons of 231 cu in.

$$P_A = \frac{0.001926 \cdot Q^{1.75}}{d^{4.75}} \text{ (turbulent flow) } \dots\dots\dots [7]$$

$$P_A = \frac{0.04916 \cdot Q}{d^4} \text{ (laminar flow) } \dots\dots\dots [8]$$

Oil B; $\mu = 20$ centistokes; $\phi = 0.86$

$$P_B = \frac{0.001035 \cdot Q^{1.75}}{d^{4.75}} \text{ (turbulent flow) } \dots\dots\dots [9]$$

In the foregoing we have been considering the friction loss in straight, smooth-bore pipes. It will be understood that the friction loss of elbows, tees, and valves is very considerable and is equivalent to many feet of straight pipe. Obviously, larger pipes are more expensive to bend; they involve larger couplings or flanges, and larger valves. The cost of valves increases rapidly with increase in size, and the whole piping layout is much less compact.

Flow Resistance of Prefill Valves. The friction loss through prefill valves is not well established; there is little, if any, published work on the subject; consequently this subject has come to be regarded as a matter of opinion rather than as a matter of fact. Mr. Ernst's reference to "fairing and streamlining" the prefill valve appears to be at variance with his later statement that "the viscosity has little or no influence on the flow resistance." Surely, if the valve is considered as a passage, then flow resistance is influenced by viscosity; on the other hand, if the valve is considered as an orifice, the flow resistance varies as V^2 , and viscosity is of no account; but one cannot streamline an orifice.

Referring to W. F. Cully's experiment, previously mentioned,¹⁴ it was shown that the limiting velocity without cavitation through a prefill valve of 3.6 sq in. was 10.2 fps, with an oil of 725 SSU, and 33.6 fps with an oil of 165 SSU, all other conditions being equal. The author's firm has made extensive research into the flow resistance of self-acting pump valves; the suction valve of its straight-in-line pump is like a miniature prefill valve. It has been found that the viscosity of the oil is a very material factor in the flow resistance of the valve. It is for this reason that the firm's pump cannot operate at high speed with a heavy-viscosity oil.

Rapid Reversal of Heavy-Press Table. Mr. Ernst contends that the reversing type of pump will operate a heavy press with a smoothness of acceleration and deceleration, and freedom from hydraulic shock not attainable by any other means; and he refers to speeds of 200 fpm or, say, 3 fps. The usual practice with a constant-delivery straight-in-line pump is to unload the pump at the end of the pressing stroke and reload for the return stroke at a predetermined rate of acceleration. This can be effected by quite a simple unloading valve, and the reversal takes about $1/4$ sec. The author is of the opinion that a more rapid reversal can be achieved, without undue shock, by means of the constant-delivery pump, and more simply than by employing all the complicated mechanism of a reversible pump. It will be noted that in the typical operating cycle previously given, the servoshift to reverse the pump takes $3/4$ sec.

Sustained Pressure and "Follow-Up." With reference to the application of constant-delivery pumps with "on-off control" to plastic molding presses, it may be sufficient to say that this is almost universal practice on self-contained presses in Great Britain, and such delicate parts as Amphenol plugs are being produced on such presses. The on-off control or sustained-pres-

sure unloading valve simply unloads the pump when the pressure reaches a set figure and loads it again when the pressure has dropped, say, 10 per cent; the loading unloading during follow-up may be 30 times per min, but the pressure in the press cylinder varies only 10 per cent. On the other hand, with a weight-loaded accumulator, the pressure may vary ± 10 per cent owing to the friction of the accumulator ram, i.e., the difference between the accumulator being pumped up and falling by gravity. Even with an air-loaded or nitrogen-loaded accumulator, there is usually a pressure difference of at least 10 per cent, due to the compression and expansion of the gas load. The advantage of the on-off control is that the pump is running idle during the cure period, with momentary load peaks 10 to 30 times per min; and the ceiling pressure can be varied to suit the job, simply by adjusting the valve.

Sustained Pressure ± 1 Per Cent. If the pressure must be maintained dead steady within fine limits, then a small constant-delivery pump blowing off at a relief valve has several advantages. For instance, with a variable-delivery pintle-type pump, operating at "near-zero" delivery, the leakage heats up the oil with consequent fall in viscosity, which further increases the leakage until equilibrium is reached; and the pintle valve becomes worn by erosion, increasing the leakage still further. On the other hand, the output of the small constant-delivery pump continues to pass through the relief valve, the same amount at the same pressure, in spite of any increase in oil temperature or lower viscosity; and, if the relief-valve spindle becomes eroded, it is more easily replaced than the pintle valve of the pump.

Compressibility. The author accepts Mr. Ernst's calculations, and he agrees that the compression-dwell or build-up of the 6000-psi press may take $1/2$ sec more than that of the 3000-psi press. The latter, having a cylinder capacity of 10,000 cu in., would be similar to the press with 22-in.-diam. ram by 25-in. stroke and a working cycle of 15 sec as just described. It would seem that the $1/2$ sec lost on compressibility at 6000 psi might be set against the $3/4$ sec servoshift to reverse the pump at 3000 psi.

Pressure-Viscosity Characteristics of Mineral Oil. As mentioned by Mr. Lamb, the viscosity of an oil increases with pressure; and it may be added that the pressure-viscosity coefficient varies with temperature. There is also some variation between different grades of mineral oil. As a rough approximation, it may be said that the viscosity will be increased $1\frac{1}{2}$ times at 3000 psi and $2\frac{1}{2}$ times at 6000 psi, both being at 100 F.¹⁵

In addition to the effect on viscosity, increased pressure tends to induce laminar flow in the critical zone, $R = 1200$ to 3000. Thus, supposing flow is turbulent at atmospheric pressure and a Reynolds number of say 2700, then increased pressure will cause the flow to become laminar, with the paradoxical result that we have reduced pipe loss at increased pressure. On the other hand, if the flow is laminar at atmospheric pressure, then the pipe loss will increase with increased viscosity due to increased pressure. Here again, therefore, the balance of advantage is with the low viscosity oil, curve B.

Fire Hazard. The difference in flash point between oils A and B is not sufficient to support Mr. Lamb's contention that there is a greater fire hazard with the lower viscosity oil. Under actual working conditions, it is the author's experience over the past 15 years that the fire hazard is negligible.

¹⁵ "The Viscosities and Compressibilities of Liquids at High Pressures," by J. H. Hyde, Proceedings of the Royal Society of London, series A, vol. 97, August, 1920, p. 250 (research at 40 C).

"The Determination of the Pressure-Viscosity Coefficient and Molecular Weight of Lubricating Oils by Means of the Temperature-Viscosity Equations of Vogel and Eyring," by Dr. A. Cameron, *Journal of the Institute of Petroleum*, vol. 31, 1945, p. 401.

¹⁴ "Modern Direct Hydraulic System," by F. H. Towler, Proceedings of The Institution of Mechanical Engineers, London, England, vol. 154, 1946, p. 189; discussion by W. F. Cully, pp. 189-190.

Power Consumption. The author is somewhat surprised by Mr. Lamb's categorical announcement that the straight-in-line pump has a "greater power consumption." This does not seem to be supported by the facts. Surely, if cooling coils have to be fitted to a press to keep the oil cool, that is direct evidence that

power is being wasted. The author's firm has been supplying straight-in-line pumps to operate self-contained hydraulic presses for the past 15 years, and not one of those presses has been fitted with a cooling coil, which is direct evidence that the power consumed is doing useful work.

Elements of Graphical Solution of Water-Hammer Problems in Centrifugal-Pump Systems

By A. J. STEPANOFF,¹ PHILLIPSBURG, N. J.

WATER HAMMER IN SIMPLE CONDUITS

Since publication of the "Symposium on Water Hammer," in 1933,² considerable progress has been made abroad in the development of the graphical method of solution of water-hammer problems. Although based upon the same fundamental relationships as the analytical solutions, the graphical method does not require an exact knowledge of the theoretical background and, at the same time, it gives a clear mental pattern of the events which enables one to use it with confidence for solution of practical problems. Early studies of water-hammer problems were carried out almost entirely in connection with water-turbine applications. Water-hammer problems in the centrifugal-pump field are more numerous and varied, but owing to the smaller size of units, the effects of water hammer were learned by experience, damage being repaired at a relatively low cost. Centrifugal-pump designers and users are little familiar with water-hammer theory. Fear of involved mathematical treatment has been a major factor to such neglect. The author attempts to present the water-hammer theory, and the graphical method of solution in its simplest terms. This will enable a pump engineer to recognize the conditions leading to water hammer, to estimate the possible maximum pressure rise and, when necessary, to provide means to reduce the pressure rise to a safe limit.

WATER hammer is defined as the change in pressure in closed conduits above or below normal caused by sudden changes in velocity of flow. The presence of a hydraulic machine such as a centrifugal pump or water turbine is not a necessary condition for water hammer, but it was the danger of damage to such machines that induced the study of water hammer. These studies were carried out almost entirely in connection with water-turbine application. Water-hammer phenomena in centrifugal-pump installations are more frequent than in water-turbine practice but, owing to the smaller size of the units, the effects of water hammer are learned by experience, and any damage is easily repaired. For that reason users and designers of pumps are little familiar with water-hammer theory.

Fear of the complicated theoretical treatment has been a contributing factor to such neglect. Fortunately, water-hammer theory is one of the rare cases in hydraulics wherein theoretical solutions agree very well with the actual tests and observations. Graphical solutions of water-hammer problems are a later development. These are based on the same fundamental relationships as the analytical solutions but are easier to grasp and, in addition to greater flexibility, possess a marked degree of elegance. It is essential for a pump engineer to be able to know when to expect water hammer, to estimate the possible maximum pressure rise, and, if necessary, to provide means to reduce the maximum pressure rise to a safe limit.

Joukowsky's Law. Joukowsky has found that the maximum pressure rise $h_{\max}$ above the normal pressure H_0 in a conduit due to instantaneous closure of the valve is given by the expression known by his name

$$h_{\max} = \frac{aV_0}{g} \dots\dots\dots [1]$$

where V_0 is the normal velocity in the conduit before the closure of the valve, a is the velocity of sound in water under the conditions existing in the pipe, which is the velocity of propagation of the pressure wave, g is acceleration due to gravity.

This equation follows from the application of the second Newton's law, which states that the force (increase in pressure at the valve) is equal to the time rate of change of momentum of the moving mass. In Fig. 1 consider a long pipe flowing full with a uniform velocity V_0 under the head H_0 . If the valve at its end is closed instantaneously, then at time t a portion of the column of length x will have its velocity reduced to zero; then

$$F = A\gamma h_{\max} = \frac{A\gamma x}{gt} \cdot V_0$$

where A is the area of pipe, and γ is specific weight of water; hence

$$h_{\max} = \frac{aV_0}{g} \dots\dots\dots [1]$$

¹ Development Engineer, Ingersoll-Rand Company. Mem. ASME.

² The AMERICAN SOCIETY OF MECHANICAL ENGINEERS published in 1933, a "Symposium on Water Hammer." This symposium summarized and reviewed the development of water-hammer theory up to that time. It also contains a complete bibliography and historical sketch. Later a "Supplement to the Report of the ASME Committee on Water Hammer" was added.

A majority of the later articles on the subject deal with the graphical method of solution of water-hammer problems. The main contributors are Schnyder in Germany (1, 2),³ Bergeron in France (3), and Angus in United States and Canada (4, 5, 6). Schnyder and Angus use a graphical method as a means for solving sets of simultaneous equations derived from the original Allievi's functions. These are difficult to follow or use without detailed knowledge of the rather involved theory. Bergeron proposed a simple interpretation of the procedure of drawing water-hammer diagrams which does not require an exact knowledge of the theoretical background and, at the same time, gives a clear mental pattern of the events which enables one to use the method with confidence for solution of practical problems. An understanding of the mechanism of pressure waves, their propagation, reflection, and addition, is essential for the intelligent use of the graphical method even in its simplest form.

³ Numbers in parentheses refer to the Bibliography at the end of the paper.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society. Paper No. 48-A-89.

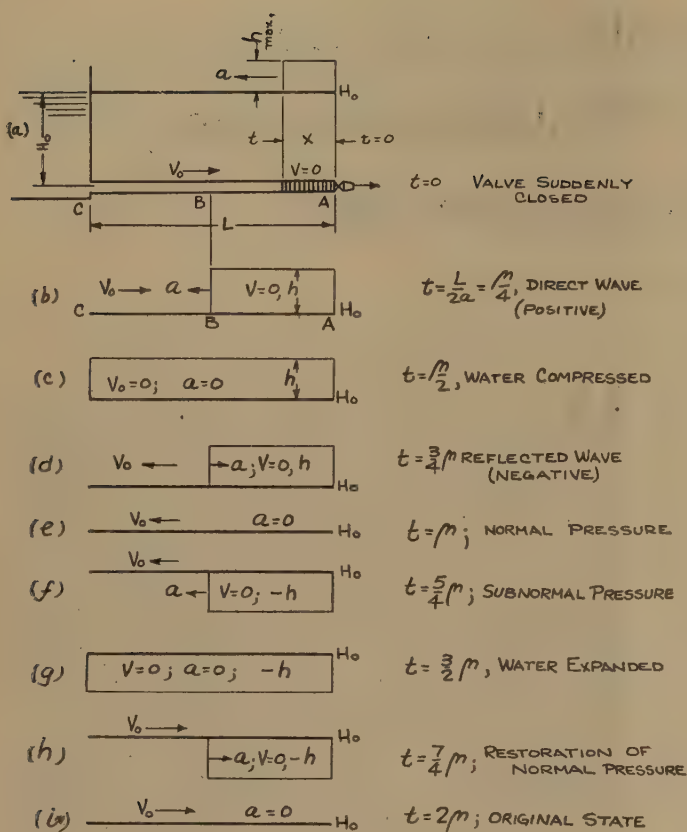


FIG. 1 SEQUENCE OF EVENTS AFTER SUDDEN VALVE CLOSURE

Here x/t is the rate at which the pressure wave travels along the pipe, and this is equal to the velocity of sound in the water within the pipe. Since the velocity V_0 in the pipe prior to the valve closure was constant and velocity of the pressure wave a is constant, the pressure at the valve remains constant. The water between the valve A , in Fig. 1(b), and the wave front will be at rest under a state of uniform compression under the head $h_{\max}$ (in addition to initial compression due to H_0), while water between the wave front point B , Fig. 1(b), and the open end of the pipe (point C) is under its original pressure at every point and moving with a constant velocity V_0 toward the valve. When the wave front reaches the open end of the pipe C , the whole pipe is completely filled with water at rest under an additional pressure head $h_{\max}$ uniform along the whole pipe length, Fig. 1(c).

However, the instant the pressure wave reaches the open end, water in the pipe will begin to expand through the open end, creating a velocity V_0 in the pipe in the opposite direction and relieving the pressure back to its original pressure, Fig. 1(d). This pressure drop travels backward from the open end to the valve with the same velocity a , and reaches the valve in a time $2L/a$ from the time of valve closure. At this instant the pipe is again under the normal pressure and the water moves with velocity V_0 toward the reservoir, Fig. 1(e). In the next instant water will continue to move toward the reservoir, bringing the slug of water adjacent to the valve to rest, at the same time the pressure at the valve will drop below its normal pressure H_0 by $-h_{\max}$, exactly equal to the pressure rise during the first interval of time $2L/a$ as the same velocity is destroyed at the same rate. The wave of subnormal pressure $-h_{\max}$ now starts from the valve, Fig. 1(f), and reaches the open end at time $3L/a$, when all water in the pipe is at rest under reduced compression due to a negative head of $-h_{\max}$, Fig. 1(g). During the next instant water will flow into

the pipe with velocity V_0 restoring the pressure in the pipe to the original value H_0 , Fig. 1(h). At the end of the second interval $2L/a$, or total time from the valve closure $4L/a$, the conditions will be the same as before the valve closure, Fig. 1(i). After that the new cycle begins and repeats itself continuously until the original kinetic energy of flow is dissipated in pipe friction, internal water friction (due to velocity rearrangement), and energy rejected into reservoir. However, the damping has little effect on the pressure of several cycles under usual conditions.

It will be noticed that while part of the water column is expanded or compressed the rest of it is at rest (or moving with a constant velocity). Thus there is no mass oscillation as a whole as has been observed, for instance, on boiler feed pumps with unstable characteristics, producing pressure surges in the discharge lines.

The time to complete a round trip for the pressure wave $\mu = 2L/a$ is called variously "period of pipe," "reflection time," "critical time," "one interval of time." The latter term will be used in this paper.

Sudden Opening of Valve. The sequence of events can be similarly traced for a sudden valve opening. The magnitude of the pressure drop and rise is given by Joukowski's Equation [1], and is the same as in the case for a sudden valve closure. However, during the first interval of time μ , pressure at the valve remains subnormal, while during the second interval the pressure is above the normal. The velocity fluctuates from 0 to $2V$, about its average velocity V , which is the final velocity of the established flow.

Partial Valve Closure. An instantaneous partial valve closure produces the same cycle of events as a complete valve closure. The pressure rise and fall is given by the

formula

$$h = \frac{a}{g} (V_0 - V) = \frac{a}{g} \Delta V \dots \dots \dots [2]$$

where V_0 is the initial velocity of flow,

V is the final velocity. The pressure diagram for the two intervals will be the same as in Fig. 1, but the velocity will fluctuate from V_0 to $V_0 - 2\Delta V$ about its average value $V = V_0 - \Delta V$, which is the final velocity of established flow.

Gradual Valve Closure. This case is usually considered as the equivalent of a series of consecutive instantaneous partial closure movements, each producing a pressure wave proportional to the increment of velocity destroyed and traveling between the valve and open end of the pipe. The pressure at any point of the pipe is the algebraic sum of all pressure waves, direct and reflected. Thus the pressure will continue to rise at the valve during the whole first interval $2L/a$ until the reflection wave arrives at the valve at this instant.

Variation of Pressure Rise Along Pipe. If a valve closure occurs instantaneously the pressure wave will travel undiminished up the pipe to the open end as shown in the diagram of Fig. 1. Fig. 2 shows the variation of pressure with time at the valve A , near the open end of the pipe C , and at an intermediate point B . At the open end C the original pressure H_0 prevails. For a gradual valve closure, when the total time of closure T , is equal to or greater than one interval of time, the maximum pressure rise occurs at the valve and from there to the open end it reduces to zero uniformly along the length of the pipe (Fig. 3, full line). Note that although the pressure wave travels undiminished along the pipe after each partial instantaneous closure of the valve, at points other than the valve there are times when

pressure remains unchanged. Thus in Fig. 2 at point *B* in the middle of the pipe the original pressure H_0 prevails during time $\mu/2$ between each positive and negative pressure wave. When series of pressure waves are traveling along the pipe as a result of successive partial valve closures some of the waves would fall in the gaps between the positive and negative pressure waves; pressure waves of opposite sign will cancel each other, and as a result, a pressure distribution as shown in Fig. 3 is obtained. If the closure time is less than one interval of time, $2L/a$, the maximum pressure rise will be transmitted undiminished to an intermediate point (a distance $Ta/2$ from the open end), and from that point to the open end the pressure rise reduces uniformly to zero (Fig. 3 dotted line).

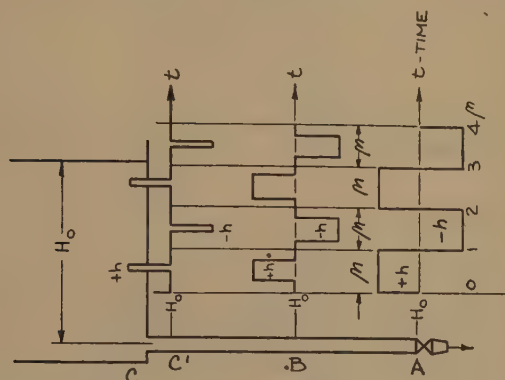


FIG. 2 PRESSURE VARIATION WITH TIME ALONG PIPE
($\mu = 2L/a$, one time interval.)

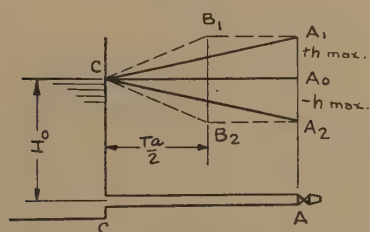


FIG. 3 PRESSURE DISTRIBUTION ALONG PIPE
(Full line $T \geq \mu$; dotted $T < \mu$.)

As soon as the valve movement ceases, the supernormal pressure at the valve will be succeeded by subnormal pressure and the pressure will fluctuate between these limits until damped out by friction, Fig. 7.

Velocity of Pressure Wave. Knowing the pipe size, thickness, and material, and the bulk modulus of the liquid, the velocity of the pressure wave can be calculated by using the formula

$$a = \frac{1}{\sqrt{\frac{\gamma}{g} \left(\frac{1}{k} + \frac{d}{eE} \right)}} \quad [3]$$

where

- γ = specific weight of liquid
- k = bulk modulus of liquid
- d = pipe diameter
- e = pipe thickness
- E = pipe material modulus of elasticity

For cold water $k = 294,000$ psi for pressures up to 500 psi; this increases as the water temperature and pressure are increased. For steel pipe $E = 29,400,000$ psi. Using these values for cold water and steel pipe Equation [3] becomes

$$a = \frac{4660}{\sqrt{1 + \frac{d}{100e}}} \quad [4]$$

A simple development of this formula is contained in the symposium (7).

It should be noted that elasticity of pipe and water serves to cushion the shock due to a sudden change in velocity of flow. Without these, or considering the water column as a solid rod, the rise of pressure would be greater, and, in fact, theoretically infinite in the case of a sudden complete closure of the valve. This follows from Equation [3] for if k and E are infinite, a becomes infinite, and hence $h_{\max}$ becomes infinite.

Pipe of Variable Diameter. In the case of a compound pipe consisting of several sections of different diameters, an "equivalent velocity" is used for calculation of the maximum pressure rise with Equation [1]. The equivalent velocity is defined as

$$V = \frac{L_1 V_1 + L_2 V_2 + \text{etc.}}{L} \quad [5]$$

where L_1 is the length, and V_1 is velocity of one section of pipe, L_2 and V_2 the same for the second section, etc.; L is the total length of the pipe. The value of the pressure-wave velocity for a compound pipe is determined by the following equation

$$\frac{L}{a} = \frac{L_1}{a_1} + \frac{L_2}{a_2} + \text{etc.} \quad [6]$$

where a_1 , a_2 , etc., are the values of the velocity of sound in the first, second, etc., portions of the pipe. The value of a is referred to as "effective" velocity of the pressure wave. If the closure time is appreciably greater than one interval (over 4 intervals) accurate results are obtained by using "equivalent" velocity of flow and effective pressure-wave velocity.

Pipe Friction and Velocity Head. In all discussions it has been assumed that the pipe velocity is so low that the pipe friction loss and velocity head are very small in comparison with the head H_0 . Therefore the head H_0 prevailed at all points of the pipe before the valve closure. Such conditions could be realized, for instance, if there is a nozzle at the end of the pipe, provided with a needle valve similar to those used with Pelton water wheels. With water turbines such conditions are approximated, and the rate of flow may be varied by wicket gates with very little or no change in pressure in the penstock.

When the pipe is not horizontal (as shown in Fig. 1), then the original head H_0 is the total head, or neglecting the friction loss and velocity head $H_0 = p/\gamma + y$, where y is the elevation of the point of pressure measurement above the nozzle, Fig. 4.

GRAPHICAL SOLUTION

In the cases of sudden valve closure and closure in a time equal to or smaller than one interval of time, Joukowsky's Equa-

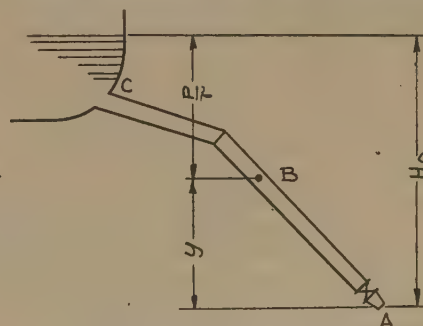


FIG. 4 TOTAL HEAD ON NOZZLE

tion [1] gives accurately the maximum pressure rise in a simple conduit which takes place at the valve. In the latter case negative (reflected) waves will not have reached the valve in this time, and the pressure rise is built up gradually to the same value as it would reach after a sudden valve closure. If the total closing time is greater than one interval, the pressure rise at the valve is equal to the sum of the supernormal pressure waves created by the valve closure and the negative waves arriving from the open end of the pipe. The maximum pressure rise can be found by determination of pressures at a fixed point (at the valve) at some arbitrary interval of time, like in the arithmetical integration method of Gibson (8) or Quick (9) and analytical method of Allievi, or by following the pressure variation of some arbitrary pressure wave as it travels back and forth from the valve to the open end of the pipe. The latter is the principle employed in the graphical method by Bergeron. In the graphical method the pressure rise at end points is determined as an intersection of the "surge characteristics" with the system characteristics given by the end conditions of the pipe, somewhat in the manner used for the determination of the operating point of a centrifugal pump discharging into a pipe line.

The graphical method is more flexible and easier to grasp than the analytical method. It permits solution of a variety of water-hammer problems not possible analytically.

System Characteristics. In any closed conduit to be investigated for water hammer, the system characteristics either are known or are estimated. This means that the relationship between the head and capacity, or velocity, at both ends of the conduit are known for a steady condition prior to the valve closure. Such a relationship can also be established for any partial valve opening. The same relationships hold at any instant during the gradual valve closure. Taking as an example a nozzle discharging freely under a head H_0 , Fig. 5 (a) with the nozzle wide open, the system characteristics for the point A, is a square parabola, marked O in Fig. 5 (b), representing velocity in the pipe (not a velocity through the nozzle) in terms of head in the reservoir H_0 . This applies to a steady flow prior to valve closure or at time $t = 0$. Such characteristics can be drawn for any valve position.

For instance, if the valve is gradually and uniformly closed in time $T = 3\mu$, the nozzle will be $2/3$ open at time $t = \mu$ and, under the same head, the pipe velocity will be $2/3$ of the original velocity V_0 . This is expressed by a parabola marked μ , with abscissas $2/3$ of their original values at any head. Similarly, at time $t = 2\mu$, the nozzle is $1/3$ open, and the system characteristic is a parabola 2μ . At time 3μ the flow is zero and the system characteristics coincide with the axis of heads. A uniform valve closure was assumed for simplicity.

There is no added complication in the procedure of the graphical method for other rates of valve closure as long as the discharge through the valve is known as a function of time. This results only in a different spacing of parabolas 0, μ , 2μ , etc. Another property of the system is that the pressure at the point C always remains equal to H_0 at all rates of flow and is not affected by the valve closure. The line $H_0 = \text{const}$ is a system characteristic for the point C. Parabolas 0, μ , 2μ , and 3μ represent a time scale on the diagram for the events occurring at point A. Such a time scale can be drawn at each $\mu/2$ or $\mu/4$ interval, or a parabola of time can be drawn for any specified time from the moment the valve closure starts. Both system characteristics, parabolas μ , 2μ , 3μ for A, and $H_0 = \text{const}$ for C, give the "end conditions" for the pipe at any given time. Pipe friction and velocity head will be neglected for the time being for simplicity and will be discussed later.

Surge Characteristics. Applying Joukowski's equation to a partial valve closure the pressure rise becomes

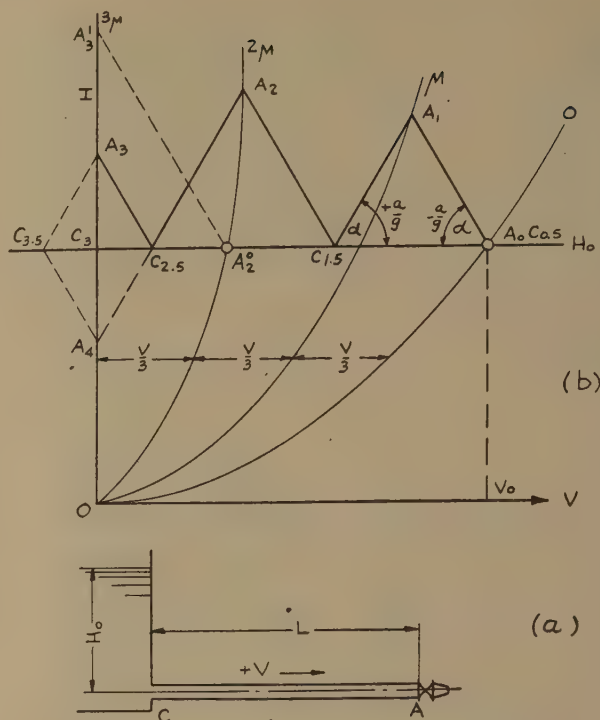


FIG. 5 VALVE CLOSURE IN THREE TIME INTERVALS

$$H_0 - H = -\frac{a}{g} (V_0 - V) \dots \dots \dots [7]$$

which shows that the pressure rise is proportional to the velocity drop; $-a/g$ being the factor of proportionality. Equation [7] is a typical straight-line equation where $-a/g$ is a "slope coefficient." The expression $-a/g$ is not a tangent of the angle α between the straight line, Equation [7], and the positive direction of axis of V , because V and H are not of the same dimensions. Therefore $-a/g$ is not dimensionless as a tangent should be. The actual slope of the line depends upon the scales used for V and H and does not affect the results. This is determined from

$$\tan \alpha = \frac{H - H_0}{V_0 - V} \dots \dots \dots [8]$$

where the numerator is the pressure rise for an arbitrary velocity drop $V_0 - V$; both expressed in inches to the scale used for the diagram. Equation [7] is a surge characteristic and represents pressure rise in terms of pipe velocities at the valve (point A, Fig. 5). Point A_0 in Fig. 5 (b), represents the initial conditions. The pressure rise will continue during the first interval μ , until point A_1 is reached on the parabola μ . At this instant the first reflected negative wave (or pressure drop) will arrive at point A, Fig. 1(d).

Fig. 6 (a) shows a simplified diagram of the pressure-wave positions at time $\mu/2$, assuming that velocity is extinguished instantaneously by the valve at equal increments at a uniform rate. Fig. 6 (b) shows a diagram of pressures along the pipe at time μ . The broken line A_1C_1 shows the supernormal pressures produced at A. These are partially canceled by the reflected negative pressure waves arriving from C. The latter are represented by a broken line C_1A_0 . All the pressure waves produced during time $\mu/2$ in Fig. 6 (a), are reflected at C and moving toward A. The net pressures along the pipe are equal to the difference between the two and are represented by a line C_0A_1 . The pressure rise A_0A_1 is the same as that due to a sudden valve closure de-

stroying the same velocity instantaneously. It is easy to see from Figs. 6 (a) and (b), the effect of the pipe length on the maximum pressure rise at the valve. The longer the pipe the longer the pressure at the valve will be built up until the first negative

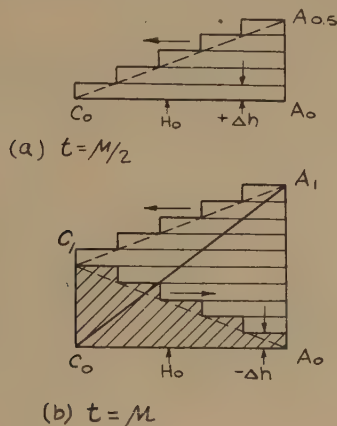


FIG. 6 PRESSURE RISE FOR GRADUAL VALVE CLOSURE

wave arrives at A , as μ is directly proportional to the pipe length. Actually, events at the valve are complicated by the fact that even with a uniform valve closure (as just defined), the velocity through the nozzle (and pipe) is affected by the pressure rise at the valve which varies with time. However, the linear relationship between velocity and pressure rise holds irrespective of time, or space.

From the beginning of the second interval it is easier to follow up the pressure variation of the pressure wave leaving A at $t = \mu$, than to keep a record of the pressure at point A . Leaving point A at this instant and moving in direction C the pressure wave will meet the negative waves reflected at C . The negative pressures are appearing at C at the same rate as positive pressures were created at A . Therefore the pressure of the traveling wave will be reduced at the same rate. The surge characteristic will change its direction, but the linear relation between pressure drop and velocity in the pipe is maintained. Its equation will become

$$H_0 - H = \frac{a}{g} (V_0 - V) \dots \dots \dots [9]$$

In Fig. 5 (b), Equation [9], is represented by the line $A_1C_{1.5}$. The pressure continues to decrease for one half of interval, while the pressure wave travels from A to C . At time 1.5μ , it will reach point C , and will change its direction moving from C to A . Now the traveling wave will meet supernormal pressure waves, hence its pressure will increase, or the slope of the surge characteristics will change again. Thus the surge characteristic will change its direction each half of interval or at intersections with parabolas $\mu, 2\mu, 3\mu$ producing points A_1, A_2, A_3 and points $C_{1.5}, C_{2.5}, C_{3.5}$ as intersections with the system characteristics for the point C . Velocity in the pipe is decreasing from V_0 to 0, or until the surge characteristic intersects the axis of H . Hereafter the pressure at A will fluctuate between A_3 and A_4 in the same manner as after a sudden valve closure. The surge characteristic, if continued after the valve closure, will be represented by $A_3, C_{3.5}, A_4, C_{2.5}, A_3$ still changing its slope each half interval of time $\mu/2$. The velocity becomes negative and positive alternately each interval of time as illustrated in Fig. 1. Points A_1, A_2, A_3 , and A_4 show the pressures at the valve (point A) at times 1, 2, 3, and 4 intervals from the beginning of the valve closure.

Note the notation in the diagram, Fig. 5 (b). Capital letters A and C show the pressures at points A and C on the pipe, Fig. 5 (a), at times indicated by subscripts given them such as A_0, A_1, A_2, A_3, A_4 , and $C_{0.5}, C_{1.5}, C_{2.5}, C_{3.5}$, and which are expressed as fractions or multiple of one interval of time $\mu = 2L/a$.

Fig. 7 shows the pressure-time curve for the point A at the valve plotted from the data in Fig. 5 (b).

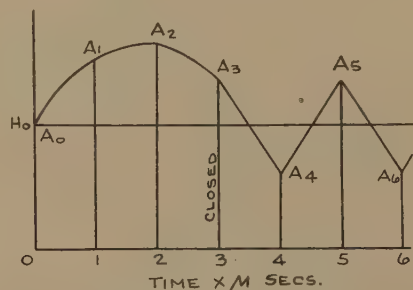


FIG. 7 PRESSURE AT VALVE A VERSUS TIME

If more points are desired, the system characteristics for the point A are drawn at half of one interval of time, Fig. 8, and the pressure variation of two pressure waves are followed up; one leaving A at time $t = \mu$ (full line), and the other leaving A at time $t = \mu/2$ (dotted line). The surge characteristics for this wave are $A_0, A_{0.5}, C_1, A_{1.5}, C_2, A_{2.5}, C_3$, drawn in the same manner, i.e. changing its direction each time it strikes the C characteristics, or A —parabolas at one interval of time spacing. Thus we have obtained additional points $A_{0.5}, A_{1.5}, A_{2.5}$. In general we could follow up any pressure wave leaving point A at any instant between 0 and μ and draw its surge characteristics. For that we would need time parabolas drawn at μ intervals from the pressure-wave departing time to locate the points of change in the slope of the surge characteristics. In this way it is possible to determine the pressure at point A at any desired instant.

Note that the slope of the surge characteristics is positive ($+a/g$) when the pressure wave travels in the direction opposite to the original flow velocity or from A to C ; the slope is negative ($-a/g$) when the pressure wave travels in the direction from C to A .

Pressure Rise at Any Point on Pipe. For practical purposes only the maximum pressure rise is of importance. In our example it takes place at the valve. However, to illustrate the flexibility and general properties of the graphical solution, the method used

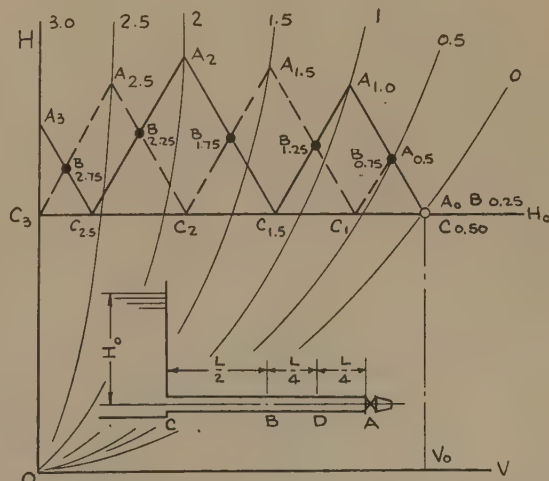


FIG. 8 PRESSURE RISE FOR TWO POINTS, A AND B

to determine the pressure at some point between A and C will be shown. For simplicity take point B , Fig. 8, midway between A and C . Note that the pressure rise created by the valve closure at A will not be felt at B until time $\mu/4$ sec. Therefore point A_0 can represent the pressure at B until time 0.25μ (marked $B_{0.25}$). Thereafter, when the pressure wave travels from A to C , it will reach $B_{0.25} \mu$ sec later. But, on its way from C to A , the pressure wave will reach $B_{0.25} \mu$ before it reaches A . On the diagram, Fig. 8, the points representing pressures at B are placed between A and C points at 0.25μ sec from either one, and are marked $B_{0.75}$, $B_{1.25}$, $B_{1.75}$, $B_{2.25}$, $B_{2.75}$, which are intersections of the two surge characteristics of the two pressure waves at 0.5μ time spacing. For any other point, say, D at $L/4$ from A , a surge characteristic should be drawn starting on line A_0A_1 and parabola 0.75μ (nearer to A_1) and changing its directions at $H_0 = \text{const}$, and parabolas 1.75μ , 2.75μ . Evidently the pressures at D will be higher than at B and lower than at A .

Bergeron's Interpretation of Events During Valve Closure. The surge characteristics, according to Bergeron, should be drawn by an imaginary observer traveling at the speed of the pressure waves and recording pressures as he moves along the pipe from one end to the other.

For drawing the surge characteristics, the observer is guided by the following rules:

- 1 Remaining at the valve (point A , Figs. 5, and 8) for μ sec, the observer will record the pressure rise there (line A_0A_1 , Figs. 5, and 8). If the valve closure continues, the observer starts moving from A to C meeting the reflected waves from C (in the same order as they were produced at A) which will reduce the pressure rise from A_1 to zero at C (point $C_{1.5}$). The observer now returns to A , recording pressures as he moves (line $C_{1.5}A_2$). The observer will continue to commute from A to C and back until the valve is completely closed.

- 2 The slope of the surge characteristic is $-a/g$ when the observer is moving in direction of the initial flow, and the slope is $+a/g$ when he moves in the opposite direction.

- 3 When the observer finds any discontinuity of his path, the surge line changes its slope. Following are a few typical examples of discontinuities: (a) Change in pipe size and thickness (different value of the velocity a in the slope coefficient a/g); (b) a branch in the conduit; (c) a hydraulic machine, such as a centrifugal pump; (d) end of the pipe.

- 4 At the end of the pipe (or each $\mu/2$ sec), the observer changes his direction, therefore the surge line changes its slope.

- 5 To obtain a desired number of points on the diagram it may be necessary to send more than one observer along the pipe. Since, during the first μ sec, the maximum pressure occurs at the valve, the first observer should stay at A μ sec to record the maximum pressure at this point. The other observers could leave at equal time spacing so that pressures at A are recorded at equal time intervals, and this results in a uniform time scale on the diagram. In Figs. 5 (b) and 8, line A_0A_1 locates the starting points of all observers (point A on the pipe) and time of their departure between $t = 0$ and $t = \mu$ sec, time scale being given by the system curves, such as parabolas 0 , μ , 2μ , etc., in Figs. 5 (b) and 8.

Valve Closure From Some Intermediate Position. If a valve is closed completely or partially from some intermediate position, the pressure rise may be considerably higher than that produced by a complete valve closure at the same rate from the wide-open position. In Fig. 5 (b), suppose the original position of the valve is two thirds closed, the operating point A_2^0 is on the intersection of the curve 2μ and H_0 . If the valve is now closed in a time μ (the same rate as before), the pressure will rise during a com-

plete interval μ , and will reach the point A_3^1 , along the dotted line $A_2^0A_3^1$ which is higher than the maximum pressure rise previously obtained. When starting from wide open (A_0), the pressure rises during a complete interval μ (A_0A_1) but the slope of the parabola μ is such that the pressure rise is lower than for closure from any intermediate position.

Effect of Pipe Friction. There are several methods used to take care of pipe friction loss in the graphical solutions of water-hammer problems. All are approximate but sufficiently accurate for the determination of maximum pressure rise at one of the extremities of the system:

- 1 In the problem shown in Fig. 5 (a) and (b), suppose the pipe friction loss from A to C is h_f , Fig. 9. This plotted against velocity in the pipe is represented by the curve h_f . In one method (Schnyder's) the pipe friction loss is subtracted from the system characteristics H_0 for the point C , and the resultant (curve R_c) is used for the construction of the surge diagram as shown in Fig. 9. This method is equivalent to an assumption that all pipe friction is concentrated at point C (for instance, a local obstruction).

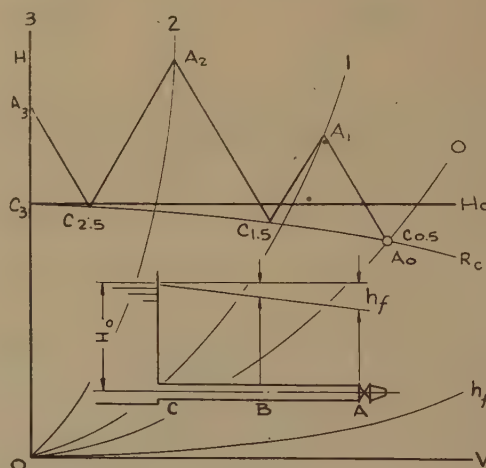


FIG. 9 PRESSURE RISE WITH FRICTION ALLOWED AT C

- 2 In the second method it is assumed that all friction loss is concentrated at the valve at point A . This means that pressures on all system curves for A , (parabolas 0 , 1 , $2 \dots$ Fig. 10), are raised by the amount of friction to produce the same velocities (curves $0'$, $1'$, $2'$). Using these new system curves for A and $H_0 = \text{const}$ for the point C , the diagram is completed following the same procedure. This method in essence is the same as that used by Bergeron.

EXAMPLES OF WATER-HAMMER PROBLEMS IN CENTRIFUGAL-PUMP SYSTEMS

Power Failure—No Check Valve. A pump A in Fig. 11 is discharging into a tank C under static head H_s and friction head h_f . The system curve for point A (pump) is R_{A0} for steady conditions before power failure. There is no check valve between the reservoir and the pump. In event of power failure assume that the pump will slow down, stop, and reverse in less than one interval of time, μ . The pump will then operate as a hydraulic turbine under head H_s at runaway speed. The pump characteristics under these conditions is R_{A1} and the operating point for final equilibrium is D . Prior to the power failure, the system curves are the pump head capacity curve R_{A0} and the pipe resistance curve R_c , which fix the operating point A_0 . The pipe velocity V is used to represent capacity. At the instant of power

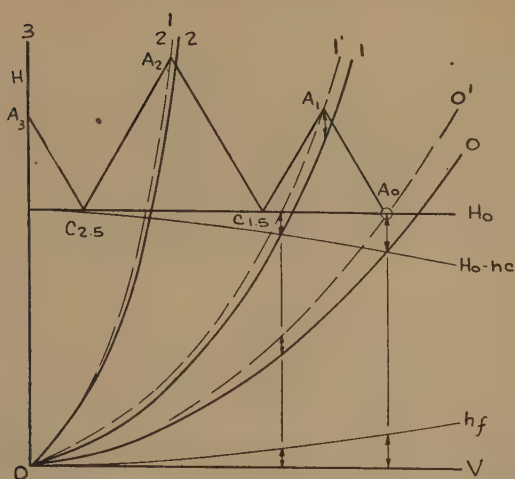


FIG. 10 PRESSURE RISE; ALL FRICTION AT VALVE A

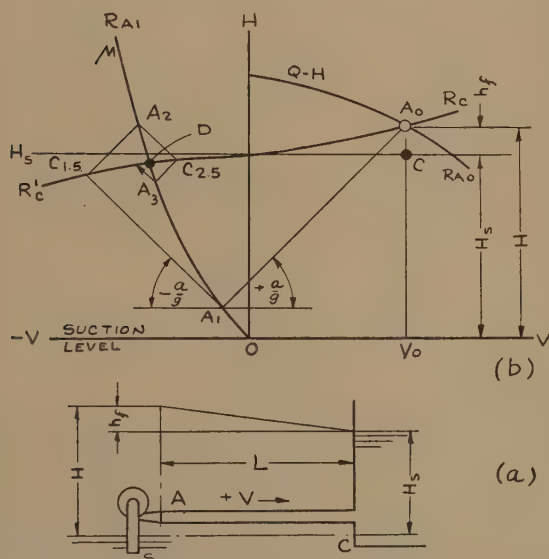


FIG. 11 POWER FAILURE WITHOUT A CHECK VALVE

failure and for one interval of time, the pressure and velocity at point A drops. Starting with the point A_0 the surge characteristic is drawn with a slope $+a/g$. The surge velocity a is calculated by means of Equation [4].

The pump will stop and reverse in a time of one interval μ (or less); after which the pump characteristic becomes R_{A1} , and all points for A on the diagram lie on this curve.

The first point is given by intersection of R_{A1} with the surge characteristic A_0A_1 .

After the pump reversal, the velocity in the pipe is negative and the characteristics for the point C are given by the curve R_{C1} .

All points for C on the diagram for different times will lie on the R_{C1} characteristics.

At time μ we send out an observer from A to C to record the pressures. The surge characteristic at A_1 will change its slope from $+a/g$ to $-a/g$ as the observer is traveling in direction of original velocity V_0 , going from A to C. When he gets to C after a time $\mu/2$ sec, the surge characteristics recorded there will give point $C_{1.5}$. Although the observer will record H_s pressure at C, this should be reduced by the amount of friction and velocity head, because the flow through the pump at A is produced

by the net pressure available at that point. In other words, we assume that all pipe resistance is concentrated at C, and the observer will record the pressure just ahead of this obstruction. Returning from C to A the observer will change the slope from $-a/g$ to $+a/g$ and will record point A_2 at time 2μ . Changing the direction again at A and going to C the observer will record point $C_{2.5}$ there. Points A_3 , $C_{3.5}$, A_4 , etc., are obtained in the same manner. Experience has shown that, even with little or no pipe friction, no dangerous pressure rise occurs. With a long suction pipe its resistance should be added to the pump characteristic for the reverse flow R_{A1} , making it steeper. This will result in a higher pressure rise in the system.

Level Receding in Tank. If the static level in the tank C is receding as the flow is reversed in the pump, the levels at times 1.5μ , 2.5μ , etc., should be known or estimated, and system characteristics for the point C are drawn for times 1.5μ , 2.5μ , etc., using different characteristics each time the observer reaches point C. Otherwise the procedure remains the same as in the foregoing example.

A Check Valve at Pump. If the check valve closes instantaneously, no water can flow past it. This means that the system characteristic R_A for the point A coincides with the axis of heads, Fig. 12. The surge characteristic $A_0A_1C_{1.5}A_2C_{2.5}$, etc., is drawn as in the previous example, and the operating points (pressures at A and C) are obtained as intersections of the surge characteristics and the system characteristics. If the check valve fails to close fast enough to prevent back flow through the pump, the surge line A_0A_1 would be extended to the left to intersect a parabola of the combined characteristic of the pump running backward and the check valve partly open, and higher pressure rise will result.

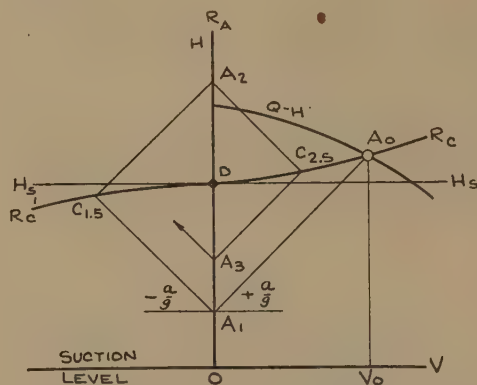


FIG. 12 POWER FAILURE WITH A CHECK VALVE

Depending on the value of a/g , the surge characteristics A_0A_1 may intersect the axis of velocities before it does the axis of heads, Fig. 13. This means the pressure has dropped to the pump-suction pressure and cannot drop any lower as the check would open and let water flow through the pump. At the end of the first time interval μ only part of the original velocity ΔV is destroyed, and the flow will continue with a velocity V_1 in the same direction. The check valve is still open. Construction of the rest of the diagram is evident from the figure. Note that a greater pressure rise is obtained if the pump operates at a lower original velocity V_0^1 , because the velocity destroyed is greater, being a maximum when A_1^1 coincides with the zero-head-capacity point. In that case the whole original velocity V_0^1 is destroyed.

Relief Valves and Surge Suppressors. A surge suppressor consists of a special relief valve of adequate capacity arranged to open at a predetermined rate under conditions which would produce a surge and before check-valve closure, after which the relief

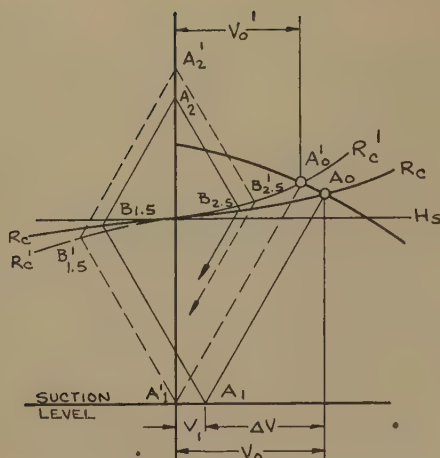


FIG. 13 PRESSURE DROP TO SUCTION LEVEL

valve slowly closes. Surge suppressors may be arranged to be actuated hydraulically, mechanically, or electrically. A number of installations were equipped with surge suppressors (13), partly to prevent reverse rotation, partly to reduce the pressure rise in the pipe in case of power failure. The method of operation of the surge suppressor will be illustrated in the example in Fig. 14, showing a pump with a check valve and surge suppressor. The system curve of the surge suppressor is shown by a parabola R_{AS} . The suppressor is set to open at a time between μ and 1.5μ . The $A_0A_1C_{1.5}$ part of the surge characteristics is drawn in the same manner as in Fig. 12. Point A_2 is placed on the parabola R_{AS} , so is A_3 . In this way the pressure rise is almost entirely eliminated. After the flow settles at the point D , the surge suppressor is gradually closed and the final point of the system moves from D to D' on the axis of heads.

In recent installations positively controlled valves combine the functions of both a check valve and a surge suppressor.

Flywheel Effect. If the pump in the first example has an appreciable rotating mass, the inertia of the rotating element will

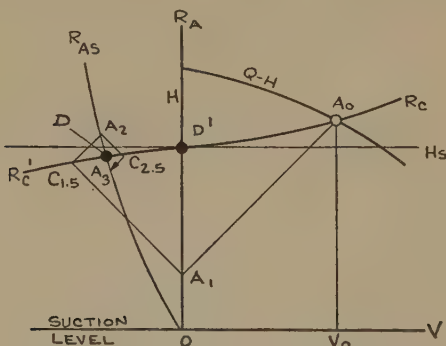


FIG. 14 PUMP WITH CHECK VALVE AND SURGE SUPPRESSOR

delay the retardation and reversing of the pump speed in event of power failure to a time greater than one interval of time, μ . The author knows of instances when centrifugal pumps were provided with flywheels so that in case of electrical power failure a steam turbine could take over the load with a minimum drop of the discharge pressure. In this case, at the end of the first interval, the pump characteristic will be a regular head-capacity curve at a reduced speed (R_{A1} , Fig. 15). Several such characteristics may be drawn at speeds corresponding to times μ , 2μ , etc., from the instant of power failure.

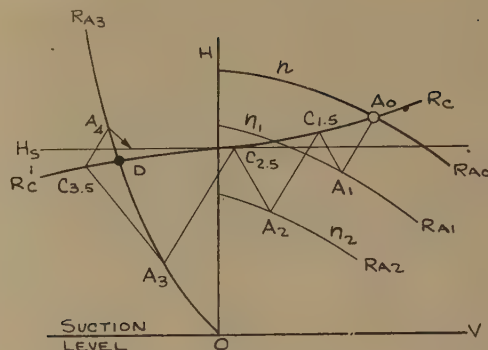


FIG. 15 PUMP WITH FLYWHEEL-EFFECT ARRANGEMENT, FIG. 11 (a)

To draw such characteristics, pump speeds at times μ , 2μ , etc., are determined from the relationship

$$T_0 = I \frac{\delta\omega}{\delta t} \dots \dots \dots [9a]$$

Torque T_0 is known at the time $t = 0$; I is the moment of inertia of the revolving mass, including the motor rotor; $\delta\omega$ is the increment of angular velocity reduction during an increment of time δt . Using $\delta t = \mu$, $\delta\omega$ is determined and thus the speed n_1 at time μ is established. The pump characteristic R_{A1} at speed n_1 is drawn. Point A_1 is determined and torque T_1 can be calculated. The pump speed at time 2μ can be determined in the same manner and lines $A_1C_{1.5}$ and $C_{1.5}A_2$ are drawn.

The pump will reach the zero speed point between times 2μ and 3μ . After that the pump will run in a reverse direction gradually increasing its speed and always remaining on its zero torque characteristic R_{A3} . The rest of the diagram is drawn in the same manner as in the first example.

If more points for a pressure-time curve at the pump are wanted the head-capacity curves are drawn for $\mu/2$ intervals, and two observers are sent out from A , one at time $t = \mu$, and the other at time $t = \mu/2$, they will record pressure at A at $\mu/2$ intervals. As a means for reduction of pressure rise, use of a flywheel is not very effective. For instance, in one case an 800-hp, 1200-rpm pumping unit having a WR^2 of 1450 lb in.² and delivering 9400 gpm at 284 ft into a 42-in. pipe 900 ft long, reversed in 0.8 sec and produced a pressure rise of 108 ft. When supplied with a flywheel with $WR^2 = 5500$, the time of reversal increased to 2.42 sec, and the pressure rise reduced to 48 ft (14). Thus, to reduce the pressure rise to 44.5 per cent, the moment of inertia of rotating mass (WR^2) had to be increased 4.8 times.

Valve Opening. Fig. 16 shows a water-hammer diagram for a pump started with the discharge valve closed, and then the valve suddenly opened. The surge diagram is drawn starting from the point A_0 and zig-zagging between the pump characteristic R_A and the pipe characteristic R_C until the final point D is reached.

Positively Controlled Valves. To avoid surge pressures due to a sudden check valve closure when a pump is shut off or when the power fails, valves with automatically controlled closure time are used. The most advantageous valve-closing time can be established from a study of the water-hammer diagrams.

In Fig. 17 (a) the pump at A discharges against a static head H_s and pipe-line resistance R_C . In Fig. 17 (b) R_A is the characteristic of the pump running in the reverse direction at zero torque; R_{V1} is the valve resistance curve at time μ , R_{V2} is the same at time 2μ . At time 3μ the valve is completely closed. In order to construct the surge diagram, the system characteristics are drawn for time μ and time 2μ by combining R_{V1} and R_A , and R_{V2} and R_A , to obtain R_{A1} and R_{A2} , respectively. R_{A3} coincides

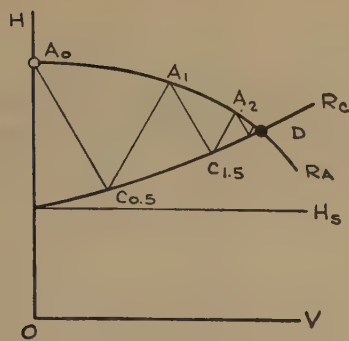


FIG. 16 SUDDEN VALVE OPENING

with the axis of heads. In Fig. 17 (c) the surge characteristics are then drawn following the usual procedure. Points $A_0, A_1, A_2, A_3, \dots, D$ give pressures at A at one interval of time spacing.

Valve Closure at End of Discharge Pipe. In Fig. 18 a centrifugal pump at A delivers water against a static head H_s to the tank at C . The pipe resistance characteristics R_{C0} determine the operating point A_0 . Now suppose the valve at C is closed in a time

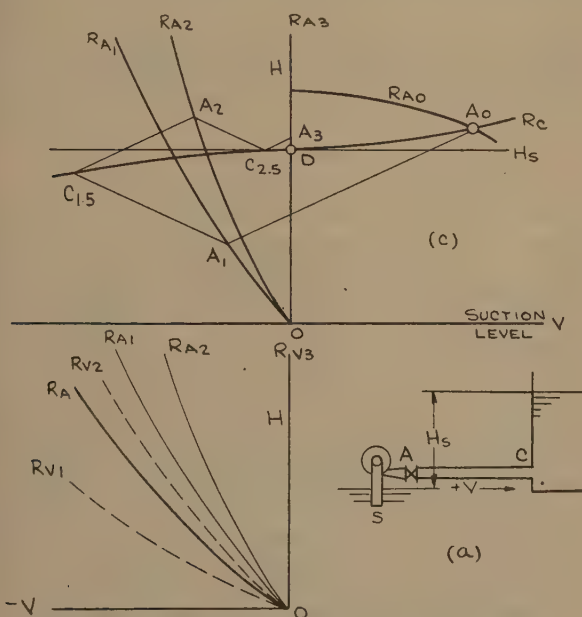
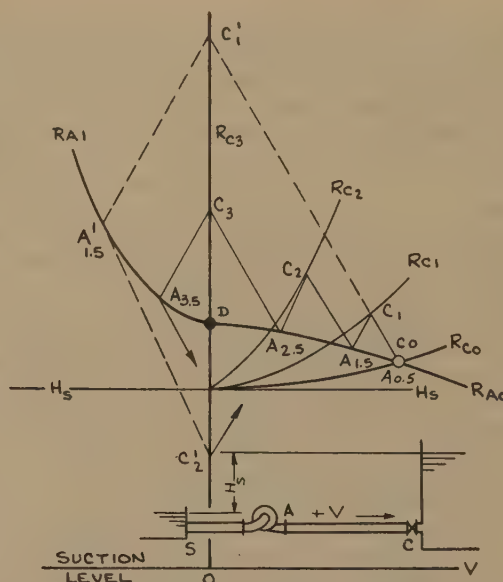


FIG. 17 POSITIVELY CONTROLLED VALVE CLOSURE

$T = 3\mu$, while the pump is running. The system characteristics for the point C with the valve throttled at times $\mu, 2\mu$, and 3μ , are R_{C1}, R_{C2} , and R_{C3} , respectively. The pump characteristic for positive flow will remain R_{A0} and for negative flow it is represented by R_{A1} . The surge characteristic $A_{0.5}, C_1, A_{1.5}, C_2, A_{2.5}, C_3$ is drawn in the usual manner, the maximum pressure rise being at C_3 . If the valve at C is closed in a time $T \leq \mu$, the surge characteristic will be $A_0, C_1', A_{1.5}', C_2'$ as shown by the dotted line.

SPECIAL PROBLEMS

Water Hammer in Suction Pipe. In all examples thus far considered, attention has been confined to water hammer in the discharge pipe between the points A at the pump-discharge flange and C at the open end of the pipe, as in Figs. 11, 17, and 18. However, all velocity changes in the discharge pipe are followed by a corresponding velocity change in the suction pipe; thus water hammer is also produced in the suction pipe and pump casing.


 FIG. 18 VALVE CLOSED IN $t = 3\mu$, FULL LINE; SUDDEN VALVE CLOSURE PUMP RUNNING, DOTTED LINE

Mechanically, the suction pipe and pump casing may be the most vulnerable parts of the whole installation. It is not often that long suction pipes are used and, as a rule, suction velocities are lower than those in the discharge pipe. Thus dangerous pressure rises due to water hammer in suction pipes are not frequent. However, cases of suction-pipe damage are on record (19). The author knows a number of cases where pump casings cracked on gasoline loading pumps due to a sudden closure of a quick-closing valve at the end of the discharge pipe. In several cases vertical turbine pumps (in a suction barrel), used for a similar service, developed buckled shafts (by compression) in addition to cracked discharge nozzles, thus indicating a higher pressure in suction than in discharge. Angus mentions cases in which the entire rotating element of a hydraulic turbine has been lifted by the pressure rise in the draft tube, which corresponds to the suction of centrifugal pumps (20).

Suction pipes, including the pump up to the discharge flange, can be investigated for water hammer in the same manner as that used for discharge pipes. The end conditions are fixed by the system characteristics. The suction level for the intake end of the suction pipe (point S in Figs. 11, 17, and 18), and the other end (A in these figures) is common for both the suction pipe (including pump) and the discharge pipe; thus the same characteristics apply. The starting point A_0 is given by the initial conditions and is also common for both pipes.

Evidently, if the object of a study is a determination of the maximum pressure rise there is no need to draw a water-hammer diagram for the suction pipe because the maximum rise will appear in the discharge pipe. However, the pressure rise produced on the discharge end of the pump will travel through the pump casing and into the suction pipe (reduced only by hydraulic losses) when not prevented by a check valve or positively controlled valve. In the case of a sudden valve closure, Fig. 18, the pressure rise C_1' will travel the full length of the suction pipe undiminished (except for hydraulic losses), the duration of super-normal and subnormal pressure decreasing from μ sec at C to zero at the free end of the suction pipe. Reference to Fig. 2 will illustrate the point. Note on this figure that at certain times (for instance μ to 1.25μ), the pressure at a point C' , the point most removed from the valve A , is higher than at the valve by

the amount of the full pressure rise h . This may help to visualize how pressure at the pump suction, with pump running, can be higher than at the pump discharge if the pressure rise is higher than the pump generated head. If the discharge valve at C , Fig. 18, is closed gradually, the pressure rise in the suction pipe and the pump casing will be lower than at the valve (see Fig. 3), depending upon the relative length of the suction and discharge pipes.

When the subnormal pressure wave passes through the pump it may reduce the pressure at the impeller eye to the vapor pressure, thus producing a cavity. If this occurs a severe blow can develop upon arrival of the supernormal pressure wave which would cause an instantaneous collapse of the vapor cavity in the same manner as that during cavitation. Formation of the cavity will interfere with the propagation of pressure waves, thus complicating matters considerably. Cavitation and parting of the water column may also take place on the discharge side of the pump if the topography of the pipe layout is such that the subnormal pressure wave is greater than the absolute pressure (above the vapor pressure) at some points on the pipe line. Bergeron (3) and Angus (6) give graphical solutions for such systems, but experimental verification of results is lacking. The sequence of events and the manner in which the high pressure rise appears under cavitation conditions is illustrated in the example which follows.

Parting of Water Column or Cavitation. Suppose in a system as shown in Fig. 19 (a), the valve at the discharge flange A , is suddenly closed, and we want to investigate water hammer in the discharge pipe. On the diagram, Fig. 19 (b), A_0 is the initial operating point, and V_0 is the initial pipe velocity. The sudden valve closure will cause a sudden pressure drop at A to absolute zero (vapor pressure) and a sudden velocity drop from V_0 to V_0' , as determined by the slope coefficient a/g of the line A_0A_0' . From that instant to the time $t = \mu$, water will continue to move at A with a velocity V_0' and the column moves a distance $V_0'\mu$ leaving a vacuous space $V_0'\mu$ long. At time μ the velocity at A drops suddenly to V_1 , and water will continue to move away from the valve an additional length $V_1\mu$. Velocity V_1 is determined by the surge lines $A_0'C_{0.5}A_1$ drawn in the usual manner. However, in the case of a sudden valve closure the physical meaning of these surge lines is different; they do not represent the pressure variation along the pipe line, as pressure travels undiminished, and pressure and velocity changes occur suddenly. But the surge lines give a time scale for the velocity variation during the retardation time and the acceleration of the water column in the opposite direction (toward the valve). At time $t = 2\mu$, the velocity at A will change suddenly from $+V_1$ to $-V_2$ and the water column will start moving toward the valve. At time $t = 3\mu$, the vacuous space between the valve and water column will have been reduced by the length $V_2\mu$. Also, at time $t = 3\mu$, the velocity will increase suddenly from $-V_2$ to $-V_3$, and the vacuous space will be reduced by an additional length $V_2\mu$ during the fourth interval of time. It is assumed, for simplicity, that the vacuous space is reduced to zero exactly at time $t = 4\mu$ when velocity $-V_3$ is suddenly arrested, causing a pressure rise to A_4 following the surge line $A_2C_{3.5}A_4$. This pressure will prevail at A for μ sec, then will fall suddenly to A_5 , after which the cycle will repeat itself at reduced velocities until all the energy in the water column is absorbed by the hydraulic losses, and the final point for settled conditions is reached at D . The pressure variation at A with time is shown in Fig. 19 (c). The height of the peak and time spacing between the peaks is reduced as the energy of the water column is wasted. Such curves were obtained experimentally by Le Conte (21) and Langevin (15).

From the inspection of Fig. 19 (b) it is evident that the maximum pressure rise at time $t = 4\mu$ is approximately equal to the

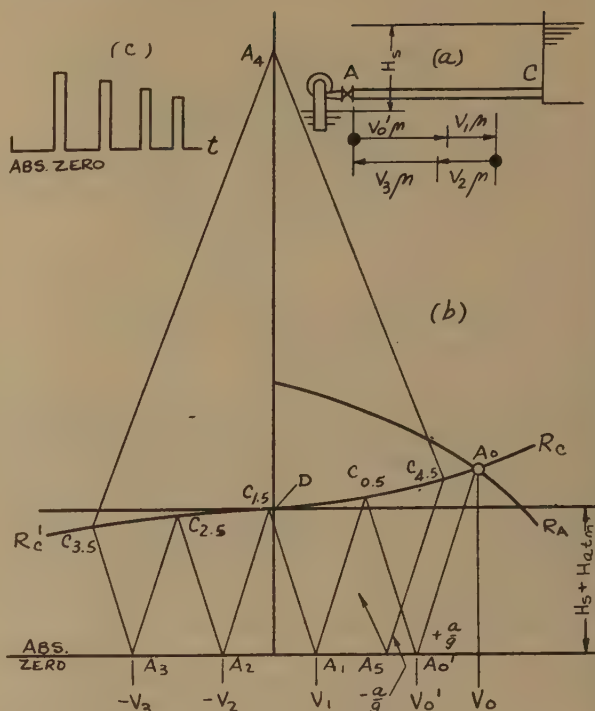


FIG. 19 PARTING OF WATER COLUMN

pressure rise due to a sudden destruction of the initial velocity V_0 , reduced only by hydraulic losses during the first cycle of events. For a frictionless flow the maximum pressure rise will be exactly equal to that due to a sudden extinguishing of the original velocity V_0 . The maximum velocity in the reverse direction will be exactly equal to V_0 , and the time of recoil of water column would be exactly the same as the time to retard the water column to zero from its original velocity V_0 . This is because the reverse flow is produced by the force due to a pressure $H_S + H_{atm}$ which is the same force that caused the deceleration of the initial flow in the pipe. The kinetic energy of the water column in the pipe is converted into the energy of position at the level $H_S + H_{atm}$ during the deceleration of the initial flow, which is converted back into the kinetic energy of the flow in the reverse direction during the second half of the cycle. If the instant of the closing of the vacuous space does not coincide with the end of one complete interval, the shape of the pressure-rise peaks in Fig. 19 (c) becomes irregular. If the valve is closed gradually in a time less than one interval μ , the same pressure rise will result, but the curve becomes sharp-pointed.

Air Chamber and Check Valve. Air chambers are used effectively to reduce pressure fluctuations at the pump check valves. The size of the chamber and air space are selected so that the air chamber will be able to store the excess volume of the expanding water and that due to the contracting pipe without raising the air pressure beyond a predetermined limit. To be effective the air vessel should be placed as close as possible to the source of water hammer. In Fig. 20 assume we are to determine the pressure variation at the pump (point A) in the case of power failure. Suppose that the physical dimensions of the system are such that the pump comes to rest, and the check valve closes, instantaneously. The height of water column in the air chamber is very small in comparison with the total head H_0 . The procedure is the same as in Fig. 12 except that the system characteristics for the point A are determined by the air chamber and not by the check valve. The surge characteristics are drawn at

If the time of one interval μ is very small, or if the surge lines are drawn at $\mu/4$, or $\mu/8$ spacing, no second approximation is necessary. The rest of the diagram is drawn following the usual procedure. By comparing Fig. 20 with Fig. 12, it will be noticed that for a sudden valve closure without an air vessel the flow in the pipe is reversed in one time interval μ , and the full pressure rise appears at the end of the second interval (point A_2 , Fig. 12). With an air chamber it takes several periods to reverse the flow in the pipe, (or extinguish the initial velocity V_0) and produce supernormal pressures. By that time a considerable portion

Air Chamber and Flywheel Effect. If a pump has an appreciable flywheel effect the system characteristics for the point A during the pump deceleration time are determined both by the pump head capacity curve and the air chamber. In Fig. 22 the pump head capacity curves are drawn for the times μ , 2μ , etc., and marked R_{A1} , R_{A2} , etc. Point A' determines the pump head H_1 and capacity (pipe velocity at the pump V_1). The pipe velocity at point A between the air chamber and point C , is greater by the amount ΔV , furnished by the air chamber as a result of the pressure drop from H_0 to H_1 , computed from $\Delta V_1 A = v_1 - v_0$ where $v_1 = v_0 H_0 / H_1$. Thus point A_1' represents the conditions between the pump and air chamber, and A_1

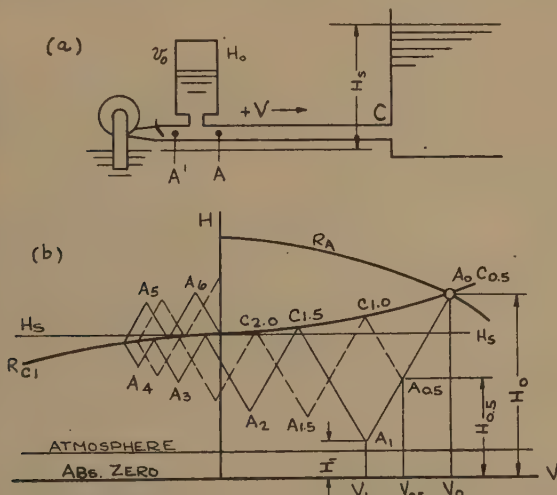


FIG. 20 AIR CHAMBER AND CHECK VALVE

Air Chambers With a Throttle. Frequently air chambers are provided with a throttle at the inlet to the chamber to increase the power dissipation and thus raise the effectiveness of the air chamber. Bergeron (15) describes a design of a throttle which

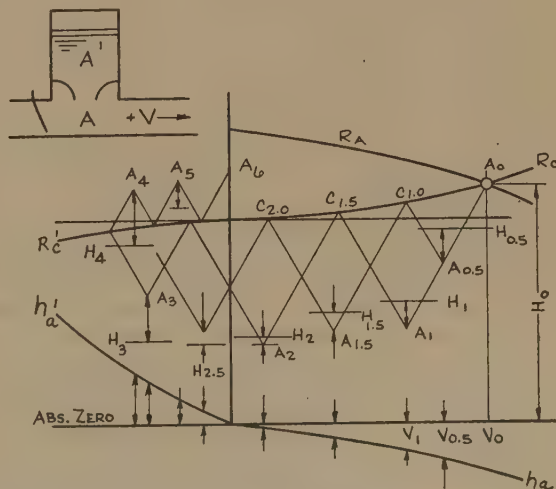


FIG. 21 AIR CHAMBER WITH THROTTLE

those for a point A on the other side of the air chamber, Fig. 20 (a). The diagram is continued from the point A_1 to locate the $C_{1.5}$ and A'_2 points. The point A_2 is obtained by adding ΔV_2 to the pipe velocity V_2 from $\Delta V_2 A = v_2 - v_1$; v_2 to be calculated by $v_2 = v_1 H_1 / H_2$. After the pump rotor comes to rest the construction of the diagram proceeds in the same manner as in Fig. 20.

Air Pockets. If the volume of the air is not sufficient to accommodate the whole volume of the expanding water column, a dangerous pressure rise may occur under certain conditions (17). Small air pockets, or traps in the pipe line or valves, may cause surprisingly great surges in the same manner. One major accident in a European low-head plant is attributed to this cause (18).

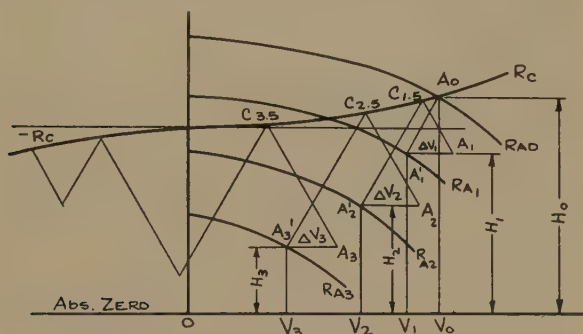


FIG. 22 AIR CHAMBER AND FLYWHEEL EFFECT

Evidently, for a small volume of air to store the whole kinetic energy of the flow, its pressure should be very high. In other words, a small air volume causes a reversal of the flow which is almost immediately stopped, producing sudden pressure rise.

EXAMPLES OF WATER-HAMMER DAMAGE

Starting and Stopping Pumps. Water hammer caused by centrifugal pumps themselves can take place in the process of stopping and starting them. Vertical turbine pumps with open impellers running in close vertical clearances with the casing seat furnish examples of damage to a pump by water hammer. In one case, a vertical pump of this type was used to circulate hot molten salt in an oil-refinery process. When started with the discharge valve closed and the discharge pipe (20 in.) air or steam-filled, a violent shock was produced which resulted in a bent 4-in. pump shaft. The difficulty was eliminated by starting the pump with the valve partly open and shaft rotating in the reverse direction. Stopping of vertical turbine open-impeller pumps used for deep-well service is always followed by pressure surges which cause the impellers to hammer against their seats while revolving backward. It is impossible to maintain close running clearances between the impeller and the casing under these conditions. Introduction of nonreversing clutches in connection with hollow-shaft motors eliminated impeller grinding but not the surges and vibration.

Stopping of pumps with the discharge valve open (automatic or due to power failure) may produce objectionable water hammer which may damage the check valve, loosen the bell-and-spigot lead joints, cause stuffing-box troubles or endanger other equipment connected to the discharge system. Sometimes noise and vibration alone, due to check slamming, may be objectionable. It is common belief that water hammer is accompanied invariably by a clearly audible blow. Severe surges have been observed when a high pressure rise was built up after the valve had closed quietly (22).

Water Hammer Originated by the System. In certain types of systems such as hydraulic descaling systems on strip mills,

hydraulic presses, refinery loading stations, and boiler-feed installations, the delivery from centrifugal pumps may be shut off suddenly while the pump is running. Destructive water hammer will result if no precautionary measures are provided by the installation. In the case of descaling and hydraulic-press systems, pneumatic accumulators serve as protective air chambers. With long suction lines, air chambers are also used on the suction to prevent breaking the water column at the pump suction. A bypass from the discharge pipe to suction (or atmosphere), while protecting the pump from overheating at zero discharge, also alleviates the hydraulic shock. Open-impeller pumps, of the vertical-turbine type, enclosed in a suction barrel, when used for petroleum-products loading service, are particularly vulnerable to hydraulic shock. In several instances pump shafts were bent (by compression) and the pump-discharge piping (cast-iron increaser) was ruptured as a result of sudden closure of the discharge valve. Instances in which volute pumps with closed impellers developed cracked casings when used on loading service are numerous. A sudden rupture of the discharge pipe produces the same effect as a sudden valve opening, resulting in a high pressure rise as discussed already. In one instance a pipe rupture on an oil pipe line caused pipe "explosions" in several other places, possibly due to a liquid-column break and subsequent water hammer at points where the subnormal pressures fall considerably below the vapor pressure.

Means to Reduce Water Hammer. The pressure rise can be reduced only by reduction of the rate of velocity destruction in the pipe. This can be accomplished by one or several of the following means:

- 1 Designing pipe systems with low original velocities.
- 2 Positively controlled valves combining functions of both a check and gate valve. The slower the closing rate, the less is the resulting pressure rise. Loss of water and reverse rotation of pumps is seldom objectionable.
- 3 Use of special check valves such as "balanced," and "loaded," or a by-pass around a check valve which can be closed slowly (manually or automatically) after the check valve is closed.
- 4 Use of air chambers, accumulators, or surge tanks. The latter are the most expensive and hardly ever justified under ordinary circumstances. One installation of this type was built by the United States Government at Granby, Colo. (23).
- 5 Relief valves are widely used for protection of various types of hydraulic equipment. When provided with a positive control these are known as surge suppressors.
- 6 When water column parting is unavoidable in a pipe line, vacuum breakers are installed to admit air which cushions the shock when the two parts of the water column join together.

CONCLUSION

The graphical solutions of water-hammer problems are just as accurate as the analytical solutions. There are numerous experimental proofs of the soundness of the basic water-hammer theory. The irregular shape of pressure-rise curves plotted from theoretical calculations were reproduced with surprising accuracy by accurate recording instruments. The accuracy of solutions depends entirely upon the accuracy of the information about the rate of flow during the valve-closure time. In centrifugal-pump applications it is not often that the rate of flow for different valve positions is known. A motorized gate valve with a uniform stem travel does most of the throttling in the last quarter of travel. Follower-ring-type valves produce a more uniform retardation of flow. With special "ported follower ring" valves, any desired rate of flow retardation (including uniform) can be obtained, but their use is very limited (4). Thus for an accurate solution of water-hammer problems (graphical or analytical), it

is necessary first to visualize the sequence of events clearly, and then to establish the end conditions accurately, including the rate of pipe-flow retardation. The drawing of the diagram then becomes a simple mechanical procedure.

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- 19 "Methods Employed to Remedy Water Hammer, etc.," by E. Bruce Ball, *Trans. ASME*, vol. 61, 1939, p. 6.
- 20 Reference (8), p. 389.
- 21 "Experiments and Calculations on the Resurge Phase of Water Hammer," by Joseph N. Le Conte, *Trans. ASME*, vol. 59, 1937, p. 691.
- 22 Reference (4); discussion by S. Logan Kerr, p. 304.
- 23 "Hydraulic Problems in Connection With the Design of Granby Pumping Plant," by E. B. Moses, *Trans. ASME*, vol. 70, p. 459.

Discussion

R. W. ANGUS.⁴ In the design of water-power plants, hydraulic engineers have long appreciated the importance of water hammer and have provided against its effects, but in the case of pumping plants and their pipe lines, the subject has only recently received much attention. If treated from the purely mathematical angle, calculations of pressure rise, due to gate closure or other cause, are long and tedious, but graphical methods have been devised which greatly simplify the calculations, and the method described in this paper is probably the best in use because the results are easily worked out on the drafting board. Unfortunately, the author has tried to cover too much ground in the space occupied, and his explanations are hard to follow by anyone not familiar with the method. The writer suggests a reference to his own paper mentioned with incomplete title in the author's reference (4).

In the writer's paper, published in 1937, he has given in an extended form a complete treatment covering both the rigid and the elastic water-column theories as applied to single lines of

pipe, but having sections of varying diameters, the method used being illustrated with many examples, a number of which are the same as the present author's. The author's reference (6) is to the writer's paper dealing with branched and compound pipes, and the two papers together cover the field of application of the method of treatment described by the author.

Since the mechanism of the events due to sudden changes of velocity is complicated, it is only to be expected that the fundamental ideas are hard to grasp. However, if one first spends the necessary time and effort to master the physical ideas involved, and gets a clear picture of what goes on in the pipe, then there is no great difficulty with the mathematics, because only an elementary knowledge of the calculus is required, and the resulting equations are very simple and easily applied. The author suggests that these be derived by Bergeron's method, which apparently avoids the mathematical approach, but with other complications; the reader must decide for himself which method appeals to him, but the writer, though not an expert in mathematics, finds its application most satisfactory in this case.

Under the subsection "Pipe of Variable Diameter," the author refers to single pipes of varying diameter and his Equations [5] and [6] are in general use, though no proof of them has ever been given, to the writer's knowledge. In most cases it is not necessary to reduce the actual pipe to an equivalent uniform pipe because the actual case usually can be easily dealt with by the graphical method given here. However, in the case, for example, where the pipe consists of alternate lengths of, say, 16-in. and 18-in. pipe, it is desirable to deal with an equivalent uniform pipe of dimensions found by the author's formulas.

Velocity head is always difficult to deal with, but fortunately it is usually relatively small compared with the total head so that it may usually be neglected. Of course where a pipe bursts, all the head in the issuing fluid is in the form of velocity head and it must be reckoned with, but there is not space enough to deal with it here. Friction loss comes under a different category; in many pumping lines, as in most water-supply piping, the design is such as to keep the friction loss to a relatively small part of the total head, but there are many other cases such as in low-head pumping, where the friction loss far exceeds the static head and where the effect of friction must be considered. A well-known example of the latter is the long oil line where the pipe is nearly horizontal and where the pump pressure, often as high as 500 to 1000 psi, is entirely taken up in friction.

The graphical method given in the paper provides no exact method of dealing with friction, and the adopted method is to assume this loss concentrated at definite places on the pipe. For example, in a line 5000 ft long with a friction loss of 50 ft, it might be assumed that 10-ft loss, due to some kind of obstruction, was concentrated at the end (entry or discharge) of each 1000-ft length, and that no loss occurred between these obstructions. Such an assumption enables friction to be dealt with by the graphical method, and, obviously, by dividing the total loss up into enough parts, the assumed pipe may be made to approach the actual one as closely as desired. In the cases the author illustrates, the friction loss appears to be relatively small, in which instance no great error is likely to result where the total loss is all concentrated at the entry end of the pipe or at its exit, but such concentration gives poor results where the friction loss is relatively high.

The author's illustration in Fig. 19 is of a case which cannot be dealt with accurately by this or any other method because of our lack of knowledge of what happens in the vacuous space. Should there be any danger of the discharge valve closing suddenly while the pump is operating, provision should be made to admit air or water on the discharge side of the valve. The graphical solution is then readily applied.

⁴ Professor Emeritus, Mechanical Engineering, University of Toronto, Toronto, Can. Fellow ASME.

Long suction pipes are by no means uncommon, those in the oil lines being often over 50 miles long. The writer examined a waterworks case with a mile of suction line. Unless the suction line is very short it should always be taken into account.

O. G. GOLDMAN.⁵ There are two basic errors in the development of the theory of water hammer:

(a) In the development and interpretation of the Joukowski equation.

(b) In the development of the equation for the velocity of the pressure wave.

In this paper is given Joukowski's law (author's Equation [1]),

$$h_{\max} = a \frac{V_0}{g} \dots \dots \dots [10]$$

The author states: "This equation follows from the application of the second Newton's law, which states that the force (increase in the pressure at the valve) is equal to the time rate of change of momentum of the moving mass" This fundamental equation is then given as

$$F = A \gamma h_{\max} = \frac{A \gamma x}{g} \cdot \frac{V_0}{t} \dots \dots \dots [11]$$

where X = length of pipe line.

Note that the foregoing equation consists of two parts, namely

$$\frac{A \gamma x}{g} = \text{mass}$$

$$\frac{V_0}{t} = \text{acceleration}$$

Now, in order to get Joukowski's equation, the "error" occurs: "Here x/t is the rate at which the pressure wave travels along the pipe line, and this is equal to the velocity of sound in the water within the pipe . . ."

The value of t in the foregoing equation is a function of V_0 and determines the value of the "rate of change of momentum," or the "acceleration" of the mass, and hence V_0/t must be treated as "one term."

Again

$$\frac{x}{a} = t_1$$

or

$$x = at_1$$

hence

$$h_{\max} = \frac{at_1}{g} \frac{V_0}{t} \dots \dots \dots [12]$$

Only when $t_1 = t$ will

$$h_{\max} = a \frac{V_0}{g}$$

In other words, Joukowski's equation is a "unit" equation and not a "general" one.

Under the section entitled, "Velocity of Pressure Wave," is given the author's Equation [3], as follows

$$a = \frac{1}{\sqrt{\frac{\gamma}{g} \left(\frac{1}{k} + \frac{d}{eE} \right)}} \dots \dots \dots [13]$$

⁵ Assistant Superintendent, San Francisco Water Department, San Francisco, Calif. Mem. ASME.

One of the fundamental equations from which this equation was derived is given in "Hydraulics" by Schoder and Dawson, and is as follows

$$k = \frac{pAL}{A(\Delta L) - 2\pi r(\Delta r)(L - \Delta L)} \dots \dots \dots [14]$$

where L = length of column or pipe line. Now the basic equation for determining the value of k is

$$k = p \frac{V}{\Delta p}$$

or

$$k = \frac{pAL}{A(\Delta L)} \dots \dots \dots [15]$$

Equation [15] is a general equation and applies to any and all materials. For predetermined conditions, namely, constant temperatures and pressures, k is a constant. Now note that the numerator of Equation [14] is identical to the numerator of Equation [15]; also that the first term of the denominator of Equation [14] is identical to that of Equation [15] but in Equation [14], the denominator also contains a minus term, namely, $-2\pi r(\Delta r)(L - \Delta L)$, which is not in Equation [15]. Only when the term $-2\pi r(\Delta r)(L - \Delta L)$ is equal to zero do Equations [14] and [15] become identical. This means that for any value of (Δr) greater than zero, the value of k increases and is not a constant. As (Δr) increases in value, the value of the denominator decreases. Hence when $2\pi r(\Delta r)(L - \Delta L)$ is equal to $A(\Delta L)$ then k becomes infinite in value, and when $2\pi r(\Delta r)(L - \Delta L)$ is greater than $A(\Delta L)$, then k becomes indeterminate.

With the development of the fundamental equations in error, the validity of the conclusions reached therefrom must be questioned.

Another point at issue is the following statement: "When the wave front reaches the open end of the pipe C , the whole pipe is completely filled with water at rest under an additional pressure head $h_{\max}$ uniform along the whole pipe length."

The writer's Equation [12] can be expressed as follows

$$h_{\max} = \frac{L}{g} \frac{V_0}{t} \dots \dots \dots [16]$$

where L = length of column in pipe line.

Now L is the length of the pipe line "measured from the open end" (in the case of a simple pipe line) "to the point of creation of the surge." Hence the magnitude of the surge pressure is directly proportional as the distance the point of observation is measured from the open end of the pipe line to the total length of the pipe line.

Let x be any point of surge-pressure observation along the pipe line, the distance being measured from the open end of the pipe line. Hence when

$$\begin{aligned} x &= L, & h &= \text{maximum} \\ x &= 0, & h &= \text{zero} \\ x &= \frac{L}{2}, & h &= \frac{1}{2} h_{\max} \end{aligned}$$

Again, under the section, "Pipe of Variable Diameter," the author states: "In the case of a compound pipe consisting of several sections of different diameters, an equivalent velocity is used for calculation of the maximum pressure rise with Equation [1]."

Referring again to the writer's Equation [12], it should be noted that it is the "change in velocity" which produces the surge pressure. Therefore whatever change in the velocity

occurs, in the particular pipe section under observation, will determine the surge pressure. An "equivalent velocity" may give a greater or lesser value than the actual.

G. D. JOHNSON.⁶ This paper is virtually a "symposium" on water hammer in centrifugal-pump systems. It summarizes most of the available information on this increasingly important subject and illustrates the application of the graphical method to a wide variety of pumping problems.

However, there is one notable omission, namely, no specific mention was made of the automatically operated, rotating, conical-plug-type valve which is now almost universally accepted as the most nearly ideal pump discharge valve. This popularity is due to negligible head loss when fully open for pumping and a discharge cutoff versus servomotor stroke characteristic that inherently minimizes water-hammer effects. Similar hydraulic characteristics are also found in follower-ring gate valves, as mentioned by the author, but superior mechanical features of the conical-plug valve have led to its widespread acceptance for modern centrifugal-pump installations. The characteristics of this type of valve have already been fully explained in the technical press.^{7,8,9} The graphical treatment applies to any system regardless of the type (or types) of valves employed, provided that a proper analysis is made of the various factors involved.

It should be noted, as pointed out by Angus,¹⁰ that the characteristic curve of discharge versus servomotor stroke for any particular valve (of any type) will be modified by the system in which it is installed, unless pipe friction be negligible. Although friction frequently can be ignored in the calculation of water hammer in hydraulic-turbine penstocks, it usually constitutes a sufficiently large percentage of any total pump-discharge head to necessitate its consideration in calculations for pumping mains. The author illustrates two methods of taking friction into account graphically, both of which yield similar results when properly interpreted.

The use of positively controlled pump discharge valves, such as the conical-plug or follower-ring types, instead of swing-type check valves, is illustrated and recommended by the author, because the maximum pressure rise resulting from emergency closure after failure of the power supply to a pump motor can be limited to any desired value by permitting a reasonable loss of water and a short period of reverse pump rotation. Closure of a swing-type check valve just as the water column comes to rest prevents flow reversal, but not pressure rise, which can almost equal the pumping head if the friction head is small and the "correct" relationships exist among the system characteristics.

If it is desired to prevent reverse pump rotation, automatic fast closure of the pump discharge valve and opening of an adequate relief valve, which subsequently closes slowly, also serves to limit the maximum pressure rise as desired. As discussed by Peabody,⁷ normal operation, with the conical-plug valve automatically opening slowly after the pump is up to speed and with the pump motor being shut off by the valve after it has closed slowly, can be effected with negligible pressure disturbances. Moreover, as mentioned by the author, a positively controlled valve is not only a superior check valve but also serves as a stop valve, thereby replacing both the swing-check and the gate valve of earlier installations.

Another important recommendation by the author is to provide for automatic admission of air at any point in a system where the water column may part, thereby increasing the parting time and cushioning the shock when the portions of the column rejoin.

JOHN PARMAKIAN.¹⁰ The following discussion applies to only a part of the paper, namely, that pertaining to the water-hammer effects in motor-driven centrifugal-pump discharge lines upon a power failure at the pump motors.

In attempting to present to pump designers the water-hammer theory for pump discharge lines in its "simplest terms," the author bases his graphical water-hammer solutions on certain simplifying assumptions which are improper for most pumping-plant installations. As a result, the correct physical phenomena associated with the transient hydraulic conditions in pump discharge lines following power failure are completely obscured. For example, in the graphical solution of the water-hammer conditions following a power failure to the pump motor at a pumping-plant installation with no check valve, the author states: "In the event of power failure assume that the pump will slow down, stop, and reverse in less than one (round-trip wave travel) interval of time μ . The pump will then operate as a hydraulic turbine under a head H_s at runaway speed."¹¹ Comparable assumptions are also made in the graphical water-hammer solutions for a pump discharge line with various types of valves or pressure-control devices.

Judging from the water-hammer analyses and available test results which have come to the writer's attention, for over 50 pumping plants with pump motors ranging from 10 hp to 65,000 hp, these assumptions cannot be justified at a single pumping plant. A careful study of recent papers on this subject¹² will indicate that a correct explanation of the physical phenomena and the graphical water-hammer analysis of pump discharge lines of any size or capacity can be obtained from the following considerations:

When the power supply to the motor is suddenly cut off, the only energy that is left to drive the pump in the forward direction is the kinetic energy of the rotating elements of the motor and pump, and that obtainable from the flow of water through the pump casing. Since this kinetic energy is usually small compared to the pumping energy that is required to maintain the flow against the discharge head, the reduction in the pump speed is rapid. As the pump speed reduces, the pump discharge and the velocity of water in the discharge line also will be reduced. As a result of the decrease in the water velocity, water-hammer waves of subnormal pressure will originate in the pipe line adjacent to the pump and will move up the discharge line. These subnormal pressure waves are then reflected at the discharge end of the pipe line, causing waves of superpressure to form which will move down the pipe line toward the pump. When these waves of superpressure reach the pump, they will cause an increase in the pumping head which exists at the pump and further reduce the pump discharge. Soon the speed of the pump is reduced to a point where no water can be delivered against the existing head. If a control valve is not present on the discharge side

¹⁰ Engineer, Bureau of Reclamation, Denver, Colo.

⁶ Hydraulic Engineer, S. Morgan Smith Company, York, Pa. Mem. ASME.

⁷ "Pump Discharge Valves on the Colorado River Aqueduct," by R. M. Peabody, Trans. ASME, vol. 62, 1940, p. 555.

⁸ "Typical Analysis of Water Hammer in a Pumping Plant of the Colorado River Aqueduct," by R. M. Peabody, Trans. ASME, vol. 61, 1939, p. 117.

⁹ "Action of Valves in Pipes," by R. W. Angus, *The Canadian Engineer*, vol. 74, March 29, 1938, pp. 28, 30 and 32.

¹¹ This assumption was suggested by Dr. O. Schnyder in 1933, who stated at that time that a thorough analytical study of water hammer in pump discharge lines had not yet been accomplished. See discussion No. 12, by Dr. O. Schnyder, "Supplement to the Report of the ASME Committee on Water Hammer," 1933.

¹² For example see, "Comparison Between Calculated and Test Results on Water Hammer in Pumping Plants," by O. Schnyder, Trans. ASME, vol. 59, 1937, or "Typical Analysis of Water Hammer in a Pumping Plant at the Colorado River Aqueduct," by R. M. Peabody, Trans. ASME, vol. 61, 1939.

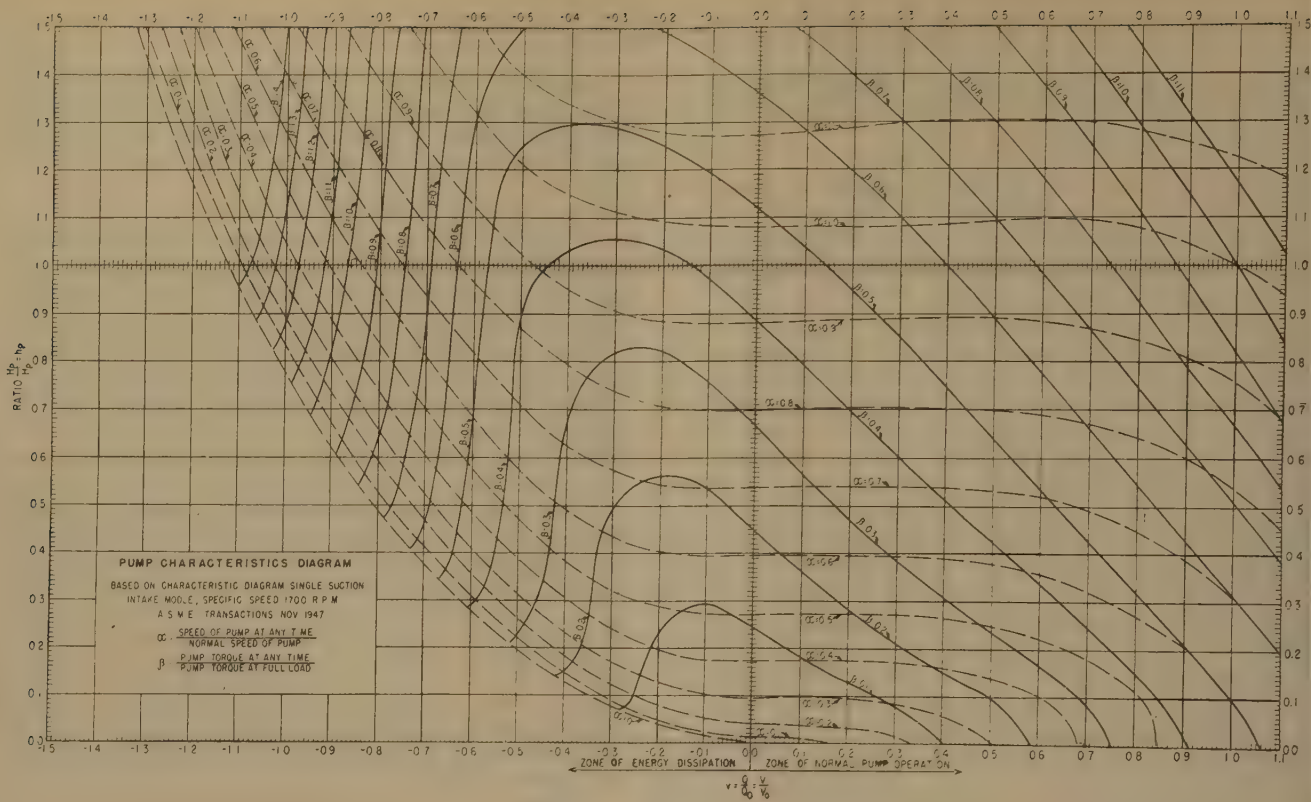


FIG. 23

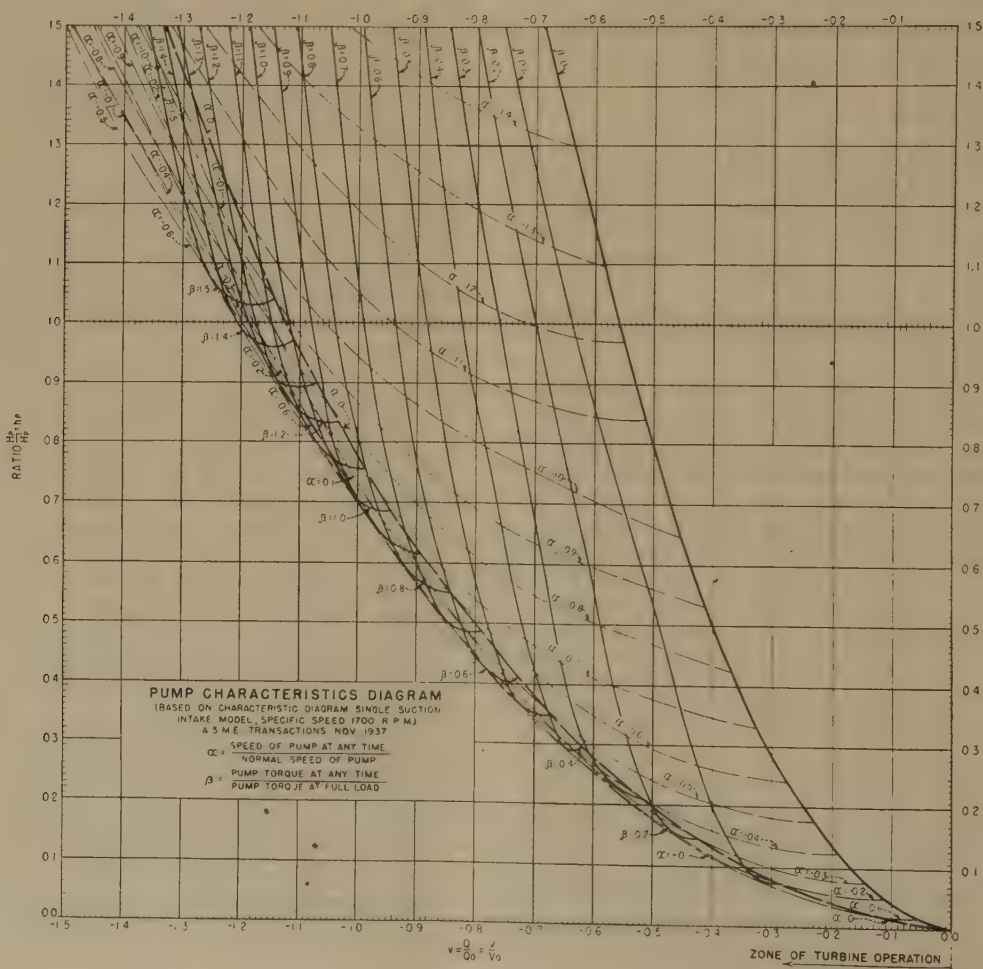


Fig. 24

of the pump to prevent reverse flow, reverse flow through the pump and the discharge line will soon start although the pump is still rotating in the forward direction. This abnormal pump operation is usually referred to as "the zone of energy dissipation." The speed of the pump will now drop more rapidly, and soon will pass through zero speed. A short time later the pump, acting as a turbine under no load, except for the inertia of the rotating parts, windage, etc., will reach a runaway speed with the pump running in the reverse direction. On most pumps the maximum reverse flow which can pass through the pump impeller is attained when the pump impeller is nearly stationary or has reached a small reverse speed. As the pump increases its reverse speed, however, and approaches the runaway reverse speed, the reverse flow which can pass through the pump is reduced. This reduction in reverse flow contributes principally to an increase in pressure or water hammer at the pump and along the discharge line.

It can be seen from the foregoing description that, after power failure at the pump motor, the flow of water and the corresponding water-hammer effects in the discharge line are determined by the flow characteristics of the pump impeller and by the inertia of the rotating elements of the pump, motor, and entrained water. In order to make a correct water-hammer analysis of the transient conditions following a power failure at the pump motor, it is necessary to derive a pump-inertia relation which can be used to determine the pump speed at successive time intervals after power failure. This relation is as follows¹³

¹³ The significance of each of the terms in this relation is indicated in the accompanying illustrations, Figs. 23-28, inclusive.

where

$$\alpha_1 - \alpha_2 = K(\beta_1 + \beta_2)\Delta t \dots \dots \dots [17]$$

$$K = \frac{91,600 Q_0 H_p'}{(WR^2)(N_0^2)\eta_0}$$

Furthermore, the graphical water-hammer solution is then performed on a set of characteristic pump-speed and torque curves which are plotted on a system of h_p and v co-ordinates. Such a set of curves for a single-volute pump for the zone of normal pump operation, the zone of energy dissipation, and the zone of turbine operation is shown in Figs. 23 and 24 on page 530.

The complete graphical water-hammer solution for an actual pumping-plant installation without a check valve, following a power failure at the pump motors, can then be performed on these charts, as shown in Fig. 25, using the conjugate water-hammer equations with the pump-inertia relation. It may be noted that for this particular installation, the maximum reverse speed for a simultaneous power failure at all pump motors is attained at about $(19L)/(2a)$ sec after power failure. The water-hammer solution is considered to be valid as long as the minimum pressure at any point in the pipe line does not fall below absolute zero. This minimum pressure condition can be examined by plotting the minimum values of the transient pressures which were obtained from the graphical water-hammer solution on the actual profile of the pipe as shown in Fig. 26.

This pumping-plant installation is equipped with standard motors with no added flywheel effect. The WR^2 of the pump motor as actually furnished was 350 lb-ft² and the WR^2 of each

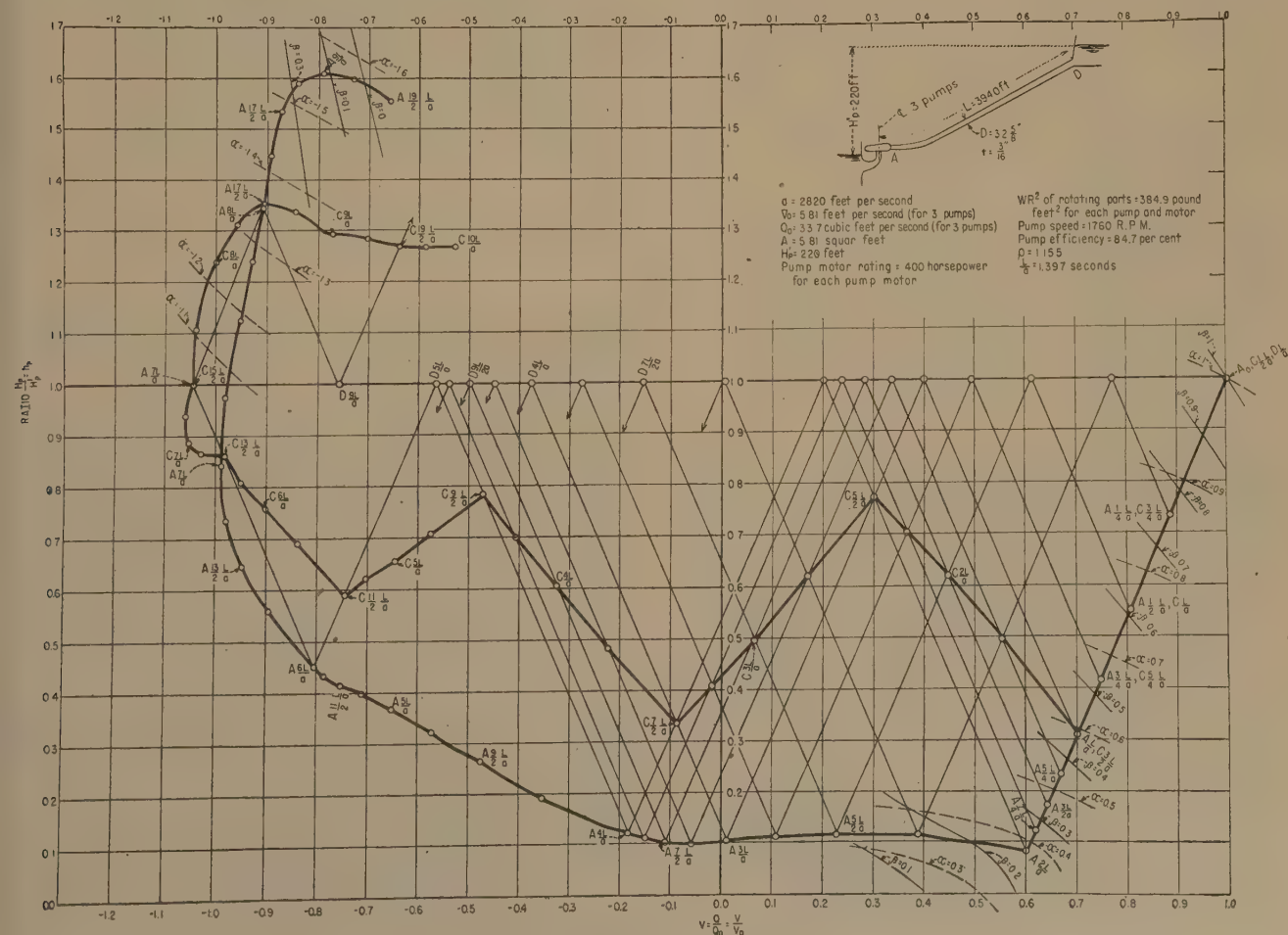


FIG. 25

pump, including the coupling and entrained water was 34.9 lb-ft². In analyzing the water-hammer effects for a pumping plant which has a very small value¹⁴ of K , water-column separation will generally be found to occur at the pump or at some point in the discharge line. Ordinarily this will result in very large water-hammer effects upon the rejoining of the water columns. Such a condition could occur, for example, at the installation shown in Fig. 25, upon the failure of the pump-shaft coupling with the flywheel effect of the motor removed.

The graphical method of water-hammer analysis which has been described may be extended to solve a large variety of pump problems with various types of pressure-control devices. For

¹⁴ Note that it is the value of K and not the value of the WR^2 alone which determines the speed changes of the pump impeller for any time interval.

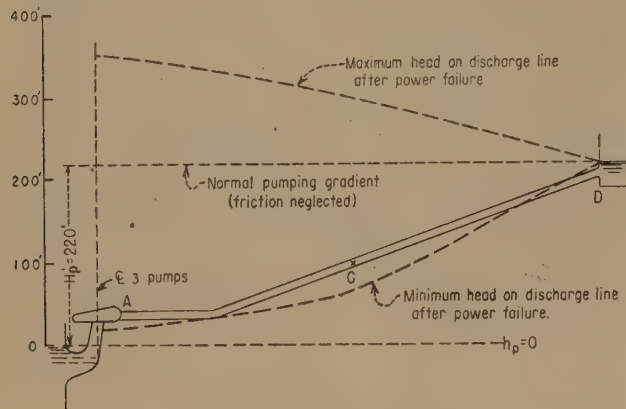


FIG. 26

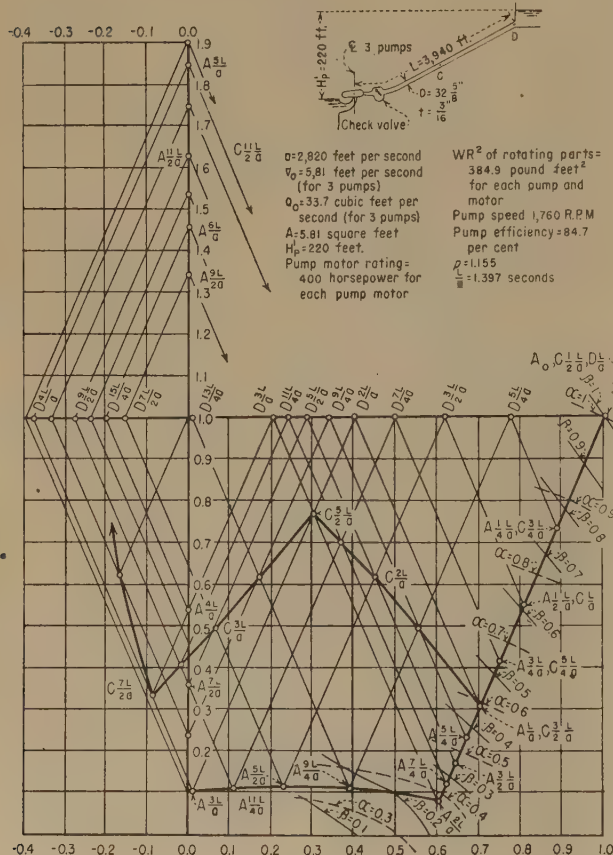


FIG. 27

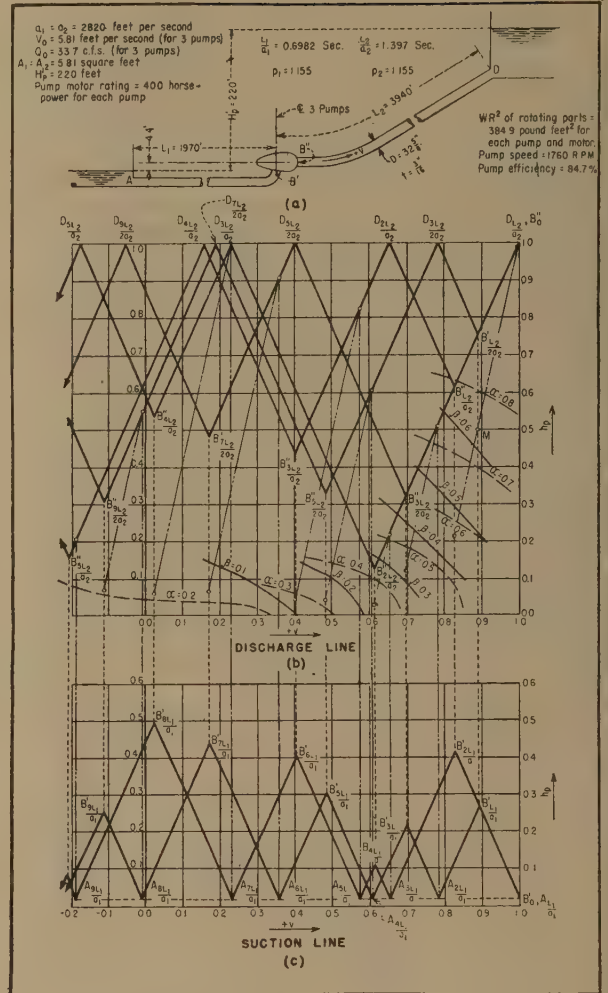


FIG. 28

example, in Fig. 27 the graphical water-hammer solution is shown for the same installation with a check valve.¹⁵ Similarly, in Fig. 28 part of the graphical water-hammer solution is shown for the same installation with a long suction line.

E. B. STROWGER.¹⁶ The author has made a worth-while contribution to the present engineering literature on water hammer. In connection with his introduction, it may be interesting to point out that the first development of a simple graphical method of determining water hammer was made by Professor Wood of McGill University in July, 1926. This contribution was published as a discussion of a paper by S. L. Kerr and the writer.¹⁷ Wood's method is based upon the Gibson formal equations for water hammer which have been shown to agree with the Allievi equations.¹⁸ While the basis of the graphical method was first

¹⁵ In this solution, the check valve is assumed to close instantly upon the reversal of flow. The actual closing time of the check valve and the corresponding water-hammer effects require an additional study and further refinement in the calculations.

¹⁶ Hydraulic Engineer, Buffalo Niagara Electric Corporation, Buffalo, N. Y. Mem. ASME.

¹⁷ "Speed Changes of Hydraulic Turbines for Sudden Changes of Load," by E. B. Strowger and S. L. Kerr, Trans. ASME, vol. 48, 1926, pp. 209-232; discussion by Prof. F. M. Wood, pp. 243-252.

¹⁸ See discussion by E. B. Strowger, of paper by Angus entitled, "Simple Graphical Solutions for Pressure Rise in Pipes and Pump Discharge Lines," *The Engineering Journal*, May, 1945, p. 266. See also op. cit., discussion by F. Knapp, p. 267.

developed by Professor Wood, he did not recognize the full importance and extension of his method. This was done quite independently by Schnyder and Bergeron in Europe, and later by Angus in Canada.

The author refers to trouble which has been experienced with boiler feed pumps where they have had unstable characteristics and where a mass oscillation of the water column produced pressure surges in discharge lines. He also cites examples of water-hammer damage and gives some helpful suggestions on how to reduce water hammer.

Distinction should be made between the mass oscillation of a column and the oscillation accompanying water hammer, i.e., a vibration of period $2L/a$. On two boiler feed pumps with which the writer is familiar, a water-hammer vibration in combination with an unstable quantity-head curve caused a severe vibration which was eliminated by changing the pump impeller to obtain a steeper head curve at low pump velocities. In this case the system resistance curve was relatively flat, the specific speed of the pumps was high, and the head-quantity curve was flat at low velocity. In fact, there was some evidence that it reversed direction. This characteristic is not a stable one, when operating under varying pressures at or near the point where the head-flow curve reverses direction, especially when the pressure rises and falls periodically due to a recurring water-hammer wave. This wave repeats itself in phase with a change in flow through the pump as the latter operates at varying points along the head-flow characteristic and may assume large pressure amplitudes.

Slow-motion moving pictures taken of pressure gages on the pump-discharge line showed square-topped waves which indicated that the vibration was water hammer and not a mass oscillation. Also, the vibration was eliminated when the discharge valve was closed and a recirculating line was opened, thus eliminating the long discharge line. The water-hammer period of the pump-discharge lines ($2L/a$) was $1/2$ sec, corresponding to a frequency of 2 per sec, and the pump vibration had this same frequency. The natural period (mass vibration) of the system, however, had a frequency of 8 per sec. Accordingly, the phenomenon involved a water-hammer vibration and not a mass oscillation of the water column.

The following changes in the pump or system were considered in order to eliminate or reduce the vibrations which occurred at low values of pump discharge:

- 1 Increase the number of stages in the pump, thereby increasing the specific speed and making the H - Q curve steeper.
- 2 Put in a large amount of friction, concentrating it at one point to dampen out the water-hammer wave.
- 3 Shorten the discharge lines to reduce the $2L/a$ value of surge interval and thereby eliminate resonance.
- 4 Put a surge chamber in the discharge line to eliminate the water-hammer waves.
- 5 Change the pump characteristics so that the H - Q curve is steep, i.e., replace impellers and diffusers of four stages.

Suggestion 1 meant a new pump, and this solution was not economical. Adoption of alternative 2 would have interfered seriously with the operation of the boiler-feed-pump controls and therefore was impractical. No. 3 was not possible on account of the plant layout. No. 4 was not workable because if air was used as the medium it would be dissolved quickly by the deaerated water, and consequently, 5 was the only practical approach to the problem. The solution to this problem was indicated from a study of the water-hammer characteristics of the system.

AUTHOR'S CLOSURE

Before commenting on some of the objections raised by several

discussers, the author would like to restate that the objective of the paper is to present a brief introduction to the subject of water-hammer theory, based on best authentic information, and the graphical method of solution in its simplest terms. Solutions of actual problems were not contemplated; on the contrary, attention was concentrated on the method itself, i.e., to establish the end conditions and draw the surge lines.

A number of topics, such as divided flow, compound conduits, distribution of losses among several points, etc., brought up by Professor Angus, were not included in the paper to conserve space, and for the reason that it is not often that a pump engineer would have use for these. On the other hand, after the fundamentals and the method are grasped by the reader, a further study of Professor Angus' papers can be followed without difficulty.

Professor Angus inquired about the validity of Equations [5] and [6] of the paper. The first one is obtained by equating the total kinetic energy of the flow of the several pipes, comprising a compound conduit, to that of one pipe with an equivalent section area A and carrying water with the equivalent velocity V

$$LA \gamma \frac{V^2}{2g} = L_1 A_1 \gamma \frac{V_1^2}{2g} + L_2 A_2 \gamma \frac{V_2^2}{2g} + \text{etc.} \dots \quad [18]$$

where A , A_1 , A_2 , are the pipe sectional areas for pipes L , L_1 , L_2 , etc. respectively; γ is the specific weight of water. But

$$AV = A_1 V_1 = A_2 V_2 = \text{rate of flow} \dots \dots \quad [19]$$

By combining Equations [18] and [19] herewith, Equation [5] of the paper is obtained.

Equation [6] expresses that the total time to travel the combined pipe length of pipe L with an effective velocity of sound a is equal to the sum of times to travel pipe length L_1 with a velocity a_1 , pipe length L_2 with a velocity a_2 , etc. Thus both Equations [5] and [6] are well grounded and have been found to give satisfactory results within the limits indicated in the paper.

The author is in agreement with Mr. Johnson in regard to the suitability of the revolving plug-type valve to control the rate of flow during the valve closure. However, in smaller sizes, these valves do not possess such refinements as streamlined plug opening, and incur losses greater than the common gate valves. In one large pipe line the plug-type valves are being replaced to reduce the hydraulic losses through the valves.

Mr. Parmakian's several examples of graphical solution of water-hammer problems, following power failure in an installation without a check valve at the pump, correspond to the author's case represented in Fig. 15 of the paper (apparently overlooked by Mr. Parmakian). A more simplified case of the same problem, based on an assumption that the pump will slow down, stop, and reverse in less than one interval of time, is dealt with in Fig. 11. There is nothing "improper" about making such an assumption to facilitate the presentation of the method itself. In practice any pump with sufficient discharge pipe length will stop and reverse in a less than one interval of time. Diagrams supplied by Mr. Parmakian (Figs. 25, 27, and 28) could be drawn without resorting to the "conjugate equations" and would look exactly the same. All three diagrams are drawn on dimensionless scales (ratios) introduced by Allievi in his analytical solution, which is an unnecessary complication for the graphical solution as pointed out by Bergeron in his discussion of reference (4) of the paper. Equation [17], used by Mr. Parmakian, is derived directly from the author's fundamental relationship as given in Equation [9a]. The contribution of the two charts of centrifugal-pump characteristics, extending beyond the normal range, is very welcome, as such data are still very scarce.

Mr. Strowger reports an interesting case of the effects of water

hammer on the behavior of boiler feed pumps with an unstable head-capacity characteristic. The author has suspected superimposition of water-hammer pressure waves on the pressure fluctuations induced by the pump but has had no opportunity to investigate it.

The author feels that he failed to impress upon Mr. Goldman the importance of a complete understanding of the events in a conduit in the case of a sudden and gradual valve closure. Without a clear visualization of the mechanism of pressure rise, pressure-wave propagation and reflection, it is possible to misinterpret the simple relationships and formulas for water hammer established a long time ago. The author went to much more detail on this matter than any of the previous authors.

Perhaps the author misled Mr. Goldman by arranging the terms of the equation reproduced by Mr. Goldman in Equation [11] the way it appeared in the preprints and later changed to

$$F = \frac{A\gamma}{g} \times \frac{x}{t} \times V_0 \quad . \quad .$$

Although, dimensionally, Equation [11] may be read (mass) $\times$ (acceleration), no such meaning was attached to it, the whole expression representing the "rate of change of momentum," a term having a quite definite meaning and physical quantity different from (mass) $\times$ (acceleration). True, in the mechanics of solid bodies the difference may be overlooked as mass is always constant, and change in momentum can be realized only by velocity change, or acceleration. Not so when a force is applied to a compressible body, like a water column in a pipe. There is no mechanism by which velocity changes can be impressed on the whole mass (or mass center) of a compressible body. In the case of a sudden valve closure, a constant rate of velocity change (from V_0 to 0) is steadily extended to an increasing mass along the pipe at the velocity of sound.

To arrange the terms of Equation [11] to read (mass) $\times$ (acceleration) leads to algebraic difficulties in the case of a sudden valve closure. Thus when $t = 0$, V_0/t becomes infinite, and mass becomes zero, and the right-hand side of the equation takes an indeterminate form $0 \times \infty$. Considering the same expression

as a rate of change of momentum, it is finite and in fact is constant.

The difference in application and interpretation of mechanical laws of solid bodies and compressible bodies, is more spectacular and instructive in the case presented in Fig. 19 of the paper. Here, after the valve is suddenly closed at A the column of water in the pipe continues to move against a constant force ($H_s + H_{atm}$) equal to absolute static pressure available at point C. Since this force is constant, one would expect that the deceleration should be constant, or velocity should be reduced gradually to zero, when all the kinetic energy of the column is converted into potential energy of position. Actually, deceleration and acceleration (velocity change) will occur in jumps ΔV each interval of time, remaining constant during each interval of time, but extending gradually to the whole mass of water column from one end to the other.

The author prefers the development of Equation [3] as it appears in Moody's article in the symposium (reference 7 of the paper) to that given in Schoder and Dawson's "Hydraulics" quoted by Mr. Goldman.

Mr. Goldman's algebra is difficult to follow due to confusion of terms and symbols. Thus he introduces a new term t_1 (time) which must be different from author's time t ; a little further, a term L differently defined twice in two lines that immediately follow Equation [16], and different from the author's notation in Fig. 1. As a result, Mr. Goldman obtained a picture of pressure distribution along the pipe different from that shown in Figs. 1 and 2 for a sudden valve closure, and Figs. 3 and 6 for a gradual valve closure.

Very frequently, in the field of centrifugal-pump application, no exact determination of the maximum pressure rise is required. Mere recognition of the existence of conditions leading to water hammer is sufficient to arrive at a procedure of starting or stopping a pump, or manipulating the discharge valve to avoid objectionable pressure rise. However, an understanding of water-hammer theory is a prerequisite for any consideration of water hammer. The author hopes that this paper, together with the discussion, will be of help to those not yet initiated in the subject.

Involute-Gear Geometry

Nondimensional Analysis and Design of Tooth Forms, Tooth Thickness, and Mesh Relationships

By C. E. GROSSER,¹ SYRACUSE, N. Y.

Definition and discussion of recently discovered fundamental gear characteristic called "belt-length ratio," to simplify gear design and generation computations, are given with necessary measurement procedures, formulas, and a graphical calculating chart. The concepts and methods are proposed as alternatives to use of conventional involute-function tables and formulas. Relationships are given in nondimensional (ratio) form so that a minimum of data are applicable to all gear sizes. A new standard for pin measurements of gears and involute splines is suggested.

INTRODUCTION

THE primary purpose of this paper is to introduce and explain the use of the graphical "belt-length-ratio" chart which is appended, and which is intended to simplify the computations necessary for the analysis of involute gears and for design for optimum performance.

It is presumed that the procedures and means here to be presented may supplant the conventional involute-gear tables and the established computation methods to a great extent. Primarily they are intended to afford short-cuts in gear calculations without loss of accuracy; and to present a more simple conception of mesh conditions, reducing the need for checking designs by accurate drawings of the gears themselves. It is recognized of course that large-scale drawings should be regarded as the most reliable means for checking a gear design and for analyzing conditions in a gear mesh whenever practicable. However, circumstances are often such that facilities for large-scale graphical analyses are not conveniently at hand.

It is expected that the devices here offered in the form of non-dimensional or "ratio" relationships will serve the purpose of helping to visualize better the general scope of gear-mesh variations. The data are presented in nondimensional form, so that complications on account of size are eliminated, and the results of the analyses have the most general application. Gear problems are attacked by reducing all dimensions to ratio numbers, entering them into the charts and formulas. The results then come out as pure numbers which are multiplied by the same size factors with which the original dimensions were reduced.

Included are the derivations of procedures for determining such individual gear properties as: (1) the characteristic number of teeth in contact N_c , which a given gear brings into the mesh with any other mating gear; (2) a belt-length ratio (gamma ratio) which describes tooth thickness, and which in combination with the corresponding gamma ratio of any other meshing gear, readily establishes the close-mesh center distance (or deduces the

backlash of the pair on any specified operating centers); and (3) a simplified system of pitch-diameter pin measurement for establishment and inspection of gear size.

Opportunities for improvement of involute-gear action over that usually obtained within the limits of the conventional "standard" tooth proportions have been more or less generally recognized, and the benefits of such refinements are now being realized to an increasing extent in commercial practice; particularly in closely limited designs and those for large production where special effort is justified to obtain optimum performance at minimum cost. This is especially true in the automotive industry. Potential benefits available from careful study of gear action are: (a) greater carrying capacity for a given center distance; (b) smoother action through greater overlap in the mesh; (c) higher ratios in coarse pitches with well-designed pinions of small tooth numbers; and (d) improvements in efficiency.

However, when departures are made from the proportions of the conventional "uniform circular pitch" systems (which have been developed as standards for stock gears and which satisfactorily meet the great majority of ordinary simple gearing problems), many problems present themselves. The standard "stock" systems are restricted by the limitation that each gear of the system must mesh with every other gear in the system. Such a restriction necessitates sacrifice of performance mostly on account of "undercut" in low tooth numbers.

General gear-design problems are involved usually with the questions: (1) what are the significant factors which govern the tooth proportions and what is the effect of each on the over-all performance of the gear drive; (2) can the gears be cut by conventional methods and with available tools; (3) is the action free of interference and fouling; (4) is there danger of the teeth becoming pointed; (5) what will be the backlash on the required center distance; and (6) how can the strengths of the teeth on pinion and gear be equalized?

The answers to these and other questions can come readily from a thorough theoretical understanding of the properties of the involute, combined with wide experience in the laying-out of gear drives which have met a variety of operating and manufacturing conditions. Lacking the experience, a designer may equip himself with fundamental knowledge of the principles and limitations of the available gear-cutting methods and tools, and gain a clear picture of the mechanism of gear action, primarily involving the geometrical relationships of involute teeth with different proportions of thickness, height, and position on the involute. Unfortunately, the tediousness and complications of ordinary trigonometric and involute-function methods tend to discourage the student of gearing from extended analytical exploration of involute properties; and the usual lack of readily available equipment for easily drawing the involute curves to large scale is a great handicap to graphical studies.

CROSSED-BELT ANALOGY AS A CONVENIENT MEANS OF ESTABLISHING TOOTH THICKNESS

The analogy between the mechanism of involute-gear action and that of two pulleys joined by a crossed belt is well known and

¹ Professor of Mechanical Engineering, College of Applied Science, Syracuse University. Mem. ASME.

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almost always used in teaching involute-gear theory, and for forming the mental picture of the gear mesh. The crossed-belt conception has been here analyzed in detail. From it has been derived a general tooth-thickness characteristic to be called the "gamma ratio," and which has shown to be an easy means for determining close-mesh center distance of any pair of mating involute gears (necessarily with the same base circular pitch) which will run together. It has also been found that by using the gamma ratio it is a simple matter to determine backlash accurately for any pair of gears on any given centers (usually a troublesome computation when dealing with gears of unequal addenda or for modified annular internal-tooth gears), and for determining cutting depths in gear generation.

Fig. 1(a) shows a 10-tooth pinion in a tight mesh with a 30-tooth gear, the angular relationship being such that a pinion tooth is placed squarely into a space of the gear. In Fig. 1(b) the same gears are shown rotated through half a pitch so that a tooth of the gear now stands in the center of the mesh centrally in a space of the pinion. In Fig. 1(c) is seen a pair of equivalent pulleys having the same diameters as the base circles of the gears, placed at the same center distance, and with a crossed-belt stretched tightly between them. It should be obvious that the gear and pulley systems are kinematically equivalent to each other. The condition of tight mesh, obtained by pressing the gears hard against each other, is similar to tightening the belt in the pulley system by drawing the pulleys apart.

At (a) may be noted the heavy band L_t , wrapped around the base circle of the pinion and extended to end at the contacting surfaces of that tooth which is in central engagement with the mating gear. The heavy band L_t may be regarded as the generating line, the ends of which trace the two involutes composing the opposite flanks of the pinion tooth. This line L_t is observed to consist in that portion of the total "crossed-belt" which the pinion brings to the mesh when a tooth is engaged on both sides.

At (b), a similar heavy band or belt L_s , is shown terminating at the sides of a pinion space, i.e., that space which contains a tooth of the mating gear; and in this situation the heavy band L_s is seen to be that portion of the total crossed-belt which the pinion brings to the mesh when its space is engaged. The difference in length between the two different position belt lines L_t and L_s is clearly equal to one base pitch P_b (sometimes called "normal circular pitch"). Therefore the portion of "belt" which any gear brings to a tight mesh fluctuates between L_t and L_s ; and in view of the fact that when one of a pair of meshing gears contributes its longer length L_t , the other gear inevitably and simultaneously contributes its correspondingly shorter L_s (and vice versa), we may say that the mean (average) length of belt which any gear contributes to a tight mesh is equal to

$$L_m = L_s + 1/2 P_b \dots \dots \dots [1]$$

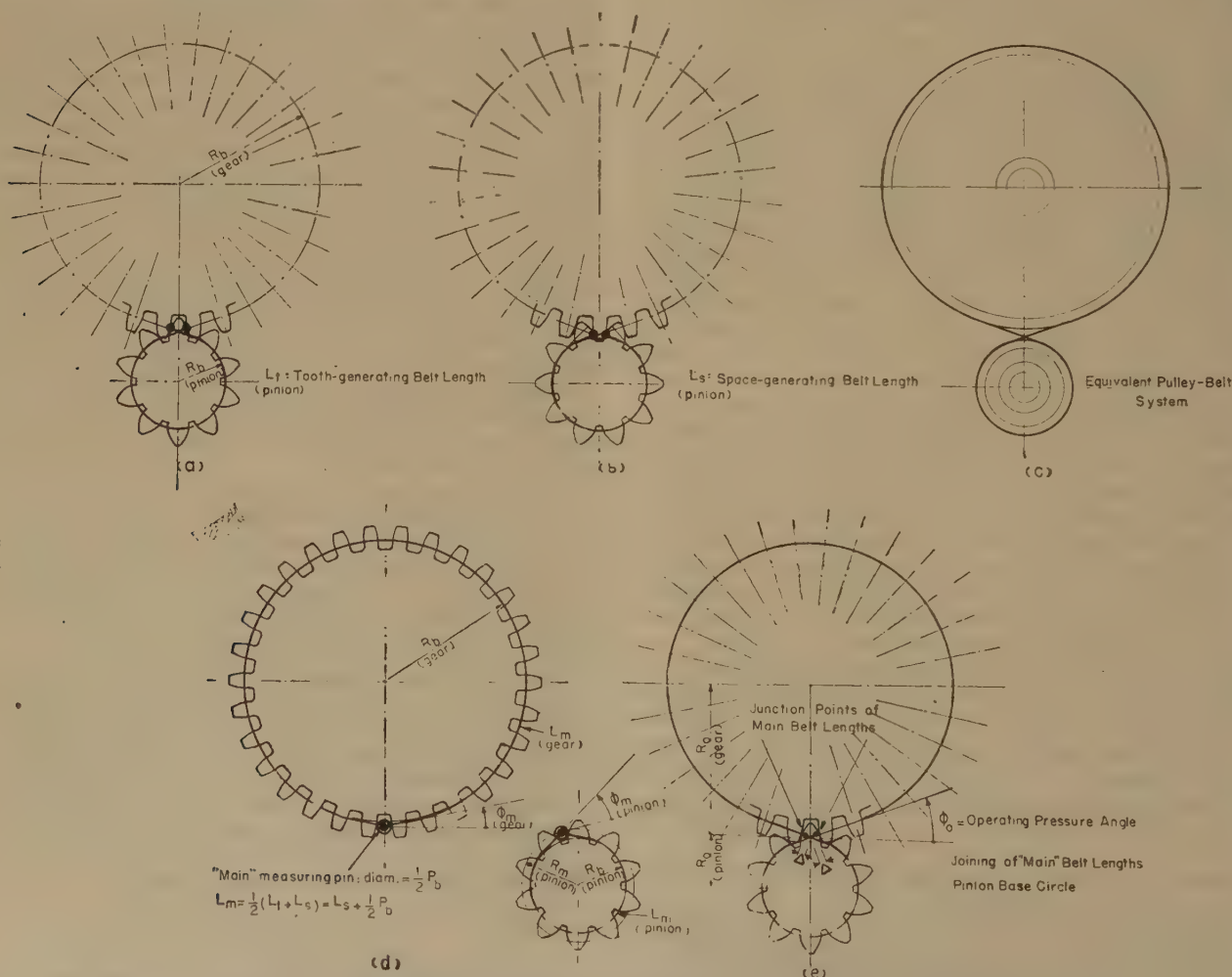


FIG. 1 CROSSED-BELT AND PULLEY ANALOGUE OF INVOLUTE-GEAR ACTION
(Demonstrating that each gear brings an invariant main belt length to a tight mesh with any other mating gear.)

Generally, $K = R/R_b = \sec \phi$, for any pitch radius.

In the Appendix is given a graphic chart which shows the relationships among pressure angle ϕ , $K = \sec \phi$, and $\gamma =$ belt length ratio for pressure angles of from 0 to 60 deg. An additional scale of $\log K$ has been added to assist in accurate computations. The chart is in graphic form to avoid the necessity for interpolation. The accuracy of the chart is such as to yield results precise to at least one part in ten thousand. It may be used easily to compute many general gear-tooth and mesh characteristics as will be shown.

GEAR-SIZE MEASUREMENT

The remarkable significance of the position taken by the main measuring pin (which has a diameter equal to one half the base pitch) is demonstrated in Figs. 3 and 4 which show several gears having different tooth thicknesses and different numbers of teeth. In each case, it is demonstrated that the "main measuring-pin center falls precisely on a pitch circle at which the tooth thickness exactly equals the tooth-space width,"

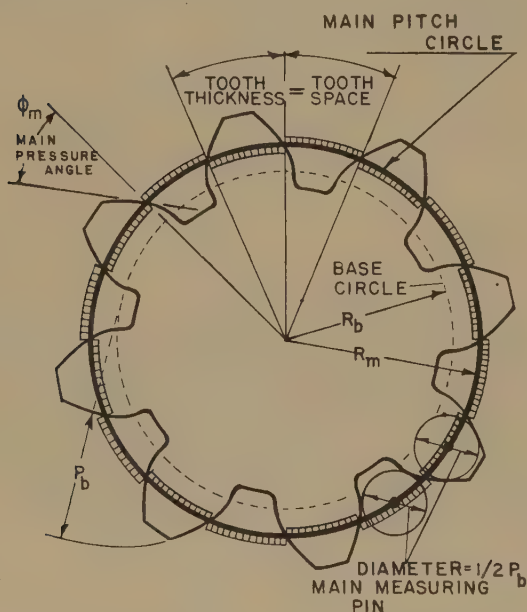


FIG. 3 DEFINITION OF MAIN PITCH CIRCLE

(For any gear there is one pitch circle upon which tooth and space are equal. Its position depends upon the center distance at which the gear is generated.)

(constituting Theorem [7], and this relationship holds for internal (annular) gears as well as for external spur gears. The proof of this is implied by Earle Buckingham,² and is given in detail by Merritt,³ although the fact has not received particular attention.

An equivalent proof may be made by referring to Fig. 2 (c) where it is seen that one end of the belt line terminated at the center of the main measuring pin has a portion of its length intercepted between the auxiliary involute z_1 , (dotted, passing through the roll center) and the adjacent tooth surface z_2 , toward the right. The intercepted belt-length interval is equal to the main pin radius, of value one quarter of the base pitch ($1/4 P_b$).

If the gear is rotated clockwise so that the auxiliary involute (dotted) coincides with the adjacent tooth surface z_2 , the necessary rotation of the gear must be one quarter of a tooth angle, since this is the amount of belt taken up by rolling onto the base circle. If the gear has turned through one quarter of a tooth angle, then the auxiliary involute z_1 , must have moved through one quarter of a circular pitch on the main pitch circle to coincide with position z_2 . Therefore the whole gear-space width must be twice as great as this (one half a circular pitch on this main pitch circle). It is obvious that the tooth thickness must also be one half a circular pitch on the main pitch circle R_m . Since this proof contains no restrictions on the number of teeth or on the proportions of the involute, the Theorem [7] is completely general.

The knowledge that the main measuring roll ($1/2 P_b$), always takes its position on that circle, where tooth and space are equal, is useful as an inspection device inasmuch as it easily establishes the position of the main pitch circle, that diameter on which is terminated the main belt length.⁴ The designations of main pitch diameter and main pressure angle for the pitch circle, having equal tooth and space widths, have precedent in their use by Vogel.^{5,6,7}

Fig. 6 reveals an important relationship between different sizes of measuring pins and the positions which they assume with

² "Spur Gears," by Earle Buckingham, McGraw-Hill Book Company, Inc., New York, N. Y., 1948, Eq. 53, p. 77.

³ "Gears," by H. E. Merritt, Pitman Publishing Company, New York, N. Y., 1942, p. 145.

⁴ A handicap encountered in the use of this particular main measuring pin (diam = $1/2 P_b$) is that for gears where the main pitch circle is to be located on full-depth-addendum teeth, the outer surface of the roll falls within the tips of the teeth. However, this disadvantage may be overcome by the use of gear-measuring micrometers having partially pointed anvils as shown in Fig. 5.

⁵ "Involutometry and Trigonometry," by W. F. Vogel, Michigan Tool Company, 1947, p. 260. For footnote 6s and 7 see p. 543.

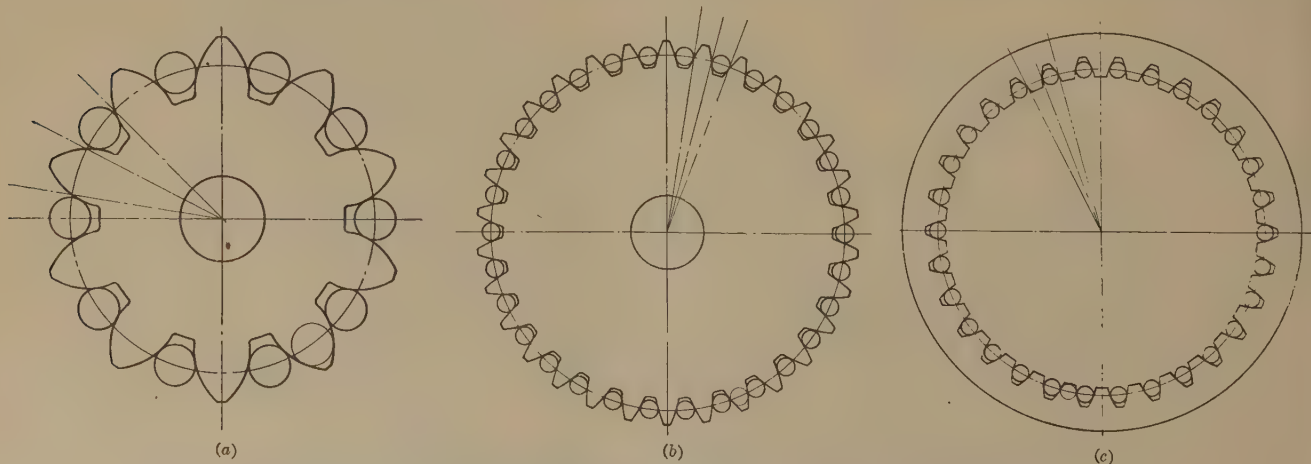


FIG. 4 EXAMPLES OF MAIN MEASURING-PIN LOCATION ON MAIN PITCH DIAMETER

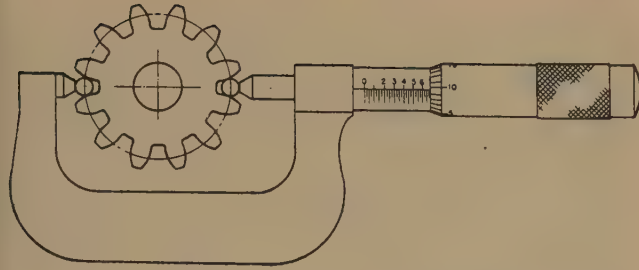


FIG. 5 SPECIAL GEAR-MEASURING MICROMETER WITH CONICAL ANVILS TO PERMIT USE OF MAIN MEASURING PIN

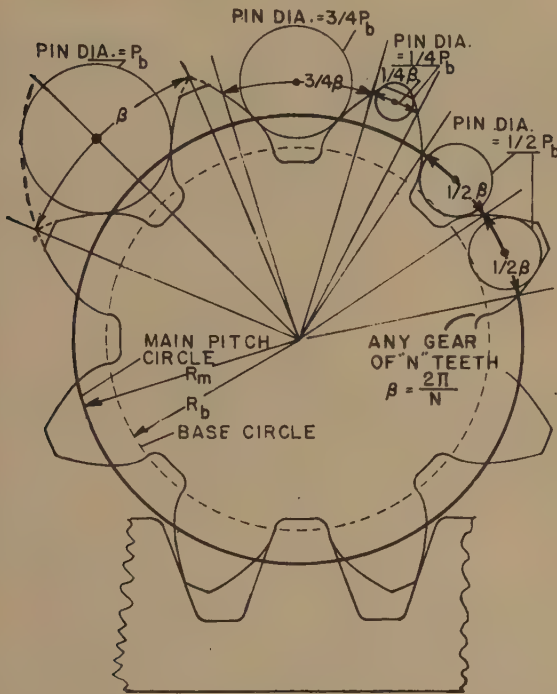


FIG. 6 DEMONSTRATION OF THEOREM THAT ANY MEASURING PIN CENTERS ON PITCH CIRCLE WHERE TOOTH SPACE IS SAME FRACTION OF TOOTH ANGLE AS PIN DIAMETER IS OF BASE PITCH

$$\left(\frac{\text{Pin diameter}}{\text{Base pitch}} = \frac{\text{Space [at pin center]}}{\text{Tooth angle}} \text{ where, tooth angle} = \frac{2\pi}{N} \right)$$

respect to the gear teeth. It may be noted that a measuring pin, having a diameter equal to a full base pitch P_b , rests with its center on the circle where the gear teeth would become pointed if sufficiently extended. Also, the center of a pin of diameter equal to $3/4 P_b$ stands on a circle on which the space occupies $3/4$ tooth angle ($3/4 \beta_t$). By a proof similar to that immediately following Theorem [7] it may be established that the pins of the same proportions will take the same relative positions on any involute gear (of corresponding base pitch); and the general theorem may be stated:

"A measuring pin will take the position such that its center falls on a pitch circle where the space angle is the same proportion of the tooth angle as the pin diameter is of the base pitch" (Theorem [8]).

Theorem [8] is useful in such cases where a measuring pin of the desired diameter is not available; in which case it is possible to translate its position in terms of the presently recommended main or even of conventional measuring pins, by the help of the gamma-ratio chart. Theorem [8] is also useful for drawing involute-gear teeth accurately. Both methods will be discussed in more detail near the end of the paper.

DETERMINATION OF TIGHT-MESH CENTER DISTANCE OF TWO GEARS BY USE OF GAMMA-RATIO CHART

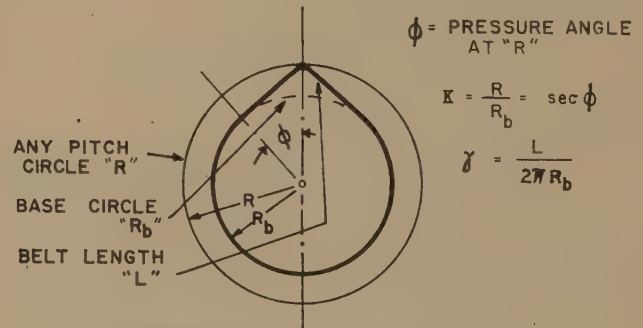
Referring to Fig. 1 (e), it is noted that the gears shown at (a), (b), and (d) have been brought back together in mesh, and their main belt lengths L_m (pinion) and L_m (gear) (shown separately at (d)), have been joined together again at the joints xx . The junction points xx are displaced from the operating pitch point p_0 by the intervals Δ . That portion of the pinion main belt length which extends from the operating pitch point p_0 around the pinion base circle and back to p_0 is shorter than L_m (pinion) by 2Δ ; and similarly the augmented belt length from p_0 around the base circle of the large gear is longer than L_m (gear) by 2Δ . The ratio γ_m (pinion) which is equal to

$$\frac{L_m}{N_p P_b} \quad \text{(pinion)}$$

in this case is greater than

$$\gamma_m \text{ (gear)} = \frac{L_m}{N_g P_b} \quad \text{(gear)}$$

This is clear from an inspection of Fig. 1(d) where it is evident that ϕ_m (pinion) is greater than ϕ_m (gear), and study of Fig. 7



FORMULAE:

R_m = MAIN PITCH RADIUS, WHERE TOOTH=SPACE

$$\gamma \text{ (MESH)} = \frac{N_1 \gamma_{m1} + N_2 \gamma_{m2}}{N_1 + N_2}$$

$$\gamma_m \text{ (CORRECTED)} = \gamma_m \text{ (UNCORRECTED)} + \frac{2 C \tan \phi_{HOB}}{\pi N}$$

$$N_c = \frac{N_{\text{GEAR}}}{2} \left[\gamma_c - \gamma_0 + \frac{\phi_c^\circ - \phi_0^\circ}{180^\circ} \right]$$

$$\gamma_p = \gamma_m + \frac{1}{2N}$$

(WHERE FORMULA IS APPLIED TO INTERNAL GEAR TEETH "N" IS NEGATIVE)

FIG. 7 MEANING OF GAMMA-RATIO-CHART SYMBOLS AND SUMMARY OF GEAR FORMULAS

(the latter of which shows the meaning of the gamma ratio, as the length of a belt stretched around a base circle and to a point on any pitch circle, divided by the circumference of the base circle). It is evident that the larger the pitch circle and therefore the greater the pressure angle, the greater the gamma ratio.

Returning to the particular analysis of Fig. 1 (e), the gamma ratio for the mesh

$$\gamma_0 \text{ (pinion-gear mesh)} = \frac{L_m \text{ (pinion)} - 2\Delta}{N_p P_b \text{ (pinion)}} = \frac{L_m \text{ (gear)} + 2\Delta}{N_g P_b \text{ (gear)}}$$

or

$$\gamma_0 = \gamma_m - \frac{2\Delta}{N_p P_b} = \gamma_m + \frac{2\Delta}{N_g P_b} \quad [9]$$

This value of the gamma ratio γ_0 is obviously intermediate between γ_m (pinion) and γ_m (gear), and so we may conclude that generally two gears which are cut so as to have main pitch circles at different pressure angles on their involutes will mesh tightly at an "operating-mesh pressure angle which is intermediate between their main pressure angles." The intermediate value of pressure angle may be determined as follows:

From Equation [9]

$$\gamma_0 = \gamma_m - \frac{2\Delta}{N_p P_b}$$

and by multiplying both sides by N_p

$$N_p \gamma_0 = N_p \gamma_m - \frac{2\Delta}{P_b} \quad [10]$$

Similarly

$$N_g \gamma_0 = N_g \gamma_m + \frac{2\Delta}{P_b} \quad [11]$$

By adding Equations [9] and [10]

$$(N_p + N_g) \gamma_0 = N_p \gamma_m + N_g \gamma_m$$

from which

$$\gamma_0 = \frac{N_p \gamma_m + N_g \gamma_m}{N_p + N_g} \quad [12]$$

where

N_p = number of teeth in pinion

N_g = number of teeth in gear

From the result, Equation [12], it is possible by reference to the gamma-ratio chart, to determine the corresponding K_0 (from which the center distance may be obtained since

$$K_0 = \frac{R_b(\text{pinion})}{R_b(\text{pinion})} = \frac{R_b(\text{gear})}{R_b(\text{gear})} = \frac{R_b(\text{pinion}) + R_b(\text{gear})}{R_b(\text{pinion}) + R_b(\text{gear})} = \frac{\text{center distance}}{R_b(\text{pinion}) + R_b(\text{gear})} \quad [13]$$

Therefore the knowledge of the main pitch-circle diameters, combined with the base-circle diameters, enables one to predict the tight-mesh center distance by the use of the gamma-ratio chart.

CALCULATION OF TYPICAL GEAR-MESH CENTER DISTANCE, USING GAMMA CHART

As an example of the procedure, let us take some values which may apply to Fig. 1. We shall say that the pinion and gear have been cut with a standard Fellows 10 DP 20-deg stub cutter in an enlarged blank, so that the specifications are as follows:

Pinion: Cut 10 teeth in an 11-tooth blank with a 10/12 DP 20-deg-pressure-angle cutter.

Gear: Cut 30 teeth in a 29-tooth blank with a 10/12 DP 20-deg-pressure-angle cutter.

This is common procedure to avoid undercut in the pinion and still operate on standard-center distance. However, if the gear cutting is done to a standard depth with a circular cutter it has been found by experience that they will not mesh tightly exactly at the standard center distance of $(10 + 30)/(2) (10) = 2.000$ in. It is assumed that a 4-in-diam 40-tooth Fellows shaper cutter is used; its main pitch radius then will be $R_m(\text{cutter}) = 2.000$ in. since these cutters are made with tooth and space equal on the nominal cutter pitch diameter.

At the standard cutting depth, the center distance for cutting the pinion will be

$$\text{center distance (cutter + pinion)} = \left(\frac{1}{2}\right) \frac{40^T + 11^T}{10 \text{ DP}} = 2.550 \text{ in.}$$

(the 11^T figure is used since we are cutting in the enlarged blank).

The kappa ratio for this cutter-pinion operation is obtained by dividing the center distance by the sum of the base radii of pinion and cutter from Equation [13]

$$K_0 = \frac{\text{center distance (cutter + pinion)}}{R_b(\text{cutter}) + R_b(\text{pinion})} = \frac{2.550}{\frac{1}{2} \left(\frac{40}{10} + \frac{10}{10} \right) \cos 20 \text{ deg}} = 1.0855$$

and referring to the gamma-ratio chart (see Appendix, chart 1), the corresponding value to K_0 is

$$\gamma_0(\text{cutter-pinion}) = 1.00723$$

also

$$\phi_0(\text{cutter-pinion}) \text{ may be read as } 22.9 \text{ deg}$$

Now reading again on Chart 1

$$\gamma_m(\text{cutter}) = 1.00475 \text{ since } \phi_m(\text{cutter}) = 20 \text{ deg exactly}$$

Applying Equation [12]

$$\gamma_0(\text{cutter-pinion}) = \frac{N_c \gamma_m(\text{cutter}) + N_p \gamma_m(\text{pinion})}{N_c + N_p}$$

or

$$1.00723 = \frac{(40)(1.00475) + (10) \gamma_m(\text{pinion})}{40 + 10}$$

from which is solved

$$\gamma_m(\text{pinion}) = \frac{(40 + 10)(1.00723) - (40)(1.00475)}{(10)} = 1.01715$$

Similarly for the gear

$$K_0 = \frac{\text{center distance (cutter and gear)}}{R_b(\text{cutter}) + R_b(\text{gear})} = \frac{\left(\frac{1}{2}\right) \left[\frac{40^T + 29^T}{10 \text{ DP}} \right]}{\left(\frac{1}{2}\right) \left[\frac{40^T + 30^T}{10 \text{ DP}} \right] \cos 20 \text{ deg}} = 1.0490$$

The corresponding values from the gamma-ratio chart 1, are

$$\gamma_0 \text{ (cutter-gear)} = 1.00320, \text{ and } \phi_0 \text{ (cutter-gear)} = 17.58 \text{ deg}$$

Solving by Equation [12] transposed

$$\gamma_0 \text{ (cutter-gear)} = \frac{N_o \gamma_m \text{ (cutter)} + N_g \gamma_m \text{ (gear)}}{N_o + N_g}$$

$$1.00320 = \frac{(40)(1.00475) + (30) \gamma_m \text{ (gear)}}{40 + 30}$$

From which

$$\gamma_m \text{ (gear)} = \frac{(40 + 30)(1.00320) - (40)(1.00475)}{(30)} = 1.00080$$

We now have the main belt-length ratios of the pinion and gear as they would come off the Fellows gear shaper (and presumably we have checked these values by actual measurement of the positions of the main measuring pins). Taking from the foregoing

$$\gamma_m \text{ (pinion)} = 1.01715$$

and

$$\gamma_m \text{ (gear)} = 1.00080$$

Equation [12]

$$\gamma_0 \text{ (pinion-gear)} = \frac{(30)(1.00080) + (10)(1.01715)}{30 + 10} = 1.00489$$

And from the gamma chart

$$K_0 \text{ (pinion-gear)} = 1.0655$$

$$\text{Tight-mesh center distance (pinion-gear)} = (1.0655) \left(\frac{1}{2} \right) \left(\frac{30 + 10}{10} \right) (0.940) \times \cos 20 \text{ deg} = 2.003 \text{ in.} \dots [14]$$

Equation [14] checks the common experience that when "corrected" gears are generated by circular cutters, additional depth of cut must be allowed over that usually provided for backlash, if the expected mesh conditions are to be obtained. In this case either pinion or gear should be shaped an additional 0.003 in. deeper before allowance for backlash is made.

On the other hand, if the same gears were hobbled with a standard hob or even if cut with a different-size shaper cutter from that

assumed in this case, the gears would mesh tightly on still different centers.

For example, if the gears are hobbled or rack-shaped to the same specifications, in cutting the 10/11^T pinion, the hob or rack would be withdrawn by 1/2 addendum from the usual position for a 10-tooth pinion. This is shown in Fig. 8.

The withdrawal of the hob (or cutting rack) by (1/2)/DP = 1/2 addendum causes the main belt length of the pinion to be increased by

$$2\delta = (2) (1/2 \text{ addendum}) \sin 20 = \frac{\sin 20}{10 \text{ DP}}$$

over that possessed by a standard (unenlarged) 10-tooth pinion.

Generally the enlargement of pinions is expressed as a correction factor which represents the withdrawal of the generating cutter (as well as the radial enlargement of the blank) in terms of a full addendum (1/DP). If, as in the case under discussion, the hob is withdrawn by 1/2 addendum, the correction factor C is 0.5.

When "enlarged" pinions are hobbled, the main gamma ratio becomes

$$\gamma_m \text{ (corrected)} = \gamma_m \text{ (uncorrected)} + \frac{(2\delta)}{(\text{base circle circum})}$$

$$= \gamma_m \text{ (uncorrected)} + \frac{(2) \frac{C}{\text{DP}} \sin \phi_{\text{hob}}}{\pi \frac{N}{\text{DP}} \cos \phi_{\text{hob}}}$$

$$= \gamma_m \text{ (uncorrected)} + \frac{2C \tan \phi_{\text{hob}}}{\pi N} \dots [15]$$

where N is the number of teeth in the gear being cut; γ_m (uncorrected) is 1.00475 for 20-deg hobs (and 1.00178 for 14 1/2-deg hobs); as may be read from the gamma-ratio chart, for these pressure angles, in the Appendix.

Applying Equation [15] to the 10/11^T pinion and 30/29^T gear to be hobbled

$$\gamma_m \text{ (pinion)} = 1.00475 + \frac{0.3640 (2) (+1/2) (\tan 20 \text{ deg})}{(\pi) (10)} = 1.01633$$

$$\gamma_m \text{ (gear)} = 1.00475 - \frac{(2) (+1/2) \tan 20}{(\pi) (30)} = 1.00089$$

These values of main belt-length ratios are appreciably different from those obtained previously by shaping with the 4-in-diam cutter.

The tight-mesh center distance of these gears may now be obtained

$$\gamma_0 \text{ (pinion-gear)} = \frac{(10)(1.01633) + (30)(1.00089)}{10 + 30} = 1.00474$$

This value of γ_0 corresponds on the gamma chart to

$$\phi_0 \text{ (pinion-gear)} = 20 \text{ deg}$$

exactly, indicating that when hobbled, the corrected gears mesh tightly on standard centers, that is, if pinion and gear are corrected oppositely to the same degree, as in this case.

It has been found that a correction factor of $C = 0.5$ is a good general figure to use for elimination of undercut in low-tooth-number pinions down to 8 teeth. Below 8 teeth full-depth teeth may become pointed.

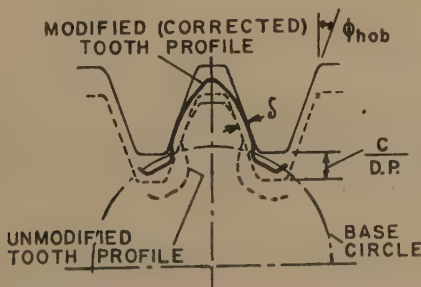


FIG. 8 ILLUSTRATING INCREASE IN MAIN BELT LENGTH AS GENERATING HOB IS WITHDRAWN FOR CORRECTION OF TOOTH PROFILE

$$(2\delta = \frac{2C}{\text{DP}} \cdot \sin \phi_{\text{hob}})$$

ESTABLISHMENT OF CONTACT RATIO BY USE OF GAMMA-RATIO CHART

It is possible to find the proportion of contact ratio which a gear brings to a mesh by the following method:

Referring to Fig. 9, the amount of belt length intercepted be-

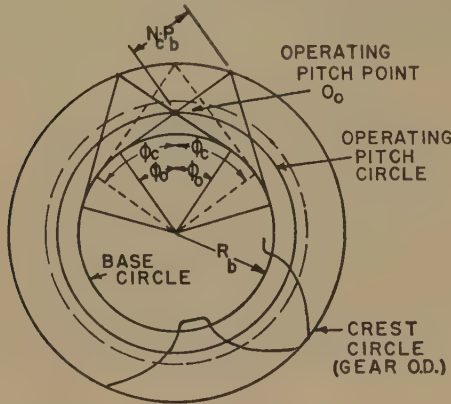


FIG. 9 CONTACT RATIO N_e CONTRIBUTED BY A GEAR OF GIVEN OUTSIDE RADIUS R_c , AND OPERATING AT A GIVEN PRESSURE ANGLE ϕ_o

$$(N_e P_b = 1/2 N P_b [\gamma_c - \gamma_o + \frac{\phi_c - \phi_o}{180}])$$

tween the operating pitch point O_o and the outside circumference of the gear is seen to be (in terms of p_b , the base pitch)

$$N_e P_b = (1/2) \left[\frac{N}{(\text{gear})} P_b \gamma_c + 2(\phi_c - \phi_o) R_b - N P_b \gamma_o \right]$$

$$= 1/2 N P_b \left[\gamma_c - \gamma_o + \frac{2(\phi_c - \phi_o) R_b}{2\pi R_b} \right]$$

or

$$N_e = \frac{N}{2} \left[\gamma_c - \gamma_o + \frac{\phi_c - \phi_o}{180} \right] \dots \dots [16]$$

where

ϕ_c = pressure angle at tooth crest, deg

ϕ_o = pressure angle at operating pitch point, deg

γ_c = gamma ratio corresponding to radius at tooth crests

N_e = number of teeth in contact between tooth tip and operating pitch point

N = number of teeth in gear

As an example of this calculation, take the contact ratio of the gear pair in Fig. 1, in tight mesh at $\phi_o = 20$ deg. For the 10/11^T pinion, the crest radius is, with 10/12 pitch stub teeth

$$\frac{R_c}{(\text{pinion})} = (1/2) \left(\frac{11^T}{10 \text{ DP}} \right) + \frac{1}{12} = 0.6333 \text{ in.}$$

$$\frac{K_c}{(\text{pinion})} = \frac{R_c}{R_b} = \frac{0.6333}{(1/2) \left(\frac{10^T}{10 \text{ DP}} \cos 20 \right)} = \frac{0.6333}{0.4700} = 1.3475$$

Referring to gamma chart 4 in the appendix

$$\gamma_c = 1.05368 \text{ and } \phi_c = 42.08 \text{ deg}$$

$$(\gamma_o = 1.00475 \text{ for } \phi_o = 20 \text{ deg})$$

Now applying Equation [16]

$$\frac{N_e}{(\text{pinion})} = \frac{(10)}{2} \left[1.05368 - 1.00475 + \frac{42.08 - 20.00}{180} \right] = 0.86^T$$

Similarly for the gear

$$\frac{R_c}{(\text{gear})} = (1/2) \left(\frac{29^T}{10 \text{ DP}} \right) + \frac{1}{12} = 1.4500 + 0.0833 = 1.5333 \text{ in.}$$

$$\frac{K_c}{(\text{gear})} = \frac{R_c}{R_b} = \frac{1.5333}{(1/2) \left(\frac{30^T}{10 \text{ DP}} \cos 20 \right)} = \frac{1.5333}{1.4096} = 1.0877$$

which corresponds on gamma chart 1, to

$$\gamma_c = 1.00751 \text{ and } \phi_c = 23.16 \text{ deg}$$

$$(\gamma_o = 1.00475 \text{ for } \phi_o = 20 \text{ deg})$$

The number of teeth brought to the contact by the gear at 20 deg pressure angle operation is then, by Equation [16]

$$\frac{N_e}{(\text{gear})} = \frac{(30)}{2} \left[1.00751 - 1.00475 + \frac{23.16 - 20.00}{180} \right] = 0.30^T$$

The contact ratio for the mesh is

$$\frac{N_e}{(\text{pinion})} + \frac{N_e}{(\text{gear})} = 0.86^T + 0.30^T = 1.16^T$$

which is rather low, indicating that full-depth teeth would be preferable for such a mesh.

DETERMINATION OF TOOTH POINTING

When corrected pinions are designed it is desirable to check that the teeth are not pointed, and how much land width exists on the crests of the teeth. This is a relatively simple procedure when Theorem [8] is used, which states that a measuring roll of diameter equal to a base pitch will stand in a tooth space with its center on the circle where the teeth would become pointed. Referring to Fig. 10, it is seen that the belt length L_p to the center of this large measuring pin must be longer than the main belt length L_m by $1/2 P_b$.

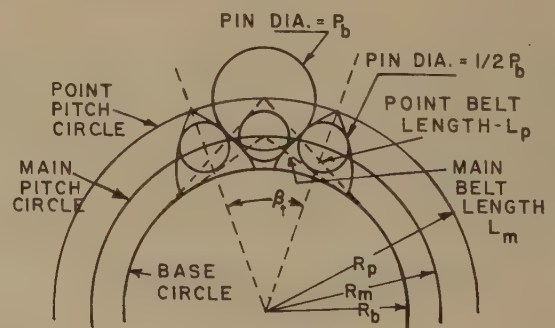


FIG. 10 DEMONSTRATING THAT THE BELT LENGTH TO TOOTH POINTS IS HALF A BASE PITCH LONGER THAN MAIN BELT LENGTH (This figure also shows how the tooth profiles may be constructed with knowledge of locations of the main pitch circle and the point pitch circle. The involute is determined by four points of location and four slope directions.)

The gamma ratio for the point location is then

$$\gamma_p = \gamma_m + \frac{1/2 P_b}{2\pi R_b} = \gamma_m + \frac{1/2 P_b}{N P_b}$$

since

$$2\pi R_b = N P_b$$

or

$$\gamma_p = \gamma_m + \frac{1}{2N} \dots \dots \dots [17]$$

Therefore if the main belt-length ratio γ_m for the gear is known (from knowledge of how it is cut), the value of K_p may be found in the gamma chart, corresponding to γ_p determined by Equation [17]. This is an extremely easy procedure for finding the point pitch radius of off-standard gears when compared with the usual method involving direct use of the involute function.

As an example of the use of Equation [17], the position of the points of the 10^T pinion in Fig. 1 are found as follows:

The hobbled $10/11^T$ pinion had a main gamma ratio

$$\frac{\gamma_m}{(\text{pinion})} = 1.01633$$

and from Equation [17]

$$\frac{\gamma_p}{(\text{pinion})} = 1.01633 + \frac{1}{(2) (10^T)} = 1.01633 + 0.05000 = 1.0663$$

From the chart 4 in the Appendix

$$\frac{K_p}{(\text{pinion})} = 1.04053$$

and the point-pitch radius

$$\frac{R_p}{(\text{pinion})} = (K_p)(R_b) = (1.04053) (1/2) \left(\frac{10}{10} \right)^{0.940} (\cos 20 \text{ deg}) = 0.660$$

in. which is well beyond the crest radius of 0.633 in. previously determined. Actually, full-depth teeth could be used safely without pointing.

Examination of Fig. 10 will reveal how by locating the point-pitch radius, the main pitch radius, and the base radius of a gear, the involute curves may be readily drawn tangent to the measuring pins there shown.

ACKNOWLEDGMENTS

Appreciation is hereby acknowledged of the value of discussions with Prof. Earle Buckingham on the general subject of gear geometry. To Mr. Azad Baloyan goes grateful credit for his interest, painstaking care, and great labor in the preparation of the graphical "gamma charts." The author wishes to acknowledge the invaluable and considerable assistance of his wife in preparing this paper for publication.

Appendix

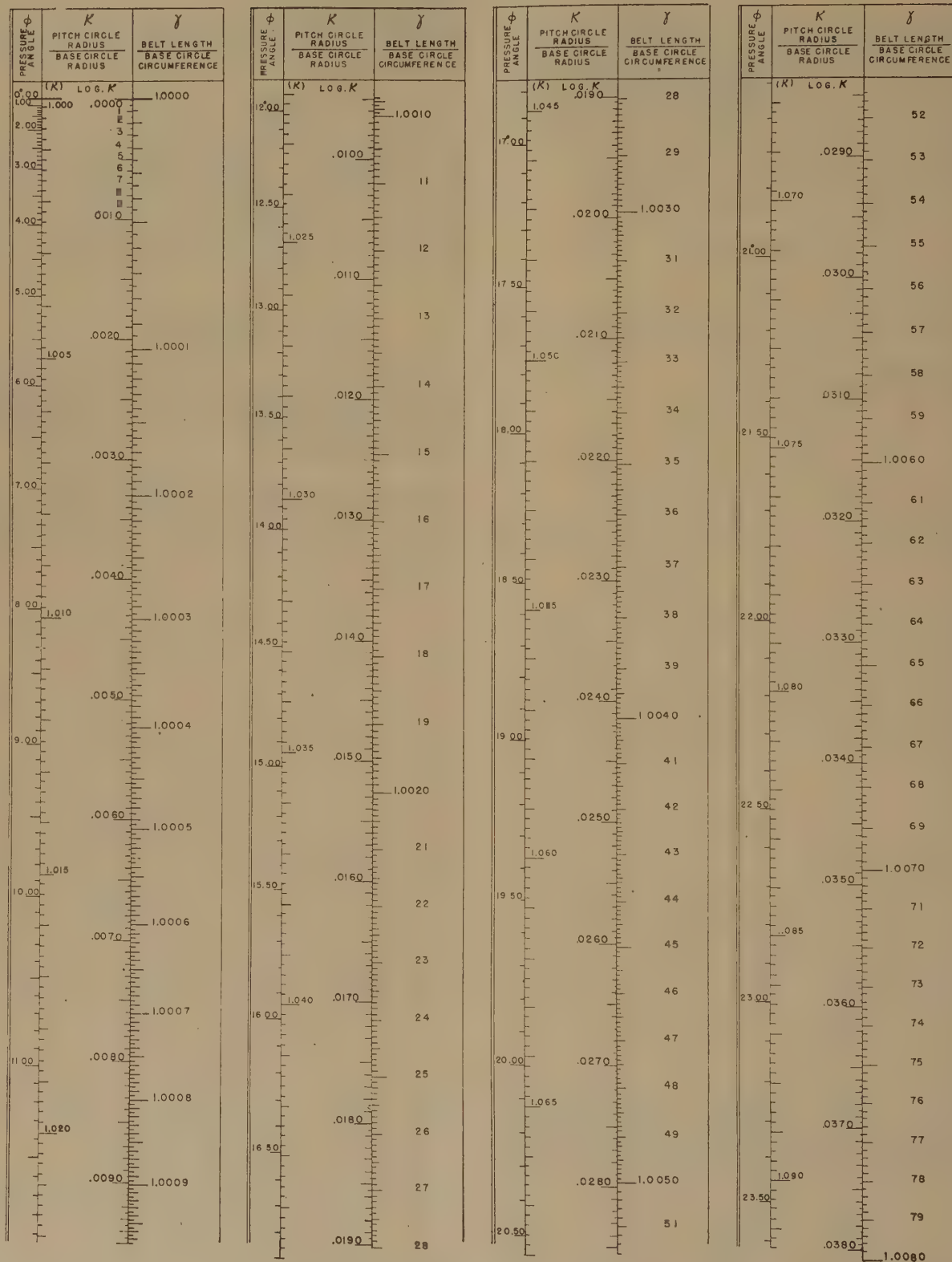
The Appendix contains a "gamma-ratio chart" which graphically relates pressure angle ϕ , ratio of pitch circle to base circle K , and ratio of belt length to base circle γ . Note that $K = \sec \phi$. Corresponding values of ϕ , K , $\log K$, and γ are located on the same horizontal line.

* "Involute Gear Calculations Simplified," by A. H. Candee, *American Machinist*, September, 1945, vol. 89, no. 14, p. 122.

† "Playing Hob With Involute Gears," by J. D. Howell, *Machine Design*, April, 1942, vol. 14, no. 4, p. 106.

GAMMA-RATIO CHART 1

APPENDIX A - GAMMA RATIO CHARTS

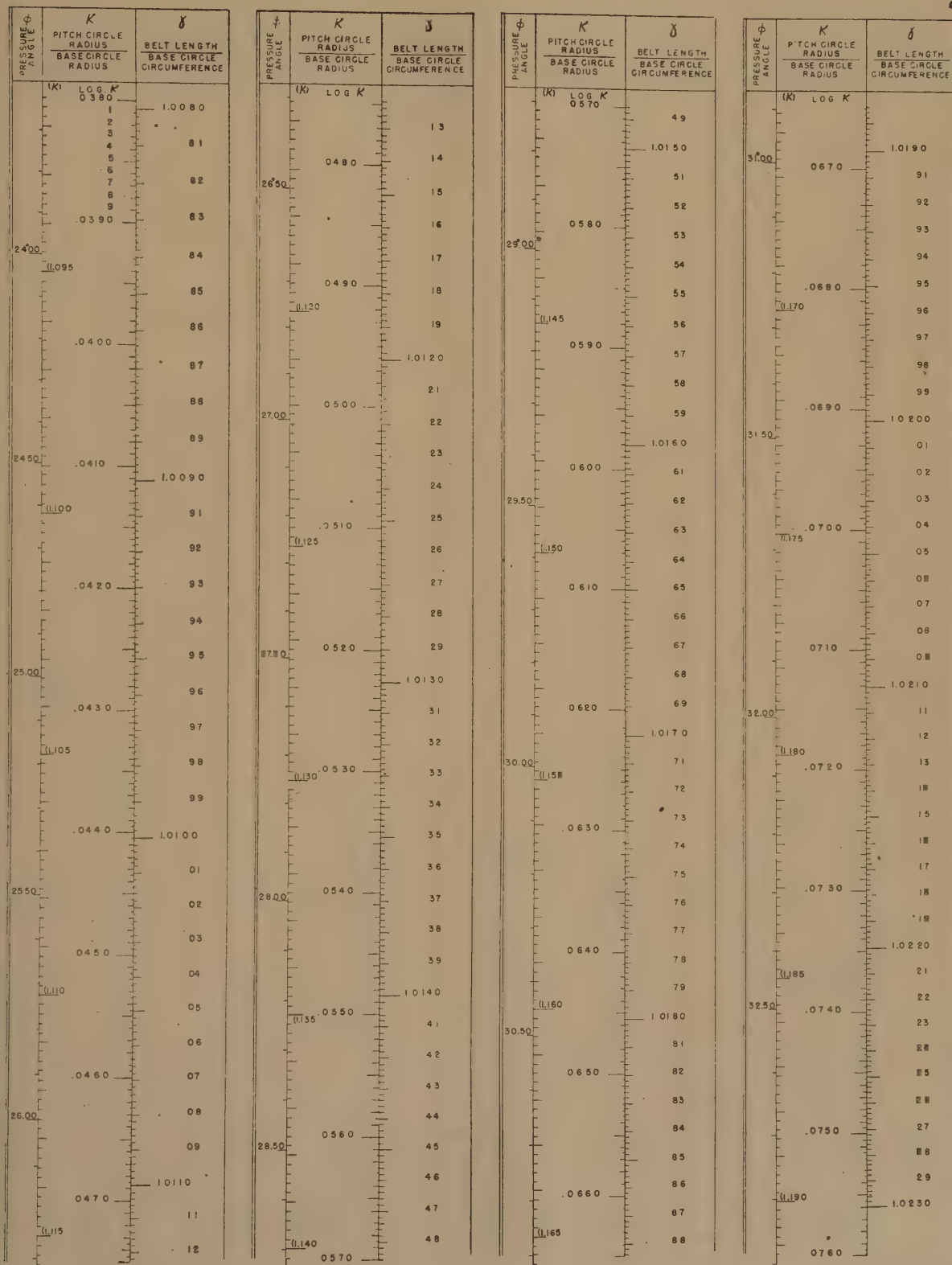


A. BALOYAN

READ CORRESPONDING VALUES OF ϕ , K , $\text{LOG } K$, AND γ ON A COMMON HORIZONTAL LINE

GAMMA-RATIO CHART 2

2

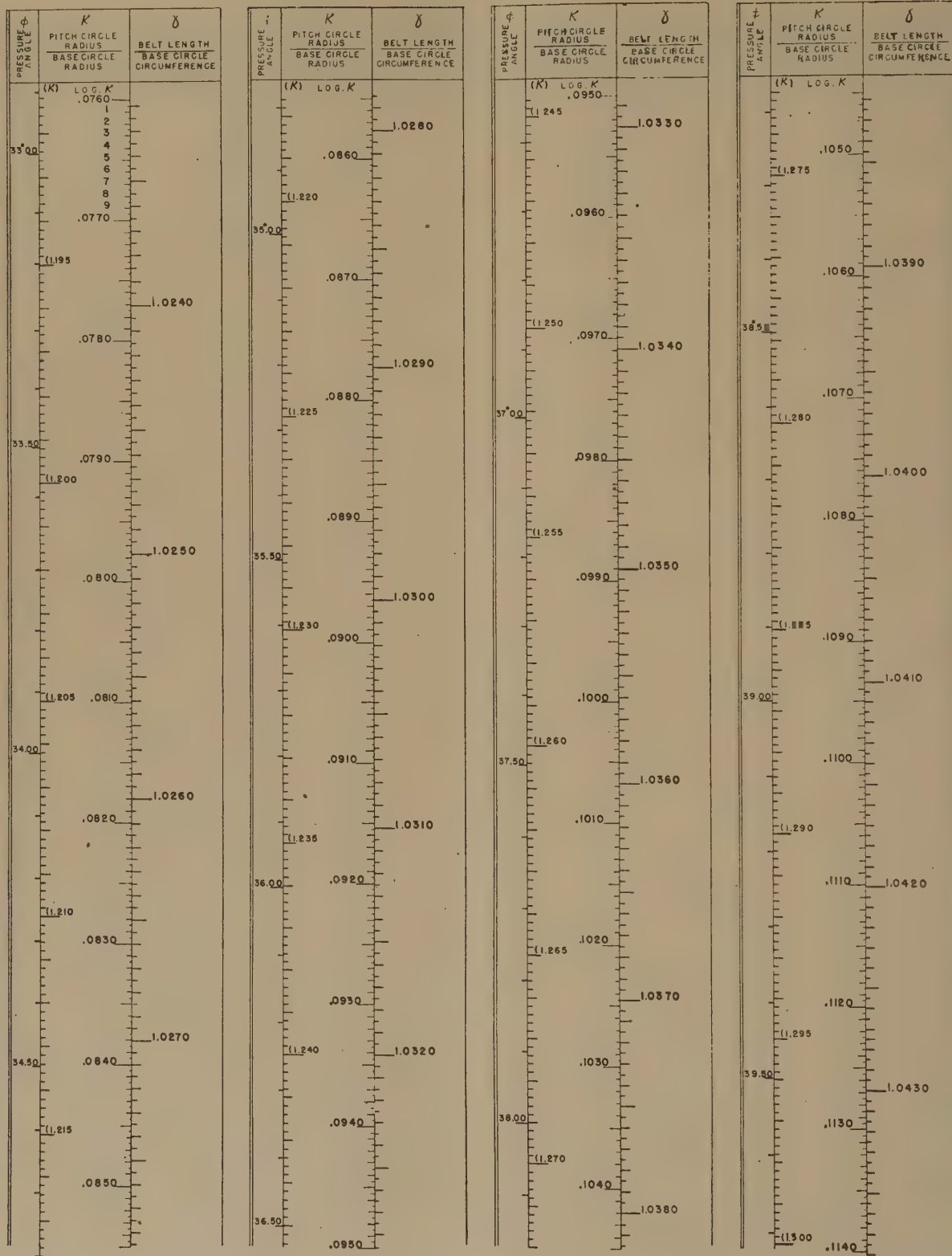


A. BALOYAN

READ CORRESPONDING VALUES OF ϕ , K , $\log K$, AND γ ON A COMMON HORIZONTAL LINE

GAMMA-RATIO CHART 3

3

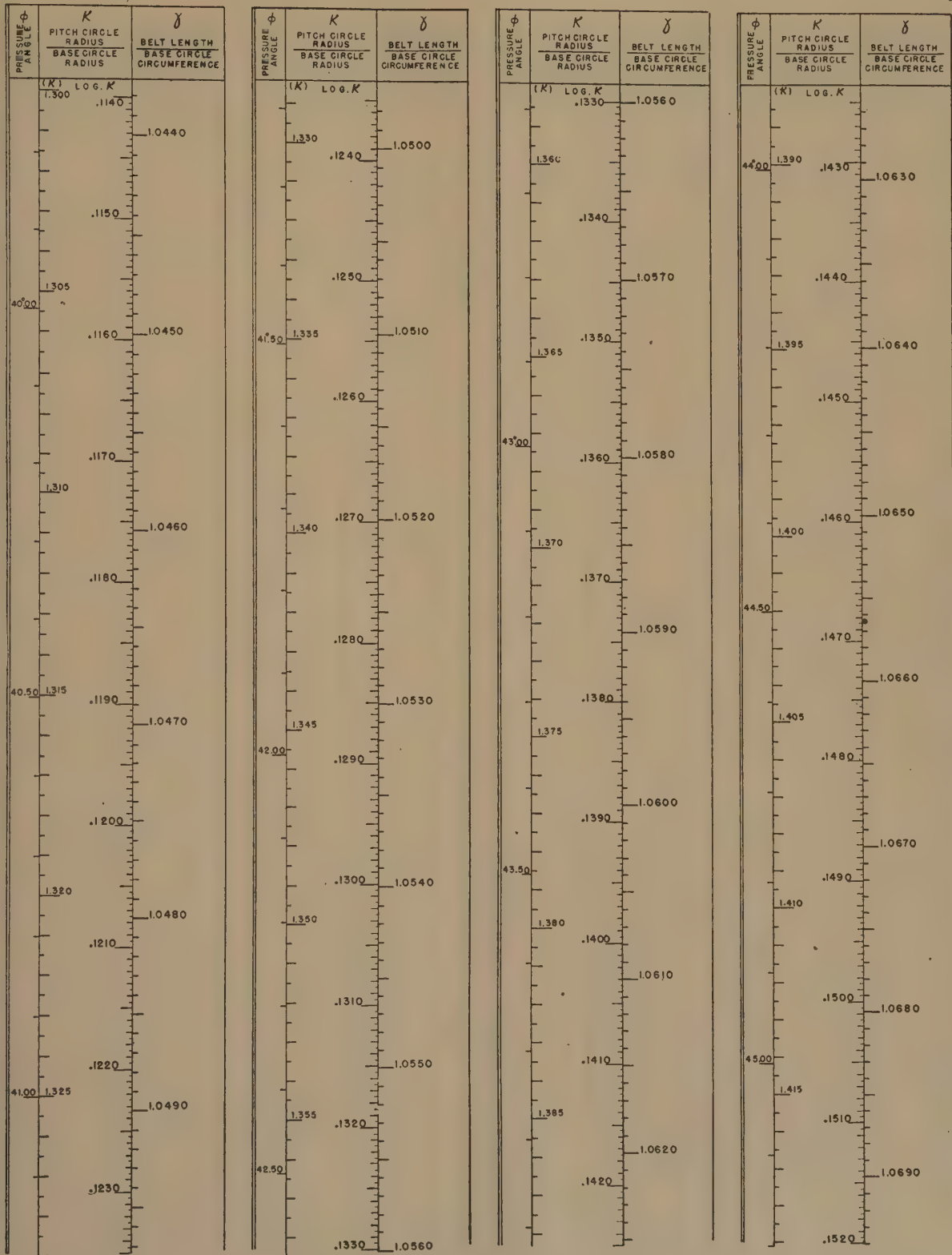


A. BALOYAN

READ CORRESPONDING VALUES OF ϕ , K , $\log K$, AND γ ON A COMMON HORIZONTAL LINE

GAMMA-RATIO CHART 4

4

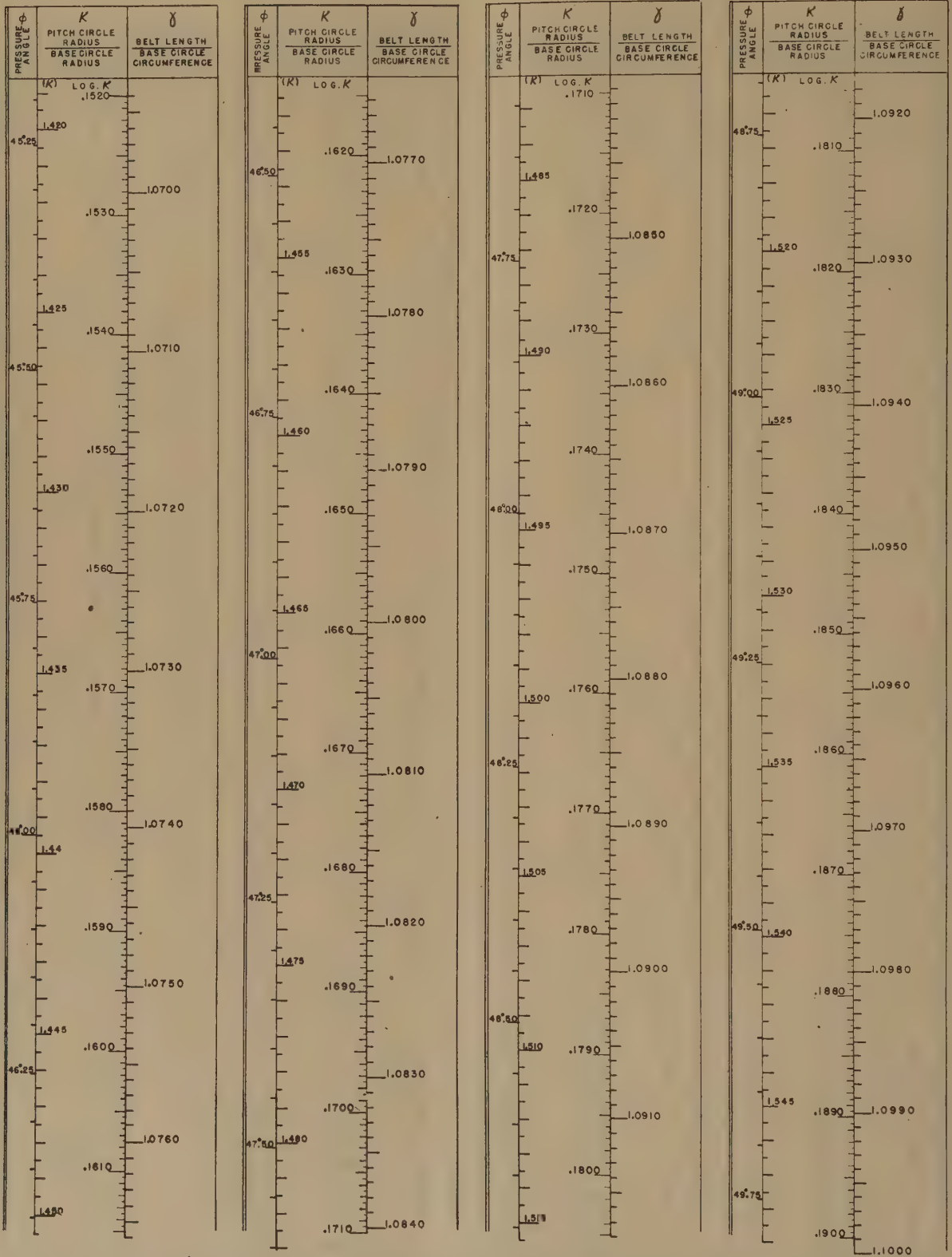


A. BALOYAN

READ CORRESPONDING VALUES OF ϕ , K , $\log K$, AND γ ON A COMMON HORIZONTAL LINE

GAMMA-RATIO CHART 5

5



A. BALOYAN

READ CORRESPONDING VALUES OF ϕ , K , $\log K$, AND γ ON A COMMON HORIZONTAL LINE

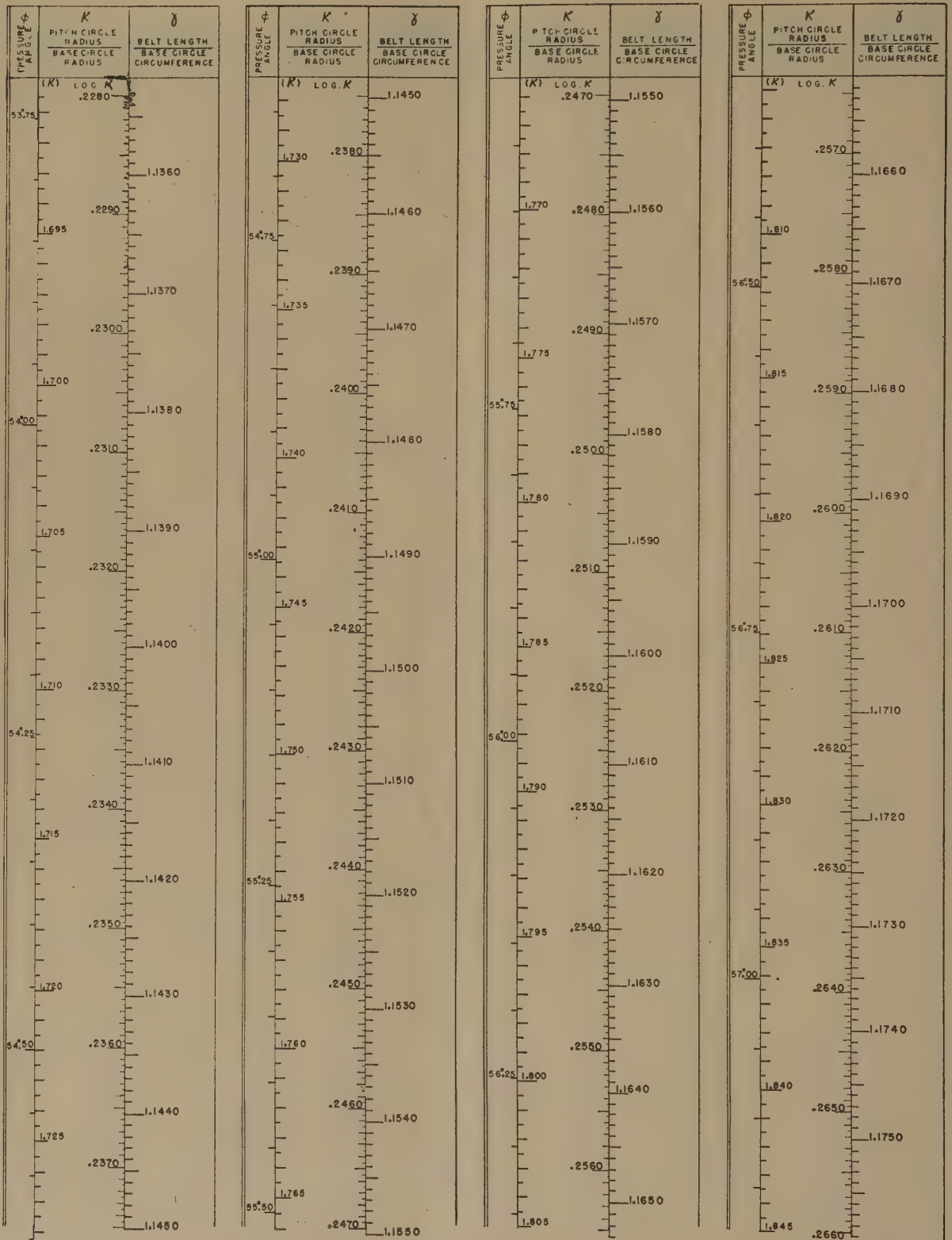
GAMMA-RATIO CHART 6

6



GAMMA-RATIO CHART 7

7

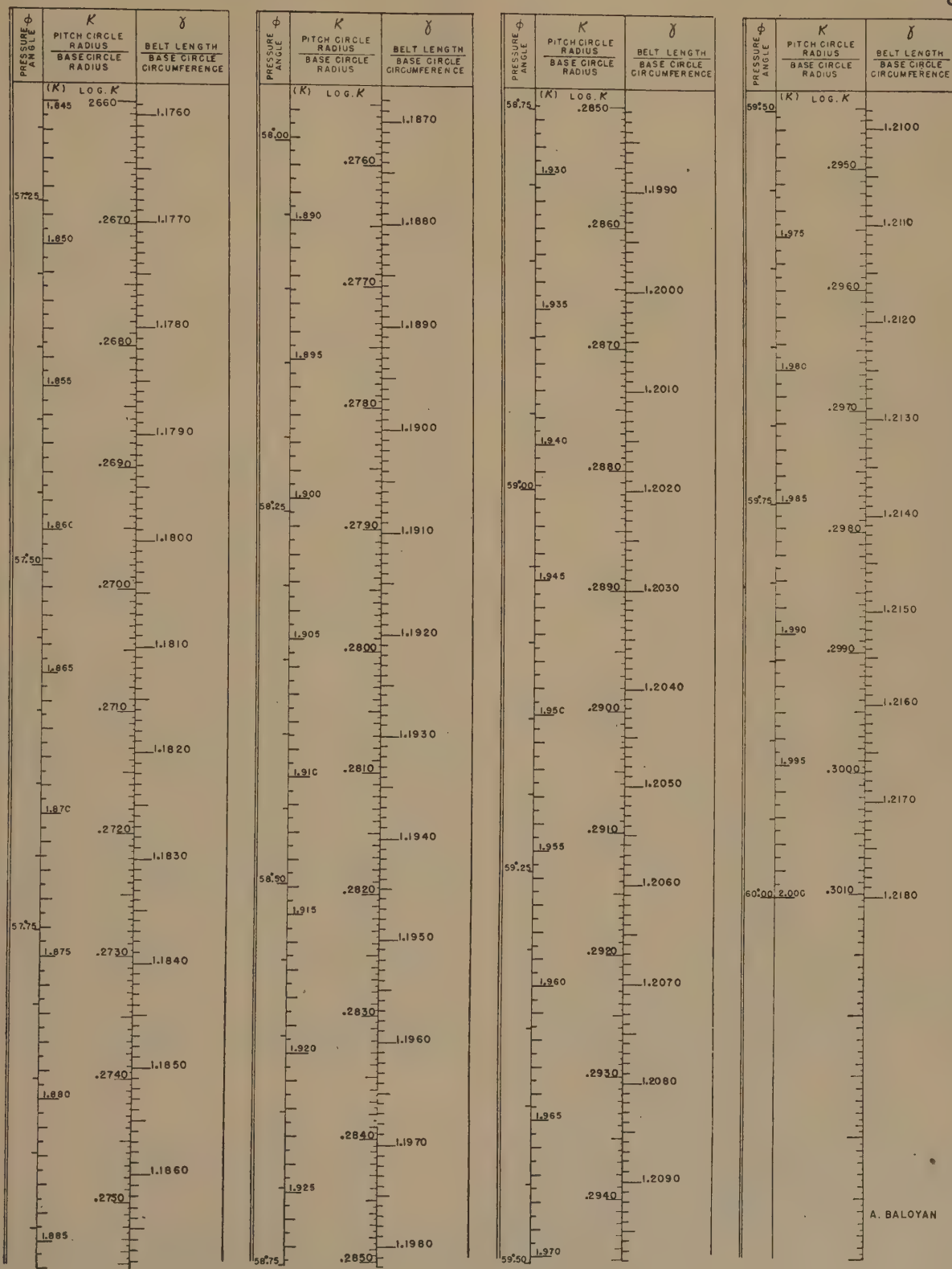


A.BALOYAN

READ CORRESPONDING VALUES OF ϕ , K , $\log K$, AND δ ON A COMMON HORIZONTAL LINE

GAMMA-RATIO CHART 8

8

READ CORRESPONDING VALUES OF ϕ , K , $\text{LOG } K$, AND γ ON A COMMON HORIZONTAL LINE

Discussion

A. E. RICHARD DE JONGE.⁸ Invited by the author to discuss this paper, the writer finds himself in a difficult position, for he had expected new facts on involute gears and how they could be improved for practical purposes. The reading of the paper left him disappointed, as nothing so revolutionary has been attempted. All that the author has submitted is a new method of calculating various involute-gear characteristics.

Many calculations on involute-gear teeth require the use of the so-called involute function $\tan \phi - \phi$. Apparently, it was the elimination of the use of the involute function which was in the author's mind. However, no method of calculation of whatever nature can get around the involute function, and the latter is bound to come up sooner or later in any such method. All that can be done and has been done in the past is to provide, by computation beforehand, tables which correlate various functions with the involute function and the pressure angle. What the author has done is to produce such a correlation of data, which he has published in the form of a "gamma-ratio chart." In order to do this and keep its size down, he found it necessary to eliminate the size factor, and this he has achieved by deriving, from well-known relations, a formula involving the involute function in nondimensional form. The remainder of the paper shows how this formula can be used to solve various problems on involute gears, and it is claimed that this formula simplifies such calculations.

It is true that the author's approach appears to be novel and as such merits attention. However, it necessitated the introduction of the rather artificial concept of "belt length ratio," as well as the setting up of a "graphical calculation chart," his gamma-ratio chart. It seems unnecessary to point out the great amount of work which the latter undoubtedly involved. In its creation lies the principal value of the paper.

Whether the author has succeeded in simplifying involute-gear calculations by his method is debatable. By the introduction of the belt length ratio, the lucidity and directness of approach to involute-gear problems suffers. Thus it remains to be seen whether the author's expectations that the "procedures and means here presented may supplant the conventional involute-gear tables and the established computation methods to a great extent," will be realized, for the practical engineer, in general, makes use of the most direct methods which he can easiest comprehend and remember, and, to the writer's mind, the method presented by the author does not fall in this category. Although the author has taken great pains to show, by a number of well-chosen examples, how his method can and should be applied in the solution of various involute-gear problems, it remains to be seen whether practical engineers will follow him in applying his method to still other problems.

While the writer is always gratified to find new solutions to existing problems, he is mindful of the fact that they should be simpler and more direct than previously known solutions. A comparison of the author's method with the one cited by him (footnote 6) shows that the latter is undoubtedly more direct and briefer. This method makes use of precalculated tables for the correlation of the various values with the pressure angle, the involute function, and other trigonometric functions of the pressure angle, but when these are known, the actual solution of involute-gear problems involving, for example, variations of gear mesh, and so forth, by this method seems to the writer to be more simple.

The use of large-scale drawings is recognized by the author to be the most reliable means for checking gear design and for analyzing conditions in a gear mesh. Indeed, by actually drawing the profiles of gear teeth in mesh with each other, the greatest clarifi-

cation is obtained, and this task is by no means as difficult as the author seems to think. The curves can easily be drawn by simple means, generally by compass and ruler, but it should be mentioned here that there exist "involutographs" which draw the involutes directly just as "ellipsographs" draw ellipses. The use of celluloid templates, or even ordinary tracing paper, allows the study of gear-generation problems and of undercutting in the simplest manner.

The writer wishes to draw attention to the fact that there still exist numerous problems concerning the generation and manufacture of involute gears of highest accuracy and means for checking the accuracy of such gears. In addition, there still remains the problem of noise suppression and the means by which noiseless gears can be obtained. In practice, slightly relieving the tooth face toward the tip has been found to improve the silent running of gears, and this can be shown theoretically to be necessary, but data on how much this relief should be do not seem to have been published. At present this is still done by trial and error. Such problems seem to the writer to be far more important than the attempts at simplifying some numerical calculations, although the value of their simplification should not be underrated.

The author has given wide space to the use of measuring pins. These have been used in practice for a long time and have been described, as stated by the author, by H. E. Merritt who has also discussed their properties. The extension of this knowledge of their properties by the author will prove a valuable aid to the technician who has to produce highly accurate gears.

Unfortunately, there are numerous numerical errors and misprints in the paper, and the omission of symbols in some figures cited in the text, which, as well as the small size of Figs. 1 and 2, prove to be an impediment to the reader.

The detailed discussion of the various problems solved by the author and their comparison with solutions by other methods would take far too much space to be permissible here, and the few general remarks made will have to suffice. As regards simplicity and directness of a method, the writer wishes to close with the well-known exhortation by Lord Kelvin: "A problem is never solved until it is reduced to the simplest terms," which, in all investigations, should never be lost sight of.

J. D. HOWELL.⁹ To understand the importance of this paper, let us take a practical example, and assume that our maintenance mechanic has brought us the gear shown in Fig. 11 of this discussion. Except for the missing teeth, it is a good gear, or at any rate, a necessary one. The mechanic knows it had 10 teeth, and its outside diameter is 13.000 in., but beyond that, he is stuck.

The first thing we do is to measure the base pitch, by method (a) in Fig. 11, if we have a micrometer or vernier caliper capable of

⁹ Manager, Design Section, Engineering Department, Willcox & Gibbs Sewing Machine Company, New York, N. Y. Mem. ASME.

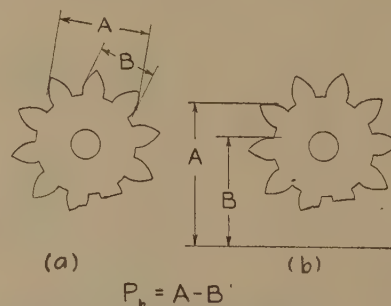


FIG. 11 MEASUREMENT OF BASE PITCH

⁸ Consulting Engineer, New York, N. Y. Mem. ASME.

getting between the teeth, or by method (b) if we prefer to use a height gage. Knowing the base pitch, we can identify the system or basic rack of the gear just as positively as we can identify a person by his fingerprint. In fact, though the writer has never come across such a table, it would be practical and very convenient to have the base pitches of all standard gear systems tabulated in an ordered series against their respective racks, identified by pitch and pressure angle. We have found the base pitch to be 2.952, and lacking a table, must use the formula.

$$P_b = \frac{\pi \cos \varphi_{std}}{DP_{std}} = CP_{std} \cos \varphi_{std}$$

Trying $14\frac{1}{2}^\circ$ and 20° for φ_{std} , we decided that the gear belongs to the 1 DP 20° PA system, though it has been modified by cutting on an 11-tooth diameter.

While the mechanic turns up the $\frac{1}{2}$ base-pitch pins to 1.4711 in. diam (we got the extra accuracy by substituting back in the formula), we find a machine shop that has the necessary shaper, cutter, or hob (it does not matter which). When we get the pins, we find the main pitch diameter, and with the formulas of this paper, obtain all the information necessary for cutting the replacement gear.

Reviewing this example, we see that the regular identification of a gear by tooth number, pitch, and pressure angle has been quite inadequate, as the two latter values cannot be measured. They are, in fact, variables dependent upon cutting and mesh. On the other hand, the base pitch is easy to measure and accurately defines the gear system. Similarly, the main pitch diameter, given the base pitch, is easy to measure, and definitely defines any cutting modification, whether it is a major modification for elimination of undercut or only an allowance for backlash. Thus with these two values and the tooth number we have a complete and practical three-parameter identification of any involute spur gear.

Some comment is due the author's γ or belt ratio, a concept that has unquestionable pedagogic value in clarifying gear geometry. This writer will admit occasional confusion over the dual role of the involute function as a function of the pressure angle and as an angle in its own right. However, the relation between $\gamma(\varphi)$ and $\text{inv } \varphi$ is linear (Equation [5] of the paper), and the writer suspects that the amazing simplification of gear formulas which the author has obtained is due less to this concept than to the concepts of main pitch diameter and main pressure angle. Thus, transforming his Equations [12], [15], [16], and [17] from the γ form to the φ form, we get

$$\text{inv } \varphi_0 = \frac{N_1 \text{inv } \varphi_{m1} + N_2 \text{inv } \varphi_{m2}}{N_1 + N_2} \dots \dots \dots [12a]$$

$$\text{inv } \varphi_{m_g} = \text{inv } \varphi_{\text{hob}} + \frac{2C \tan \varphi_{\text{hob}}}{N_g} \dots \dots \dots [15a]$$

$$N_c = \frac{N_g}{2\pi} (\tan \varphi_c - \tan \varphi_0) \dots \dots \dots [16a]$$

$$\text{inv } \varphi_p = \text{inv } \varphi_m + \frac{\pi}{2N} \dots \dots \dots [17a]$$

which seem to the writer just as simple as the original forms. Also, to the writer, the use of K for $\sec \varphi$ seems to have no value and to add unnecessary confusion.

These last points, however, are mere technicalities. The writer wishes to congratulate the author on a paper of importance to all engineers interested in gearing.

MICHAEL VASVARI.¹⁰ The author has given a valuable explanation of involute-gear geometry, but the writer feels that the calculation by this method is somewhat lengthy and complicated. Such calculations may be accomplished more easily, more rapidly, and more exactly by using the conventional involute functions and formulas, than by the present method and by the "gamma-ratio chart."

The author shows his system in an example and concludes in Equation [14] that the center distance of the discussed pair of gears, which is corrected and cut to standard depth by a Fellows circular cutter, will be 0.003 in. larger than the standard center distance. He concludes: "In this case either pinion or gear should be shaped an additional 0.003 in. deeper before allowance for backlash is made."

By coincidence the result in this example checks with the general practice, but the conclusion drawn is not theoretically correct. Pinion or gear should not be cut to the same additional depth, and this depth is not equal to the increase of the center distance.

If corrected gears are generated by a Fellows circular cutter, it is necessary to cut pinion and gear to an additional depth, and this depends only upon the number of teeth of the cut element and of the cutter, and on the value of the correction.

This is based on the following calculation:

A noncorrected pair of gear has a standard center distance

$$a_0 = \frac{N_p + N_g}{2DP}$$

and a pressure angle, which is equal with the cutter angle α , whether they are hobbled, rack-shaped, or generated by a Fellows circular cutter. In the foregoing formula, N_p = number of teeth of pinion; N_g = number of teeth of gear; DP = cutter pitch; α = cutter angle.

If we will give the pinion (C_p)/(DP) correction and the gear (C_g)/(DP), then the new pressure angle ϵ

$$\text{inv } \epsilon = \frac{2 \tan \alpha (C_p + C_g)}{N_p + N_g} + \text{inv } \alpha \dots \dots \dots [18]$$

which is the first base equation of the involute gears. (This is the same as Equation [12] in the paper.)

The new tight-mesh center distance is

$$a = a_0 \frac{\cos \alpha}{\cos \epsilon} \dots \dots \dots [19]$$

b_p and b_g are the base circle radii, and $b_p + b_g = a_0 \cos \alpha = a \cos \epsilon$.

Equation [19] is the second base equation of the involute gears. (This is the same as the Equation [13] of the paper.)

This new center distance is not equal to the sum of the standard center distance plus the corrections, but less

$$a \neq a_0 + \frac{C_p + C_g}{DP}, \quad a < a_0 + \frac{C_p + C_g}{DP}$$

This difference is very little, consequently for the calculations we must use seven-place logarithmic tables to get exact results. (The chart used by the author is only to five places.)

Base Equations [18] and [19] herewith, are applicable, if we wish to cut corrected gears with the Fellows circular cutter. Of course in this case $C_c = 0$, because the cutter is not corrected.

The calculation of the author's example will be as follows:

¹⁰ Mechanical Engineer, New York, N. Y.

Additional Depth of Pinion: (Cutter 10 DP, $\alpha = 20^\circ$)

$N_p = 10$	$C_p = 0.5$	Pitch radius of pinion.....	0.5 in.
$N_c = 40$	$C_c = 0$	Pitch radius of cutter.....	2.0 in.
Standard center distance....			2.5 in.

$$\text{inv } \epsilon = \frac{2 \times 0.5 \times \tan 20^\circ}{10 + 40} + \text{inv } 20^\circ = 0.0221838$$

$$\epsilon = 22^\circ 43' 15.9''$$

$$a = 2.5 \frac{\cos 20^\circ}{\cos 22^\circ 43' 15.9''} = 2.54688 \text{ in.}$$

$$a_0 + \frac{C_p + C_c}{DP} = 2.5 + \frac{0.5}{10} = 2.55000 \text{ in.}$$

Additional depth = difference = 0.00312 in.

Additional Depth of Gear:

$N_g = 30$	$C_g = -0.5$	Pitch radius of gear.....	1.5 in.
		Pitch radius of cutter.....	2.0 in.
Standard center distance....			3.5 in.

$$\text{inv } \epsilon = \frac{2 \times (-0.5) \times \tan 20^\circ}{30 + 40} + \text{inv } 20^\circ = 0.0097049$$

$$\epsilon = 17^\circ 24' 23.5''$$

$$a = 3.5 \frac{\cos 20^\circ}{\cos 17^\circ 24' 23.5''} = 3.446763 \text{ in.}$$

$$a_0 + \frac{C_g + C_c}{DP} = 3.5 - \frac{0.5}{10} = 3.45000 \text{ in.}$$

Additional depth = difference = 0.003237 in.

AUTHOR'S CLOSURE

The author wishes to acknowledge with grateful appreciation the thoughtful comments and criticisms put forth by Messrs. de Jonge, Howell, and Vasvari in their discussions.

In response to Mr. de Jonge's general statement that the concepts of "main belt length" and "main belt length-to-base circle (gamma) ratio" do not provide new facts on involute gears; it should be of interest to note that, in the author's experience, these notions, which embody in a single specification the complete definition of the thickness of a gear tooth throughout its length, have enabled relatively inexperienced designers to grasp

more easily the intricacies of modified gearing. This has been found to be especially true in dealing with such special designs as "face," "taper," and off-standard internal-gear combinations.

It is recognized that the imaginative device of "belt length" is perhaps not needed by the expert gear designer (such as Mr. de Jonge) who, as the result of long experience, has no difficulty in visualizing the implications of an alteration from standard proportions in terms of the involute-angle function. Also the belt method certainly is not intended to compete with the highly specialized short-cut procedures used professionally for problems often encountered in small tooth-number pinion alteration. But the "belt ratio" has been found to promote more rapid understanding of basic gear-meshing principles, particularly in student training. To the engineer who does not specialize in gear design, and only needs to apply himself occasionally to involute geometry, it is hoped that the "gamma ratio" will provide a common and simple basis for tackling all manner of gear problems directly and with confidence.

Mr. Howell has presented a strong argument for adoption of base pitch as a standard in gear specifications, which is most gratifying. The facts that conventional circular (or diametral) pitch systems do not possess the necessary flexibility and cause unnecessary near duplication of form, tools, and standards, are becoming increasingly plain to students of gearing.

Mr. Howell's criticism of the designation of $\sec \phi$ by K (kappa) as being confusing is acknowledged, but it is perhaps justified on the basis of short-hand simplification of notation.

Mr. Vasvari has apparently misunderstood the meaning of the author's statement that pinion or gear must be cut to additional depth for operation on standard centers when generated by circular cutters. There is a small theoretical difference involved, as Mr. Vasvari has so ably demonstrated in his computations. This difference was ignored since it was known to be of less significance than tolerable gear-cutting error, since the cutting pressure angles and the operating pressure angle are very near the same value and therefore extra cutting depth has almost exactly the same effect as difference in operating centers.

The graphical chart will yield results uniformly within errors of 0.0001 in. per in., which makes unnecessary the use of seven-place tables for practical gear computations. The charts also eliminate the need for interpolation, which promotes speed in calculations.

Film-Pressure Distribution in Grease-Lubricated Journal Bearings

By GUNTHER COHN¹ AND JESS W. OREN²

Measurements of the pressure developed in the load-carrying film of a grease-lubricated journal bearing show that grease can operate under hydrodynamic conditions. In a testing fixture, constructed for the purpose, the magnitude and the distribution of that pressure is investigated using a cup grease under conditions of copious feed. The graphical analysis of the results shows a grease dome similar to that of oil, but flatter and extending over a larger arc.

INTRODUCTION

IN the course of experimental work carried on at The Pennsylvania State College, in co-operation with The Texas Company, it was established that a grease-lubricated journal bearing could, under certain conditions, operate in the fluid-film region with low coefficients of friction and low grease-feed rates. The results of this investigation and the early uses of grease are described in a forthcoming paper.³

While a theory of oil lubrication for journal bearings is well established and applied in design, knowledge of the mechanism of grease lubrication is limited and publications along this line are practically nonexistent.

Briefly, lubrication analysis may be divided into three broad fields: those of (1) friction, which determines the power information; (2) running position, which establishes the minimum film thickness; and (3) pressure distribution, which defines the film outline.

Falling into the last division, the investigation described in this paper was undertaken to determine whether a hydrodynamic pressure exists in a grease-lubricated journal bearing, and if so, to obtain its magnitude and distribution.

The experimental results reported herein were obtained with a testing machine similar in principle and construction to that of S. A. and T. R. McKee.⁴ However, in this design the fixture has been made adaptable to lubrication by either oil or grease.

DESCRIPTION OF APPARATUS

A vertical steel shaft *A* containing a ground journal, 1 in. long, is attached to a drill-press arbor through a flexible coupling and is aligned by two steady bearings, Figs. 1 and 2. One of these is located in the top support bracket *C*, and the other in the base of the main frame of the fixture which is bolted to a special drill-press table. The bearing *B*, a 300-deg babbitt-lined brass shell, 2 in. long, is permitted to assume its own running

position. In order to confine the lubricant, oil seals are fastened to its top and bottom.

A single pressure tap hole, $\frac{3}{32}$ in. diam, in the middle of the bearing surface, is used for all pressure traverses.

Circumferential traverses are made by rotating the bearing in a yoke ring *D*. As the load is applied to the yoke, its circumferential point of application remains fixed. A stop prevents the yoke from turning, and the bearing is held in the desired position by tightening a set screw. For ease of rotation, the bearing rests on a Timken cone, the cup of which is held in a container provided with three leveling screws *E*. The angular bearing position is shown by a pointer *F*, which moves over a scale engraved on the drill-press table.

Longitudinal traverses are made by moving the journal past the pressure tap hole in the bearing. This is accomplished by means of the drill-press feeding mechanism which can be locked by a set screw. An indicator *G*, fastened to the testing fixture, points to a scale on the shaft which denotes the longitudinal position. The load is applied to a threaded rider *H*, which is moved axially by means of a lead screw, held by a fork fastened to the yoke ring. By this means mid-point loading is achieved at all times so that an axial bearing motion is simulated. The lead screw is used as a micrometer to obtain a particular setting. When once it is set, the friction in the threads holds the rider in position.

Adjusted by a turnbuckle, a spring balance applies the load. The lubricant is fed through a street-ell into the top of a deep, 60-deg channel which completes the bearing. Any excess, in the case of grease, is permitted to escape through a small vent at the bottom.

Properties of the test lubricants are shown in Table 1.

The pressure tap hole, at right angles to the axis of rotation, is plugged on the outside and an axial hole in the bearing shell connects with the gage through copper tubing. This is the arrangement used for tests with oil. In early tests with grease, a vaseline plug was used to separate the lubricant from the oil in the gage line. Later the tap hole was covered with a diaphragm with which steadier conditions were achieved. This necessitated a balanced-pressure system because of the small deflection of the diaphragm.

The diaphragm is a rayon-enforced synthetic rubber, 0.02 in. thick. It is cut with a punch, cemented into the $\frac{1}{4}$ -in. enlargement of the tap hole and held in place by a spacer. Finally, the hole is sealed and plugged. Flexible tubing, permitting bearing rotation, connects the axial hole to the stationary oil system, a series of valves, the measuring gage, a hand pump which supplies the back pressure, and an oil sump, Fig. 3. Great care in installing the diaphragm is required to prevent displacement of the disk or crushing of its edges. All lines in the oil system are carefully filled with oil.

TEST CONDITIONS AND PROCEDURE

The various test conditions are listed in Table 2. When testing oil, the machine was assembled without the grease cup and with the vent plugged. Oil was introduced into the bearing until its level was above the journal. The pressure gage was connected directly to the bearing. After the axial location of the

¹ Research Engineer, The Franklin Institute, Laboratories for Research and Development, Philadelphia, Pa. Jun. ASME.

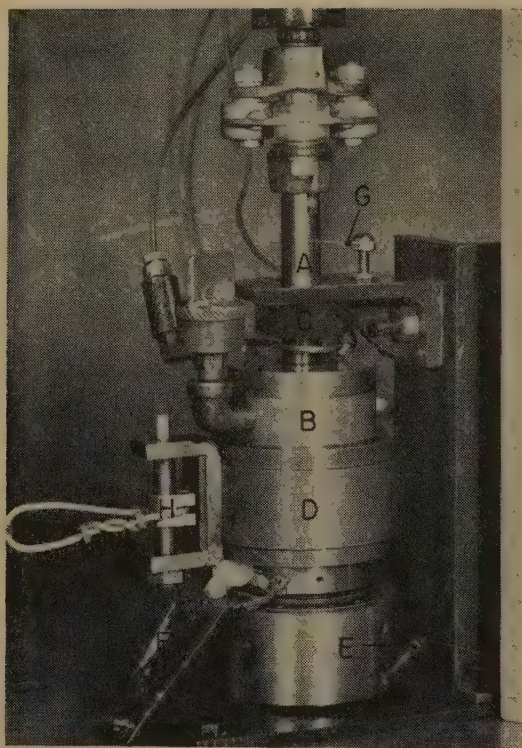
² Research Engineer, Armstrong Cork Company, Lancaster, Pa. Jun. ASME.

³ "Grease Film Lubrication of Journal Bearings," by L. J. Bradford and E. M. Barber. In preparation for publication at a later date.

⁴ "Pressure Distribution in Oil Films of Journal Bearings," by S. A. and T. R. McKee, Trans. ASME, vol. 54, 1932, pp. 149-165.

Contributed by the Special Research Committee on Lubrication and presented at the Annual Meeting, New York, N. Y., November 28-December 3, 1948, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society. Paper No. 48-A-31.



A, Journal
B, Bearing
C, Top-support bracket
D, Yoke ring
E, Leveling screws of bearing support
F, Circumferential pointer
G, Axial position indicator
H, Rider; point of load application

FIG. 1 TESTING MACHINE

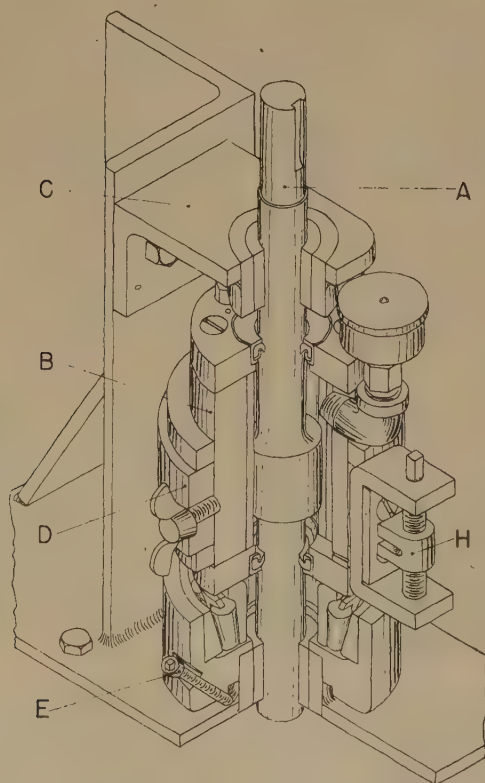


FIG. 2 CUTAWAY SECTION OF TESTING MACHINE
(Parts are as designated in Fig. 1.)

journal and the point of application of the load were adjusted to the desired position, the machine was started and the load applied. Pressures were read at eleven longitudinal positions, 0.1 in. apart in the center of the bearing and 0.05 in. apart at the ends, designating the locations as 0.05L, 0.1L, 0.2L, and so forth. The circumferential intervals varied depending upon the shape of the expected curve; the steeper the curve, the smaller the intervals.

Sufficient pressures were recorded to plot a distribution curve, thus covering about 200 deg of bearing arc. Both circumferential

TABLE 1 PROPERTIES OF TEST LUBRICANTS

Grease:	
NLGI No. 2 grade ball and roller-bearing grease	15
Calcium and sodium soap (per cent approx)	85
Mineral oil (per cent approx)	268
Penetration at 77 F (ASTM)	256
Worked	296
Unworked	52.8
Dropping point, deg F	6.54
Values of extracted mineral oil	0.9094
Kinematic viscosity (cs at 100 F)	+20
(cs at 210 F)	
Specific gravity at 60/60 deg F	
Pour point, deg F	
Oil:	
300-Sec viscosity paraffin oil	52.7
Kinematic viscosity (cs at 100 F)	6.54
(cs at 210 F)	0.8973
Specific gravity at 60/60 deg F	+25
Pour point, deg F	413
Flash, C.O.C., deg F	463
Fire, C.O.C., deg F	

TABLE 2 TEST CONDITIONS

Journal: nominal length = 1 in.
nominal diameter = 1 in.

Test no.	Speed, rpm	Diametral clearance, in. per in.	Load, psi	Lubricant, (see Table 1)
1	250	0.0082	40	Oil
2	250	0.0082	40	Grease
3	250	0.0071	40	Oil
4	100	0.0071	40	Grease
5	250	0.0071	40	Grease
6	500	0.0071	40	Grease

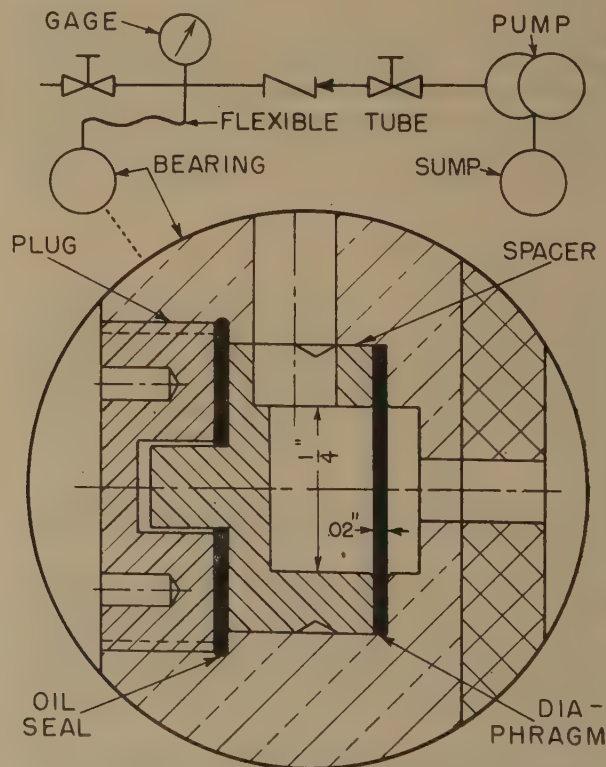


FIG. 3 DETAILS OF PIPING AND DIAPHRAGM

and longitudinal changes in position could be made without stopping the machine.

As even slight misalignments of journal and bearing caused unsymmetrical longitudinal distribution, leveling screws *E*, were added to the bearing support to obtain correct alignment. An oil test was performed, using the method described, taking spot readings at the 0.2*L* and the 0.8*L* position. The bearing was then tilted by means of the leveling screws until as nearly identical curves as possible were obtained for the two locations.

When operating with grease, the diaphragm was installed, the pressure system filled with oil, and the bearing packed with grease to prevent air pockets. The machine was then assembled with the grease cup in place and the vent open. Several cupfulls of grease were fed into the bearing and the load was applied after locations had been adjusted as for an oil test and the machine had been started. With the bearing in a circumferential position of low pressure, the oil system was pumped up to a back pressure of about 10 psi less than the expected film pressure. While the bearing was turned to the desired position, the gage read the back pressure until the film surpassed it. In other respects the testing procedure was the same as that for oil.

The difference in physical properties of oil and grease necessitated the use of different methods of assuring a copious supply to the active bearing arc. When oil is used, the journal is submerged, and the lubricant flows into the leading portion of the bearing angle by gravity and capillary action. In the case of grease, the moving journal tears the adjacent film of grease from the feeding channel and carries it into the active bearing arc, creating an empty space between the journal and the grease supply. To assure a copious feed, it was necessary to fill this space with lubricant as fast as the journal carried it away. This was accomplished by continually pushing the grease past the moving surface of the journal by manual operation of the grease cup and exuding the excess grease through the vent.

PRESENTATION OF RESULTS

Oil was tested in these experiments (1) to establish the functioning of the fixture, and (2) to obtain a basis for comparison with the grease. For proper comparison, the oil chosen has very similar properties to that extracted from the grease, Table 1. Fig. 4 shows the circumferential pressure distribution at the 0.5*L* location of an early test. This is a typical oil curve which checks with the fluid-film theory of a finite bearing and with the results found by McKee.⁴ In the direction of rotation there is a slow rise to a large positive peak, then a rapid drop to a slight negative peak, thus establishing the proper functioning of the machine. A combination gage, -8 to +20 psi, was used to obtain the low points.

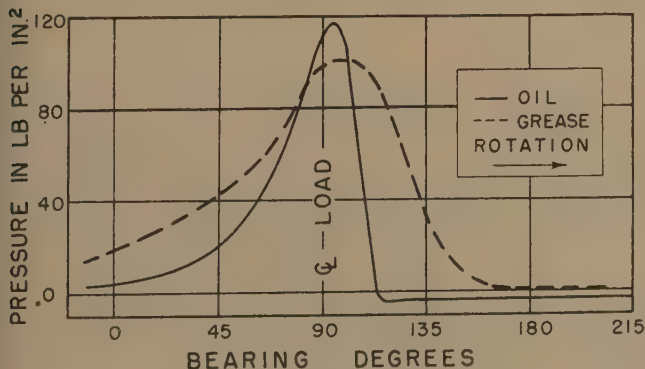


FIG. 4 CIRCUMFERENTIAL PRESSURE DISTRIBUTION IN JOURNAL BEARING
(0.5*L* location; clearance 0.0082 in. per in.; load 40 psi; speed 250 rpm.)

Fig. 4 shows that grease develops a hydrodynamic film, the pressure distribution of which is similar to that of the oil. This traverse was taken with a vaseline plug instead of the diaphragm and with mechanical alignment only, which accounts for the difference in circumferential location of the pressure peaks.

A complete comparative study of oil and grease at 250 rpm is shown in Fig. 5. Actual test points are plotted in the circumferential pressure curves *A*, obtained by using the diaphragm-back pressure method under copious lubrication. Negative pressures were not obtained since only one gage (0 to 200 psi) was used throughout. It was felt that this would not detract from the value of the results, as the investigation is primarily a test of grease. All curves rise slowly to a peak at the "off" side of the line of application of the load and then fall off more rapidly. The grease curves are lower and extend over a greater arc but are similar in shape to the oil curves, namely, skewed sine curves. The maximum pressure with oil is about $3\frac{1}{2}$ times the applied load, or about $2\frac{1}{4}$ times the load with grease at a bearing temperature of approximately 75 F. All pressure peaks fall at an angle of about 110 deg, proving proper alignment of journal and bearing.

Curves *B* are the cross-plots against bearing length *L*, chosen at various intervals which depended upon the rate of change of curvature of the pressure-distribution plots. The most striking difference between the oil and the grease may be observed at the position at right angles to the line of action of the load where the grease curves are much higher, indicating that grease has less end leakage than oil. The arrangement of the longitudinal curves about the median plane of the journal (0.5*L*) can be seen in the curves of average pressure *C*. Here the 0.1*L* curve is plotted from the average points of the 0.1*L* and 0.9*L* location, and so forth.

For a check with the applied load, curves *D* were plotted by integrating the longitudinal curves and plotting the mean pressure thus obtained at the angular positions projected on the diameter at right angles to the line of action of the load. In the case of grease there is a good correlation but in the case of oil, the value obtained by integrating is low (88 per cent of the applied load). This is due to the failure to measure the negative pressure which helps carry the load and the inability of the gage to record low pressures accurately. A better check was obtained with the early oil curves of which the 0.5*L* plot is shown in Fig. 4.

A series of tests were made at 100 and 500 rpm in addition to that at 250 rpm so as to study the effect of speed upon pressure distribution of grease. Curves from the 0.05*L* to the 0.5*L* location are plotted in Fig. 6.⁵ The most apparent effect is a decrease in the leading-edge angle and an increase in the trailing-edge angle of the film as the speed increases. This phenomenon agrees with that found by other investigators with oil.⁶ No significant changes in magnitude or position of the pressure peak were found when operating with grease. In their study of oil, S. A. and T. R. McKee⁴ found that the maximum pressure increased with an increase of speed. Here it can be seen that speed has very little effect upon the pressure distribution in a grease film.

The curves of integrated pressure for the three speeds are shown in Fig. 7.

SUMMARY

The validity of the results obtained on this fixture was established in tests made with oil as a lubricant. With grease it was desired to determine whether a hydrodynamic pressure exists

⁵ Detailed test data are given in the theses by the authors (refer to "Acknowledgments").

⁶ "The Prediction of Bearing Performance," *Lubrication Magazine*, The Texas Company, New York, N. Y., vol. 18, December, 1932, pp. 133-144.

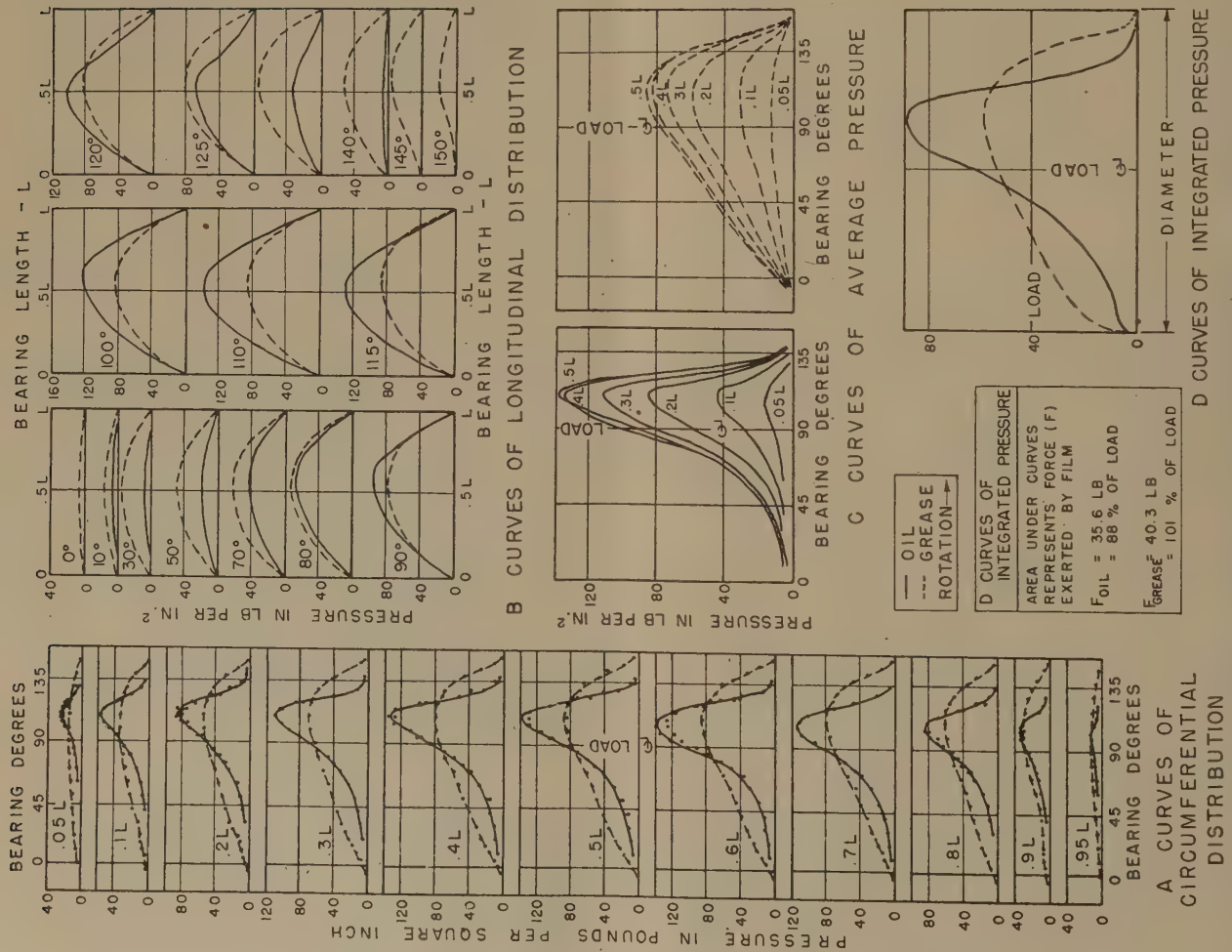


Fig. 5 CURVES OF HYDRODYNAMIC PRESSURE IN JOURNAL BEARING
 (Clearance 0.0071 in. per in.; load 40 psi; speed 250 rpm.)

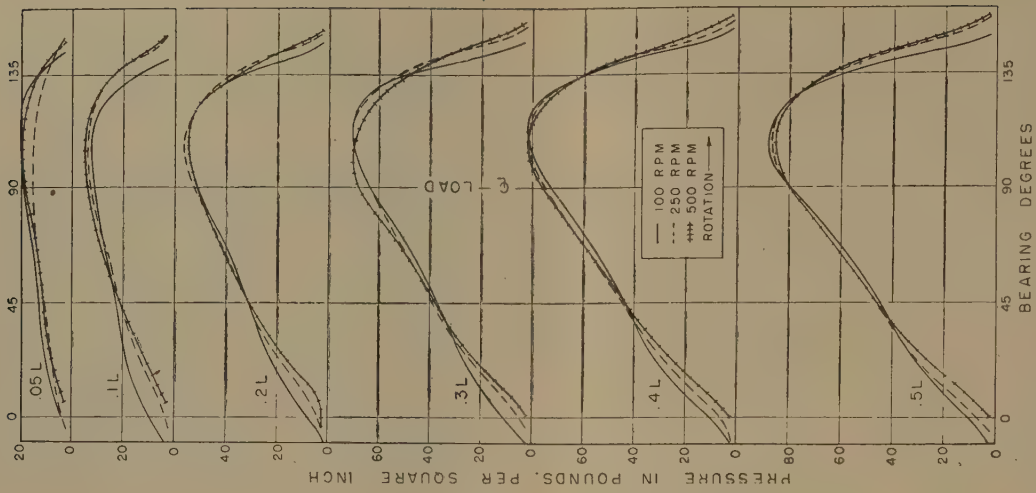


Fig. 6 CIRCUMFERENTIAL PRESSURE DISTRIBUTION IN JOURNAL BEARING
 (Comparison curves of grease films at three speeds; clearance 0.0071 in. per in.; load 40 psi.)

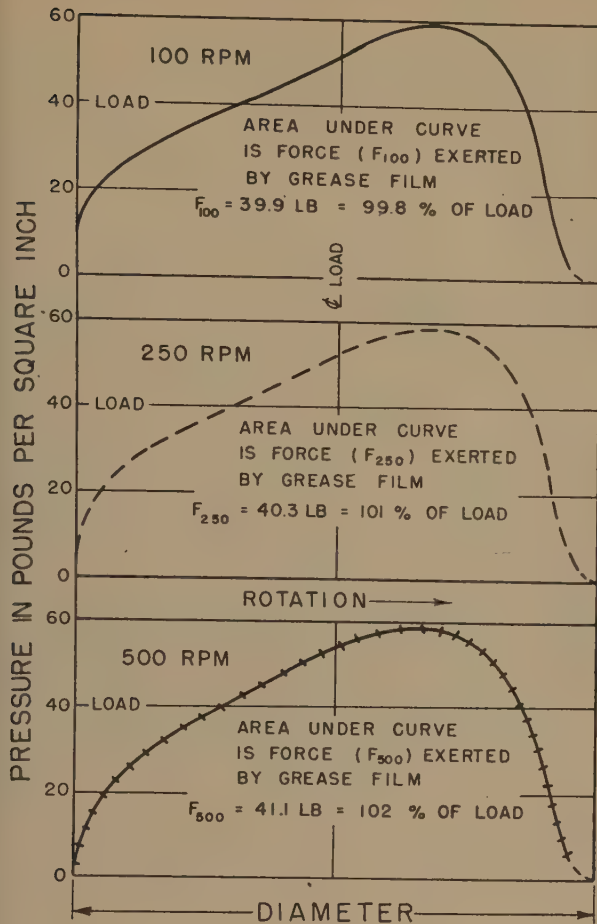


FIG. 7 CURVES OF INTEGRATED PRESSURE
(Grease; clearance 0.0071 in. per in.; load 40 psi; speeds 100, 250, 500 rpm.)

in the film of a journal bearing and to determine its magnitude and distribution. It has been shown that such a pressure exists.

The distribution and the magnitude of the pressures is shown in the various figures. Proper alignment of the bearing has been proved by the alignment of the peak pressures at the "off" side of the bearing and the symmetry of the curves about the median plane of the journal. A diaphragm-back pressure method was used successfully, and the method of a copious grease supply gave consistent and correct readings, as shown by the integrated check plots. The curves obtained with grease are similar to those with oil and therefore substantiate a fluid-film operation.

The results obtained in this investigation apply only to the conditions mentioned, in particular to a cup grease under copious feed.

CONCLUSIONS

From the results of this investigation, the following conclusions are drawn:

- 1 Grease can operate under hydrodynamic conditions.
- 2 Curves of pressure distribution of the grease film are flatter and extend over a greater arc than those of a comparable oil, showing that grease has less end leakage.
- 3 Speed affects the pressure distribution by shifting both leading and trailing edges of the grease film, but has little other effect.

The authors hope that these experiments will be of use as a basis for scientific analysis and will lead in time to a development of a theory of grease lubrication.

ACKNOWLEDGMENTS

This paper is based on data contained in two theses presented by the authors in partial fulfillment of the requirements for the degree of Master of Science at The Pennsylvania State College in June, 1947, and August, 1948, respectively.

The authors wish to express their appreciation to Prof. L. J. Bradford under whose supervision this work was carried out, and to The Texas Company for facilitating this investigation.

Discussion

MAYO D. HERSEY.⁷ A few notes on plastic lubrication have been published by A. G. M. Michell⁸ and the writer.⁹ Michell concluded that the performance of a grease-lubricated bearing would be similar to that of an oil-lubricated bearing but "less efficient." The yield-shear stress of the grease would oppose the motion with a force of constant magnitude in addition to the hydrodynamic stresses. The writer's study, based upon dimensional analysis, was prompted by his experiments of an early date at the Massachusetts Institute of Technology. Tests using a plastic lubricant gave an apparent intercept of 0.015 on the coefficient-of-friction axis, as compared with 0.002, using Newtonian oils. Both types of lubricants gave the same slope when plotting against Z/NP , where Z is the mobility reciprocal, N the speed, and P the load.

The authors' observations on film pressure are of special value in supplementing von Schroeter's experiments,¹⁰ which were limited to friction measurements. Their discovery that the pressure distribution differs from that in an oil-lubricated bearing does not seem to confirm Michell's theory, and thus invites tentative explanations. Do the authors find that the film remains uniformly plastic or that the oil separates out? Would it be possible to identify their grease more completely by giving approximate values for its yield-shear stress and mobility, as von Schroeter did? Or better still, determining its true rate-of-shear against shear-stress curve by the Weissenberg-Rabinowitsch method, as done by Norton, Knott, and Muenger,¹¹ and by Blott and Samuel.¹² It is to be hoped that this interesting paper may form, just one part of a more comprehensive investigation.

P. M. STAFFORD.¹³ The authors have established the applicability of the hydrodynamic-film principle to grease-lubricated journal bearings with copious feed. This fact may be considered an intermediate step in an attempt to determine the suitability of grease as a lubricant for journal-bearing applications. It appears that the tests were made under condition of low speed and very light load.

Indications are that more stringent operating conditions, particularly high-speed operation would result in higher working temperatures than for corresponding oil-film lubrication, and the problem of heat dissipation is further aggravated by the inherent

⁷ Research Associate in Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass. Fellow ASME.

⁸ "Progress of Fluid Film Lubrication," by A. G. M. Michell, Trans. ASME, vol. 51, 1929, MSP-51-21, p. 157.

⁹ "Theory of Lubrication," by M. D. Hersey, John Wiley & Sons, New York, N. Y., 1936, pp. 91-92.

¹⁰ "Die Schmierung von Gleitlagern mit konsistenten Fetten" (Lubrication of Plain Bearings With Plastic Greases), by von Hellmuth von Schroeter, *Petroleum Zeitschrift*, vol. 30, no. 15, 1934, pp. 1-20.

¹¹ "Flow Properties of Lubricants Under High Pressure," by A. E. Norton, M. J. Knott, and J. R. Muenger, Trans. ASME, vol. 63, 1941, pp. 631-643.

¹² "Flow Characteristic of Lime-Base Greases," by J. F. T. Blott and D. L. Samuel, *Industrial and Engineering Chemistry*, vol. 32, 1940, pp. 68-72.

¹³ Associate Professor of Mechanical Engineering, McGill University, Montreal, Que., Can.

lack of fluidity of the grease, compared with even the higher viscosity oils.

There is no indication of comparative surface condition of the mating surfaces for oil and grease lubrication and the writer understands from the authors that the duration of the test was not sufficient to make such observations. Comparative figures of wear for various operating conditions and degree of surface finish could be obtained in the same series of tests. Sommerfeld's curve relating the coefficient of friction to a combination of viscosity, speed, load, and clearance ratio would be expected to hold. The lower the viscosity of the grease and the larger the radial clearance—which, presumably is required—the greater will be the reduction of Sommerfeld's constant, thus reducing the coefficient of friction in the stable zone, but increasing it very rapidly in the boundary zone.

Being fully aware of the fact that the authors have limited their study to the film-pressure distribution, the writer suggests that the points mentioned are sufficiently important to be included in subsequent investigations, thus presenting a more complete picture of the problem at hand.

The writer shares the authors' hope that their experiment may be used as a basis for a scientific analysis of the problems involved, but, rather than leading to a theory of grease lubrication, it is suggested that the existing theory of hydrodynamic-film lubrication can be extended to cover grease lubrication.

AUTHORS' CLOSURE

It is very gratifying that Messrs. Hersey and Stafford agree with the authors' position that this preliminary investigation should be followed up by a more extensive study. The several suggestions for further parameters are appreciated.

Fig. 8 of this closure shows the "apparent" rate-of-shear against

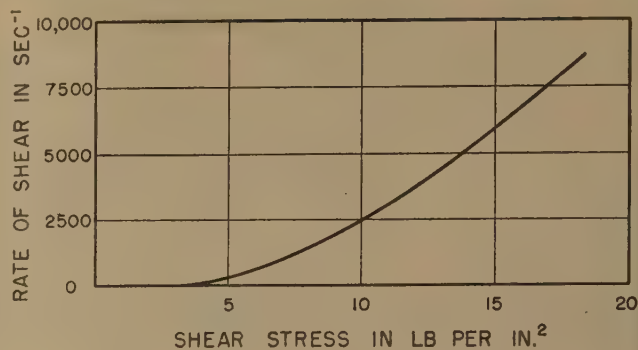


FIG. 8 RATE-OF-SHEAR VERSUS SHEAR-STRESS
(For grease at 77 deg F.)

shear-stress curve.¹³ The rate is "apparent" because of the "plug" effect which exists in the capillary-flow-type viscosimeter. While the film did not remain uniformly plastic, measurements were not made to determine whether or not the oil separation exceeded that found in the fresh grease.

Since the coefficient of friction was not measured, Michell's statement pertaining to efficiency will not be considered. Furthermore, his theory of identical film-pressure distribution for oil and grease presupposes the same viscosity. When tested, these two lubricants undoubtedly operated at different viscosities as there was no provision for the control of film temperature.

No attempt has been made to analyze the results of this fundamental study. It is hoped that a future paper will correlate experiment with the theory of grease lubrication in detail.

¹⁴ From data supplied by The Texas Company.

Viscosities and Densities of Lubricating Fluids From —40 to 700 F

By C. M. MURPHY,¹ J. B. ROMANS,² AND W. A. ZISMAN³

The viscosities and densities of a large group of high-boiling synthetic and petroleum fluids were investigated over a wide range of temperatures. These were selected because of their promise for use at extreme temperatures or their ability to supply fundamental data for viscometry and lubrication. The variation of density with temperature was practically linear over the range investigated. The ASTM viscosity-temperature chart, extended from —100 to 700 F for this work, proved very satisfactory for analysis of the data obtained. Deviations from linearity at the extreme temperatures were studied and classified and nearly all could be explained on the basis of certain physical or chemical transformations shown to occur at the extreme temperatures. The linear portions of the graphs are explained qualitatively in terms of the ability of many types of linear molecules to coil as helices. Convexity at high temperatures is believed general, caused by the disappearance of the helical configuration under conditions of violent thermal agitation.

INTRODUCTION

THE viscosity is generally considered the most important single characteristic of a lubricating fluid. There is a dearth of information as to the viscosities of petroleum and synthetic oils at high temperatures such as may be encountered in gas turbines and turbojet engines. Without such data it is impossible to ascertain the reliability of the many empirical or theoretical equations proposed for relating viscosity with temperature.

Controversy exists as to whether the absolute viscosity η , or the kinematic viscosity ($\mu = \eta/\rho$, where ρ = density) of a lubricant is the more significant in many applications. Since at ordinary temperatures the densities of a large number of petroleum oils do not differ greatly from unity, their kinematic and absolute viscosities do not differ significantly. Because the density of fluids usually decreases with increasing temperature, the difference in magnitude between the kinematic and absolute viscosities of petroleum oils may be considerable at high temperatures, and even greater differences will be observed in denser fluids such as perfluorocarbons. Furthermore, since the temperature coefficients of viscosity and density are not comparable, the coefficients of the absolute and kinematic viscosities may be very different.

FLUIDS INVESTIGATED

A variety of synthetic and conventional mineral-base oils were

studied over the temperature range of —40 to 700 F. The silicones studied include commercial samples of the polymethyl-, polyethyl-, and poly(methyl-phenyl) siloxanes (1, 2, 3, 4).⁴ The poly(methyl-phenyl) siloxanes arranged in the increasing order of aromaticity (5) are DC 550, DC 703, and DC 702. Infrared spectra of the DC 710 fluid have not been obtained, but from other properties it is believed to have a higher phenyl to methyl ratio than DC 550.

The polymeric ethylene glycols were obtained from Carbide and Carbon Chemicals Corporation (6), and the polymers of 1, 2-propanediol from the Dow Chemical Company (7). The latter glycols have methyl branches, while the former have not. Similar polymers are the "Ucon" fluids (8, 9, 10, 11, 12), the LB series fluids being derivatives of 1, 2-propanediol, while the HB series is derived from copolymers of ethylene glycol and 1, 2-propanediol. Measurements on the Ucon fluids were made with and without the recommended antioxidant.

Petroleum oils of high and low viscosity index (VI) were also studied. The low VI oils (Navy Symbol 2135 and 2250) (13) were procured from the Socony-Vacuum Oil Company and The Texas Company, respectively, and the high VI oils (Grades 1065 and 1120 of Specification AN-O-8) (14) from The Texas Company. Diester oils were represented by the *n*-butyl, 2-ethylhexyl and 1-methyl-4-ethyloctyl (or undecyl) esters of sebacic acid (15). A fraction of a fully fluorinated petroleum oil (designated as Fluoro Lubricant FCD-331), which has a boiling range of 266 to 302 F at 10 mm (16), was procured from E. I. du Pont de Nemours & Company. Two polymers of monochlorotrifluoroethylene (17, 18, 19, 20) (designated as Fluorolube-Standard and Fluorolube-Light Grease) were obtained from Hooker Electrochemical Company.

EXPERIMENTAL METHODS

In order to minimize difficulties with nonreproducibility at high temperatures, all of the fluids were stripped of volatiles by passing them dropwise from a reservoir through a 2-cm-diam glass column filled to a height of 50 cm with glass helices. The column was maintained at 220 F, and the pressure within the system was 5 mm mercury. A countercurrent of dry carbon dioxide swept the volatile products from the system into a trap immersed in a dry ice acetone mixture. Because of excessive volatility it was necessary to strip the halocarbons at 165 F. Several of the silicones were slightly turbid as received and were clarified with activated Fuller's earth.

Densities were determined in a dilatometer of approximately 12 cc capacity made by sealing to a small Erlenmeyer flask a precision-bore capillary of approximately 2.25 mm diam. Calibrations were made at 77 F with mercury, and all weights in air were reduced to weights in vacuo (21). At the other temperatures from —40 to 700 F, the volumes were calculated from the coefficient of cubical expansion of "Pyrex" glass ($5.33 \times 10^{-6}/\text{deg F}$). An excess of the fluid to be investigated was introduced into the reservoir, the system was evacuated to a pressure of 5 mm of mercury, and the oil was heated to facilitate the removal

⁴ Numbers in parentheses refer to the Bibliography at the end of the paper.

¹ Senior Chemist, Lubrication Section, Naval Research Laboratory, Washington, D. C.

² Associate Chemist, Lubrication Section, Naval Research Laboratory.

³ Head, Lubrication Section, Naval Research Laboratory. Mem. ASME.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society. Paper No. 48—A-147.

of dissolved and entrained air. When foaming stopped, the reservoir was inverted and the dilatometer filled with the liquid to a height which would insure oil being in the capillary at the lowest temperature. Runs on all fluids were begun at the lowest temperatures to reduce drainage errors to a minimum at the higher temperatures. Determinations were made at increasing temperature increments until there was evidence of decomposition or the vapor pressure was so great that liquid condensed on the cooler parts of the capillary. Densities were calculated from the weight of the fluid in vacuo, and the values for intermediate temperatures were obtained by graphical interpolation.

All kinematic viscosities were determined in Cannon-Fenske modified Ostwald type viscometers according to ASTM Method D 445-46T (22), and Federal Standard Method 30.5 (23). Conventional water thermostats which maintained the bath temperature within ± 0.05 deg F of the desired temperature were used over the range of 77 F to 210 F. The low-temperature determinations were made in a Precision Scientific Company bath using "Methyl Cellosolve" for the bath liquid and dry ice as the coolant. The temperature, controlled within ± 0.05 deg F, was measured by the emf from a calibrated 6-junction thermopile. The high-temperature bath was made of Pyrex glass, insulated with 2 in. of glass-fiber cloth, while the bath liquid was DC 550 silicone fluid. Two double windows permitted observation of the instruments. Except for the use of a 3-junction iron-constantan thermopile, the same temperature-measuring and control system was used.

Each viscometer was calibrated at 100 F with two reference fluids obtained from the National Bureau of Standards, and the calibration constants agreed within 0.1 per cent. As the viscometers were filled at room temperature it was necessary to apply a temperature correction factor for tests at other temperatures, since the hydrostatic head of liquid will vary with the temperature and the coefficient of expansion of the liquid (22, 23). The temperature correction factors were calculated for each temperature interval, and the values are given in Table 1. From -40 to 210 F the factors agreed within 0.1 per cent for all fluids, but divergence occurred at higher temperatures. Petroleum oils had the smallest and halocarbons the greatest temperature correction factors.

TABLE 1 TEMPERATURE CORRECTION FACTORS

Temp, deg F	Petro- leum oils	Ucons and poly- glycols	Silicones and diesters	Fluoro- lube standard	Fluoro lubricant FCD-331
-40	1.006	1.005	1.006	...	...
-20	1.005	1.005	1.005	...	...
0	1.004	1.004	1.004	...	...
32	1.003	1.003	1.002	1.003	1.003
77	1.001	1.001	1.001	1.001	1.001
210	0.995	0.995	0.995	0.994	0.994
250	0.993	0.993	0.993	0.992	0.991
300	0.991	0.990	0.990	0.989	0.988
350	0.989	0.988	0.987	0.986	0.984
400	0.986	0.985	0.984	0.983	0.980
450	0.984	0.982	0.981	0.979	0.976
500	0.981	0.979	0.977	0.975	0.970
550	0.978	0.975	0.974	...	...
600	0.975	0.972	0.970	...	...
650	0.972	...	0.967	...	...
700	0.967	...	0.963	...	...

The calibration constant is also slightly affected by the surface tension of liquids, and it is necessary to apply a correction for the differences in effective head caused by differences in surface tension (23). Barr (24) has discussed the surface-tension corrections and has shown that a correction factor for the change in head due to capillarity must be applied for any but small capillaries. The correction factor depends on the form of the meniscus. The petroleum calibrating liquids had surface tensions of 31.5 ± 1 dynes per cm at 77 F, Ucon fluids and the polyglycols had values of 30 to 32 dynes per cm, and the diesters had values of

29 to 30 dynes per cm. However, the densities of these liquids are all sufficiently close to unity to make the surface-tension corrections less than 0.1 per cent.

It is well known that the surface tensions of most liquids vary linearly with the temperature until close to the critical temperature (25). Winchester and Reber (26) found that the temperature coefficient of surface tension of several petroleum oils between 180 and 600 F was constant, the average value being 0.033 dynes/cm/deg F, and the maximum variation from the mean was 0.0017 dynes/cm/deg F. Unpublished investigations of the authors' laboratory reveal that temperature coefficients of surface tension of the Ucons, polyglycols, and diesters are approximately 0.03 dynes/cm/deg F. Differences between the surface tensions of the petroleum oils and the fluids mentioned will remain practically constant since the surface tensions are decreasing at the same rate. Corrections due to surface-tension differences, therefore, will become even smaller at the elevated temperatures.

Surface tensions of the other fluids investigated, at 77 F, were approximately 23 dynes per cm for the halocarbons, 19.5-21.0 for the polymethylsiloxanes, 23 to 25 for the polyethylsiloxanes, and from 20 to 26 for the poly(methyl-phenyl) siloxanes (5), depending upon the phenyl to methyl ratio. Correcting for the meniscus, the surface-tension corrections become 0.2 to 0.4 per cent for the silicones and halocarbons, the latter having the largest values. As the temperature coefficients of surface tension (5) are approximately 0.037/deg F for polymethylsiloxanes, 0.036/deg F for polyethylsiloxanes, 0.044/deg F for the poly(methyl-phenyl) siloxanes, and 0.027/deg F for the halocarbons, appropriate corrections can be made at temperatures other than 77 F. The coefficients of surface tension with temperature of the halocarbons were not linear, and it is possible that other fluids also deviated from linearity at high temperatures. This would not introduce an appreciable error in the calculated corrections as the differences in surface tension become smaller at the elevated temperatures.

Viscosities were determined at -40 , -20 , 0, 32, 100, 130, 210, 250 F, and at 50-deg F increments thereafter up to temperatures where reproducible results could not be obtained. Above 400 F it was impossible to obtain efflux times reproducible within 0.2 per cent with some fluids. Viscometers were selected so that the kinetic energy corrections were negligible except at very high temperatures, where some of the fluids had efflux times less than the minimum (23). In order to minimize the effect of atmospheric oxygen, the bath was blanketed with carbon dioxide, discharged from a funnel inverted over the top of the bath, thus permitting the viscometers to breathe an inert atmosphere. Prior to filling, the viscometers were swept out with carbon dioxide.

DISCUSSION OF RESULTS

Densities. The densities of the fluids investigated are accurate to ± 0.1 per cent and are arranged in Table 2 in order of increasing viscosities for each group of homologous fluids. Graphs of the density-temperature data can be fitted to straight lines without an error of over 0.2 per cent for the range of temperatures given in the second from the last column, and the resulting slope $-(d\rho)/(dt) \times 10^4$ is given in the preceding column. The cubical coefficient of expansion at 77 F was calculated from $(d\rho)/(dt)$ and the density at 77 F.

Except for the halocarbons the density-temperature graphs deviate from linearity only very slightly at the extreme temperatures. The graph for Fluoro Lubricant FCD-331 is not linear anywhere, but the two polymers of trifluoromonoethylethylene behaved linearly up to 300 and 450 F, respectively. In general, $(d\rho)/(dt)$ and the cubical coefficient of expansion decrease as the viscosity increases in any homologous series of fluids. The latter

TABLE 2 DENSITIES OF FLUIDS

Identification	Density, Grams/cc. at °F.				-(dp/dt)x10 ⁴	Applicable Range, °F.	Coefficient* of Expansion x 10 ⁴
	-40	77	210	550			
SILICONES							
Polymethylsiloxanes							
DC 500 "A"	1.011	0.950	0.886	0.720	4.86	32 to 600	5.12
DC 500 "B"	1.022	0.962	0.898	0.737	4.74	32 to 600	4.93
DC 500 "C"	1.026	0.965	0.902	0.743	4.71	32 to 600	4.88
DC 200 "D"	1.030	0.969	0.906	0.749	4.67	32 to 600	4.82
DC 200 "E"	1.031	0.970	0.908	0.751	4.65	32 to 600	4.79
Polyethylsiloxanes							
DC 400 "A"	1.003	0.952	0.895	0.792 _e	4.31	-40 to 450	4.53
DC 400 "C"	1.042	0.994	0.941	0.825 _f	4.02	-40 to 500	4.04
Poly(methyl-phenyl)siloxanes							
DC 702	1.107 _b	1.071	1.008	0.845	4.74	0 to 500	4.43
DC 703	1.117 _b	1.081	1.022	0.869	4.52	0 to 500	4.18
DC 710	1.125 _b	1.092	1.037	0.898	4.13	0 to 550	3.78
DC 550	1.104 _b	1.070	1.016	0.876	4.09	77 to 600	3.82
POLYALKYLENE GLYCOLS AND DERIVATIVES							
Polyethylene Glycols							
Polyethylene Glycol 200	1.175 _a	1.122	1.062	0.895	4.50	-40 to 300	4.01
Carbowax 1000		1.098 _d	1.064	0.937 _f	4.33	130 to 400	3.94
Polypropylene Glycols							
Polyglycol P-400	1.053 _a	1.007	0.945	0.784	4.69	-20 to 400	4.66
Polyglycol P-750	1.047 _a	1.004	0.945	0.794	4.44	-20 to 550	4.42
Polyglycol P-1200	1.045 _a	1.002	0.944	0.798	4.33	-20 to 550	4.32
Polypropylene Glycol Derivatives							
Ucon LB-100	1.022	0.970	0.911	0.762	4.41	-40 to 550	4.55
Ucon LB-100-X	1.027	0.975	0.917	0.767	4.41	-40 to 550	4.52
Ucon LB-250	1.042	0.992	0.935	0.790	4.25	0 to 550	4.28
Ucon LB-250-X	1.048	0.996	0.939	0.794	4.27	0 to 550	4.29
Ucon LB-550-X	1.037 _b	0.994	0.939	0.797	4.19	77 to 550	4.22
Poly(ethylene-propylene) Glycol Derivatives							
Ucon HB-100-X	1.070	1.017	0.958	0.805	4.47	-40 to 450	4.40
Ucon HB-260	1.086 _a	1.033	0.976	0.828	4.34	32 to 550	4.20
Ucon HB-260-X	1.090 _a	1.036	0.979	0.830	4.36	32 to 550	4.21
Ucon HB-600-X	1.095 _a	1.051	0.994	0.849	4.27	77 to 550	4.06
PETROLEUM OILS							
Navy Symbol 2135	0.937 _b	0.909	0.861	0.736	3.64	0 to 500	4.00
Navy Symbol 2250	0.954 _c	0.937	0.890	0.765	3.61	32 to 500	3.85
Grade 1065	0.890 _c	0.874	0.828	0.709	3.49	32 to 550	3.99
Grade 1120	0.903 _c	0.887	0.841	0.725	3.44	32 to 700	3.88
DIESTER OILS							
Di(n-butyl) sebacate	0.950 _c	0.932	0.874	0.722	4.35	32 to 450	4.67
Di(2-ethylhexyl) sebacate	0.962	0.913	0.860	0.719	4.09	-40 to 500	4.48
Di(undecyl) sebacate	0.942	0.896	0.844	0.708	3.92	-40 to 450	4.38
HALOCARBONS							
Fluoro Lubricant FCD-331	2.083 _c	2.040	1.903	1.561 _f	11.15	(g)	5.47
Fluorolube - Standard	1.994 _c	1.953	1.832	1.550 _f	9.07	32 to 300	4.64
Fluorolube - Light Grease		1.975 _d	1.898	1.594	8.91	130 to 450	4.51
a - Density at -20°F.							
b - Density at 0°F.							
c - Density at 32°F.							
d - Density at 130°F.							
e - Density at 450°F.							
f - Density at 500°F.							
g - Average value; graph curves							

a - Density at -20°F.
b - Density at 0°F.
c - Density at 32°F.
d - Density at 130°F.

e - Density at 450°F.
f - Density at 500°F.
g - Average value; graph curves

* At 77 F.

is highest for the Fluoro Lubricant FCD-331 and the polymethylsiloxane of lowest viscosity, and it is least for the petroleum oils and the most aromatic silicones.

The densities of the diesters agree within the limits set by the lesser accuracy of our earlier work (15), while those for the silicones and polyglycols and derivatives agree well over the comparable temperature range with values already reported (4, 6, 7, 9, 12, 27). The deviations from linearity of the density versus temperature graphs obtained with the petroleum oils are not necessarily in contradiction to the results reported by Winchester and Reber (26), since their oils were very viscous fractions having viscosities above 30 centipoises at 210 F.

Kinematic Viscosities. Viscosities are given in Table 3, and all values reported from -40 to 400 F are believed accurate to within 0.2 per cent relative to the accepted value of water at 68 F (1.007 centistokes). At temperatures above 400 F, the viscosities of some of the fluids were not reproducible with this precision and are so indicated by the number of significant figures in the value reported.

Due to gel formation and pyrolysis within the 30 to 40 min required for measurements, the viscosities of the polymethylsiloxanes could not be determined accurately above 550 F. Turbidity was noticed at 500 F, but it decreased as the fluid was cooled to room temperature. It became pronounced at 550 F, and the phenomenon was not reversible since a slight turbidity was apparent when the fluid was cooled. Such turbidity effects are attributed to the precipitation of higher polymers formed at the high temperatures (28). Evidence that cracking took place (28) at 550 F was revealed by a 1 per cent decrease in the viscosity at 250 F. Polyethylsiloxanes A and C became cloudy at temperatures below zero, which is believed due to the precipitation of high polymers. Gel formation was evident at 450 F, while at 500 F it fouled the capillaries. The poly(methyl-phenyl) siloxanes become cloudy at 0 F, the turbidity increasing with the aromaticity. These fluids were the most heat-stable of the silicones, and it was possible to measure viscosities at 700 F on all but DC 710. No clouding at high temperatures was observed with any of these fluids, and only DC-710 formed gel below 700 F.

A reversible color change was exhibited by the inhibited Ucon fluids which became marked at around 500 F. Around 300 to 350 F viscosity decreases with time of 0.2 per cent to 0.3 per cent were observed on the uninhibited Ucon fluids but this disappeared at higher temperatures. The same effect was observed at about 50 deg F higher with the inhibited fluids. No difficulties with clouding or precipitation were encountered at low temperatures with glycols or Ucon fluids but vapor-locking difficulties limited going to higher temperatures.

The petroleum oils caused lacquering above the liquid level at temperatures above 500 F, but it was possible to determine the viscosities at 700 F on all oils except Navy Symbol 2250 which vapor-locked at that temperature. None of the petroleum oils was studied below the cloud points. Dibutyl sebacate vapor-locked at 600 F as did the di(2-ethylhexyl) sebacate at 650 F. Diundecyl sebacate was changing chemically at 550 F, for the viscosity increased 1 per cent with each pass through the viscometer. Nothing unusual occurred with these diesters at low temperatures. It was impossible to obtain viscosity measurements on Fluoro Lubricant FCD-331 above 500 F, because it vapor-locked in the viscometer. Fluorolube "Standard" and "Light Grease" decomposed at 550 F and 600 F, respectively, as evidenced by increases in times of efflux for each pass through the viscometer.

Evans (29) investigated a number of equations relating the change of viscosity with temperature as applied to hydrocarbons and petroleum oils. He concluded that the three outstanding

equations were the ASTM equation, a modification of the Andrade and Silverman equation, and a modification of Batschinski's equation. Though empirical, the ASTM equation fits the data for hydrocarbons with remarkable accuracy over the rather narrow temperature range investigated (32 to 212 F) and has only two arbitrary constants. The other two equations, though having some theoretical justification, contain three arbitrary constants, making them tedious and unwieldy to use. As a large number of theoretical equations, attempting to relate viscosity with temperature, refer to the absolute viscosity and contain a term which is a function of the molecular weight, they are not readily adaptable for work with most lubricating fluids.

The ASTM equation in the chart form of Method D 341-43 (22) is used widely for estimating the viscosities of petroleum oils at temperatures from 300 F to the cloud point, and it has been used with success for synthetic oils such as silicones (4), Ucons (12), and diesters (15). The well-known Walther equation is as follows

$$\log \log (\mu + c) = A \log T + B$$

where μ is the viscosity in centistokes, c is a constant which Walther evaluated for petroleum oil as 0.95, A and B are constants characteristic of each liquid, and T is the absolute temperature. The ASTM equation and chart differ only in that c is a constant varying from 0.75 at 0.4 centistokes to 0.6 at 1.5 centistokes and above. The use of this equation in chart form obviates the necessity of determining the constants A and B .

As the ASTM chart does not cover the temperature range of this investigation, it was necessary to combine the C and D charts to include the viscometric data and extend the range at both the high and low temperatures. Extended charts were printed on the same scale as the ASTM chart by the Nautical Chart Branch of the Coast and Geodetic Survey.

Other choices of c in the Walther equation have been made. Values varying from 0.6 to 0.95 have been introduced into the equation to extend the viscosity range of the chart below 1 centistoke, while the particular value assigned was determined by the ability of the equation to predict the viscosity of the fluids of interest to the investigators. In general, the larger values of the constant c produce more linear graphs in the low-viscosity region. Objections to the choice of a varying term c by the ASTM in its chart (D341-39) have been raised. No difference in the dimensions of this chart and one based on a value of c equal to 0.6 can be observed above 1.5 centistokes, but below 1 centistoke the differences become increasingly great, and at 0.4 centistoke the latter chart extends to minus infinity. When c has a value of 0.75, the dimensions of this chart do not differ significantly from that of the ASTM chart above 3 centistokes, but it is approximately 16.5 per cent smaller over the range of 3 to 0.4 centistoke.

The data from Table 3 were plotted on these extended ASTM charts, and the resulting curves will be found in Figs. 1 through 4. Outstanding features of these graphs are listed and compared in Table 4 which also gives the approximate temperature limits for the essentially linear region of each graph. With the exception of Fluorolube-Standard the graphs of all fluids are linear for a temperature interval varying from 140 to 450 F. In any homologous series of fluids this interval increases with the viscosity (or average molecular weight). In the last column of Table 4 brief remarks are given covering the nature and amount of the deviation from linearity at the extremes of the curves.

There is no curvature in the low-temperature region, and the graphs are convex at high temperatures for the polymethylsiloxanes, petroleum oils, and diesters. This behavior of petroleum oils at high temperatures agrees with the only available other work (30). The polyglycols and the lower-viscosity Ucon

TABLE 3 KINEMATIC VISCOSITIES OF FLUIDS

Identification	Viscosity, Centistokes °F															Kinematic V. I.		
	-40	-20	0	32	100	130	210	250	300	350	400	450	500	550	600		650	700
SILICONES																		
Polymethylsiloxanes																		
DC 500 "A"	119.8c	78.8	56.9	35.8	17.11	13.0a	7.16	5.54	4.16	3.20	2.52	2.028	1.641	1.358				207
DC 500 "B"	329. c	215. a	152.8	94. a	44.6	33.8a	18.21	13.98	10.40	7.93	6.19	4.90	3.96	3.22				169
DC 500 "C"	564. c	360. a	259.	160. a	74.7	56. a	30.2	23.28	17.20	13.07	10.15	8.09	6.48	5.26				155
DC 200 "B"	1320. c	840. a	608.	375. a	173.7	130. a	69.9	53.4	39.7	30.1	23.46	18.56	15.01	12.3				139
DC 200 "C"	2144. c	1380. a	980.	610. a	279.	208. a	111.8	85.5	63.1	47.9	36.9	29.0	23.19	17.7	11.			
Polymethylsiloxanes																		
DC 400 "A"	360. c	178. a	100.3	65.7	35.51	9.16	4.09	3.00	2.178	1.613	1.309	1.035						218
DC 400 "B"	5460. c	2100. a	960.	296.	151.7	30.7	11.52	7.92	5.35	3.78	2.80	2.15						159
DC 400 "C"	8130. c	3200. a	1182.	493.	98.6	62. a	22.95	15.56	10.26	7.05	5.28	3.84						152
Poly(methyl-phenyl)siloxanes																		
DC 702			657. c	113.8	17.29	10.2	4.08	2.90	2.046	1.522	1.176	0.940	0.765	0.630	0.532	0.446	0.38	199
DC 703			2600. c	308.	34.2	19.3	6.84	4.74	3.25	2.377	1.865	1.426	1.111	0.935	0.780	0.65	0.55	152
DC 710			18300. c	1555	122.0	68.9	21.05	14.50	9.56	6.84	5.17	3.97	3.15	2.6				144
DC 550			2960.	612	95.4	56.2	20.68	14.34	9.77	6.99	5.22	4.01	3.16	2.53	2.066	1.698	1.42	152
POLYALKYLENE GLYCOLS and DERIVATIVES																		
Polyethylene Glycols																		
Polyethylene Glycol 200	29600.	4780.	1164.	221.0	25.5	13.69	4.34	2.88	1.922	1.376	1.044	0.828	0.674	0.561	0.48			75
Carbowax 1000						66.4	18.43	12.14	7.99	5.66	4.36	3.46	3.0					
Polypropylene Glycols																		
Polyglycol P-400	21920.		3780.	464.	33.5	16.30	4.64	2.99	1.985	1.431	1.084	0.862	0.701	0.581				26
Polyglycol P-750	32300.		5280.	681.	51.6	25.5	7.33	4.75	3.15	2.228	1.709	1.339	1.090	0.88				111
Polyglycol P-1200	44100.		7170.	930.	73.4	36.7	10.77	7.00	4.84	3.30	2.493	1.95	1.62	1.28				130
Polypropylene Glycol Derivatives																		
Ucon LB-100	12000.	2269.	623.	136.3	19.59	11.3a	4.16	2.91	2.040	1.545	1.191	0.99	0.80	0.67				124
Ucon LB-100-X	16570.		760.	158.3	20.87	12.1a	4.23	2.94	2.070	1.555	1.217	0.985	0.80	0.67				125
Ucon LB-250			2422.	456.	53.9	30.1a	9.97	6.78	4.69	3.41	2.63	2.08	1.68	1.44				128
Ucon LB-250-X			3010.	534.	57.5	30.8a	10.13	6.86	4.68	3.43	2.63	2.102	1.69	1.5				144
Ucon LB-550-X			9410.	1359.	121.7	63. a	18.90	12.53	8.35	5.85	4.50	3.53	2.87	2.3				140
Poly(ethylene-propylene) Glycol Derivatives																		
Ucon EB-100-X	1976.		554.	110.0	21.72	12.6a	4.77	3.36	2.381	1.784	1.411	1.110	0.948	0.799				156
Ucon EB-260	6500.		1667.	362.	49.7	29. a	10.00	6.93	4.75	3.53	2.74	2.21	1.79	1.55				154
Ucon EB-260-X	7800.		1966.	406.	52.3	29.5a	10.10	6.96	4.77	3.54	2.74	2.19	1.82	1.55				151
Ucon EB-600-X	31000.		6900.	1226.	134.2	71. a	23.14	15.61	10.41	7.58	5.73	4.54	3.67	3.2				143
PETROLEUM OILS																		
Navy Symbol 2135			15010.		68.5	28.8a	6.70	4.17	2.62	1.829	1.358	1.057	0.846	0.698	0.583	0.496	0.42	29
Navy Symbol 2250			8080.		159.1	62. a	9.77	5.52	3.25	2.166	1.586	1.207	0.953	0.775	0.643	0.542	0.46	19
Grade 1065			1748.		120.6	53. a	12.37	7.49	4.60	3.11	2.259	1.728	1.366	1.113	0.923	0.780	0.66	101
Grade 1120			8800.		369.	140. a	24.69	13.78	7.72	4.92	3.44	2.52	1.940	1.544	1.263	1.05	0.90	95
DIESTER OILS																		
Di(n-butyl)sebacate			19.63		6.10	4.27	2.085	1.593	1.213	0.953	0.773	0.640	0.543	0.461				151
Di(2-ethylhexyl)sebacate	1410.	1445. a	187.	60.9	12.61	7.96	3.32	2.406	1.702	1.353	1.054	0.862	0.716	0.609	0.515			154
ndecylsebacate	8990.	2199.	706.	161.7	23.03	13.9	4.68	3.26	2.285	1.697	1.344	1.067	0.865				138	
HALOCARBONS																		
Fluoro Lubricant FCD-331			5210. c		49.0	17.16	3.28	1.916	1.168	0.788	0.562	0.423	0.334					-694
Fluorolube - Standard			2690. c		86.9	31.7	5.61	3.20	1.886	1.221	0.869	0.650	0.51					-172
Fluorolube - Light Grease						72.0	26.6	10.53	5.30	3.11	2.059	1.446	1.074					

a Interpolated value.

c Sample cloudy.

FIG. 2 POLYGLYCOLS AND DERIVATIVES
(Viscosity-temperature charts modified from
ASTM, D 341-39.)

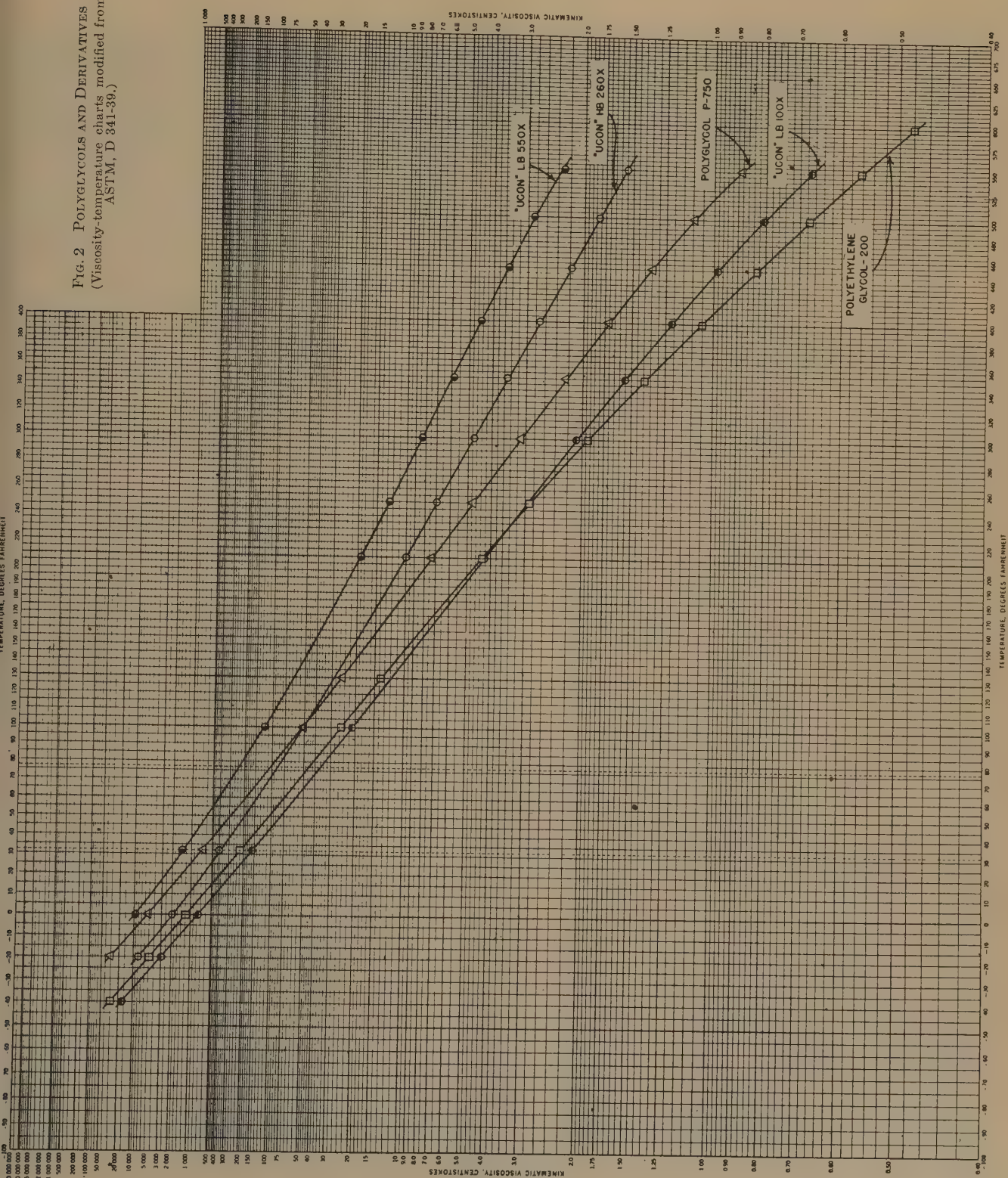


FIG. 3 PETROLEUM OILS AND DIESTERS
(Viscosity-temperature charts modified from
ASTM, D 341-39.)

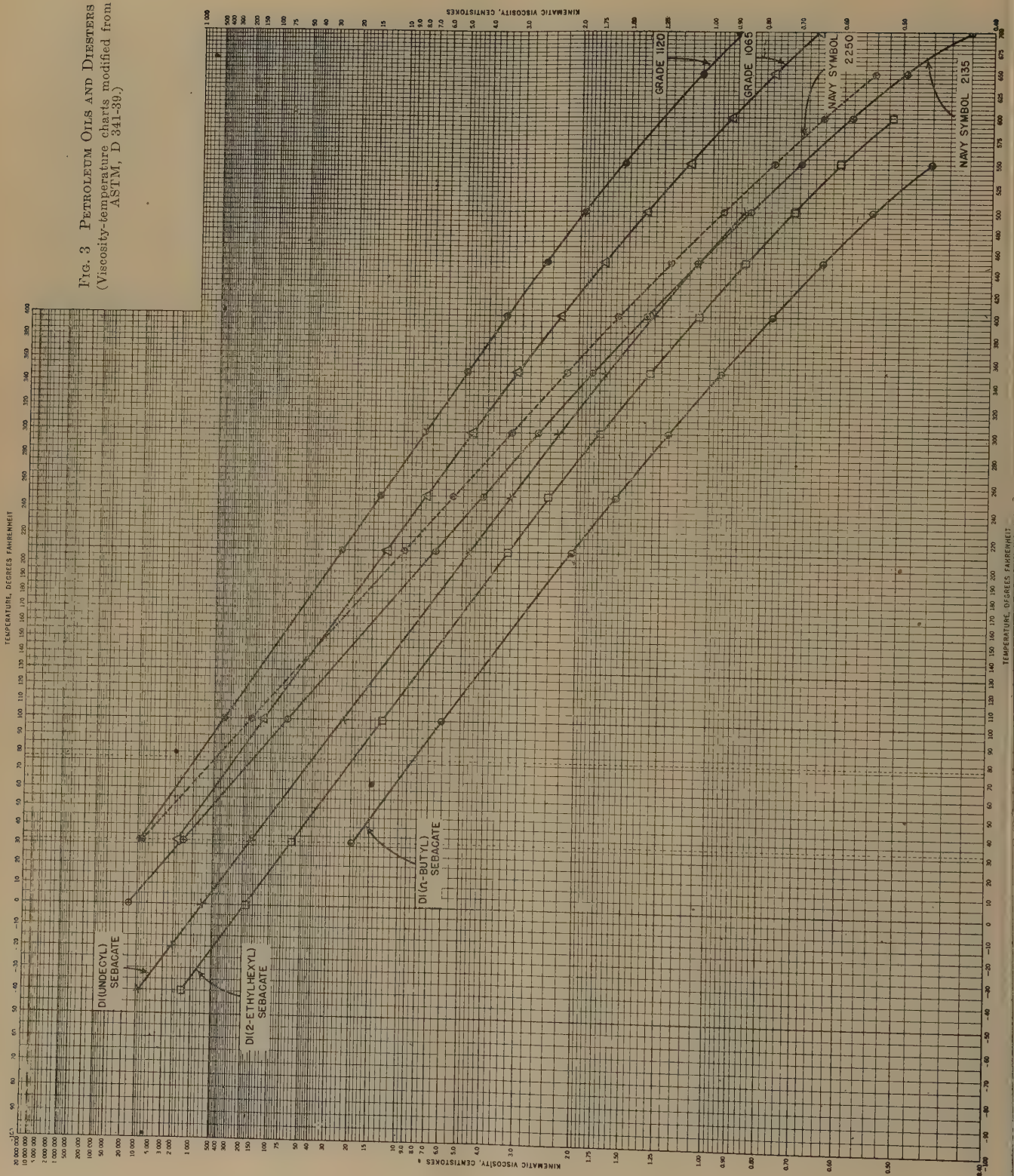


FIG. 4 HALOCARBONS
(Viscosity-temperature charts modified from
ASTM, D 341-39.)

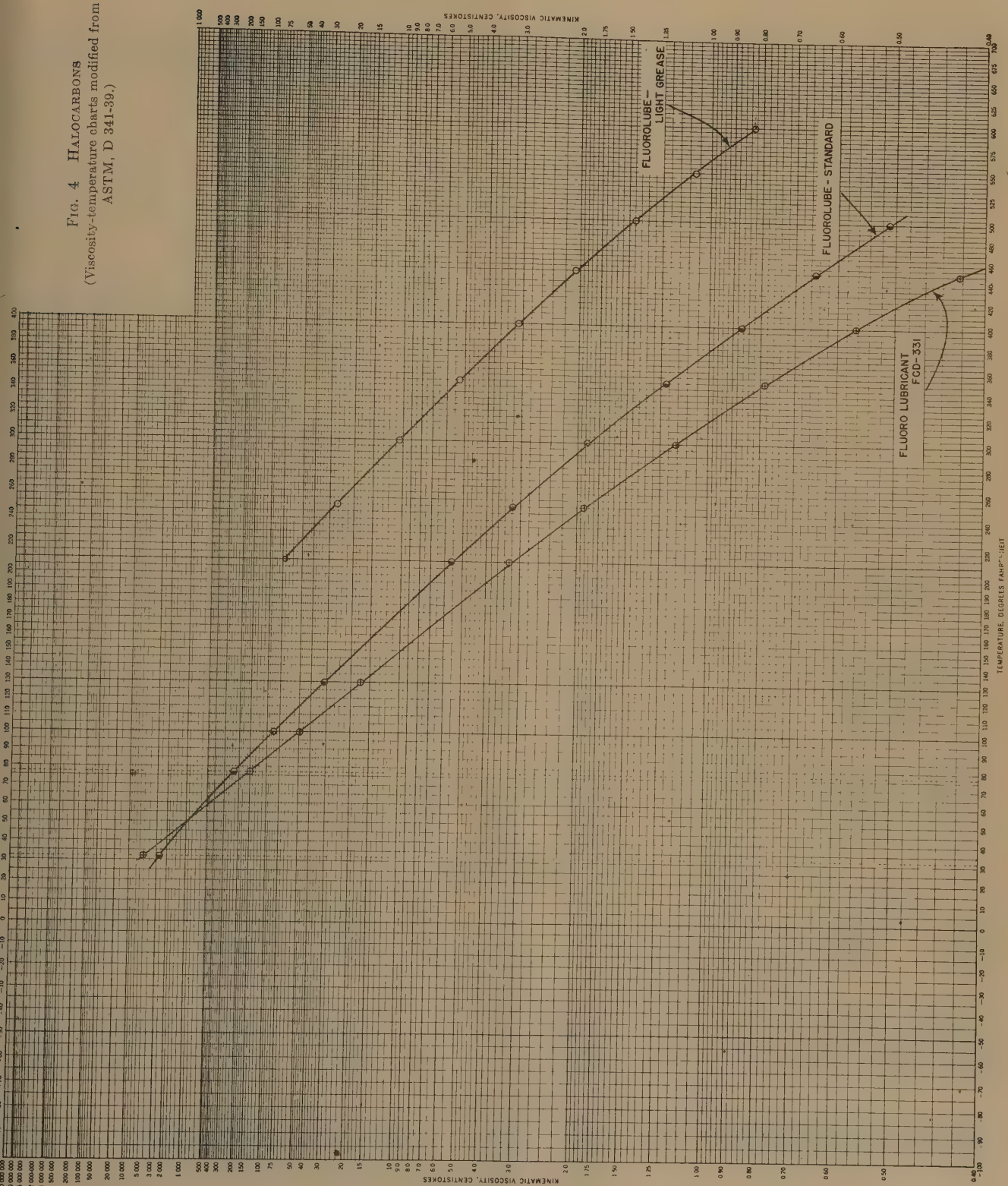


TABLE 4 OUTSTANDING FEATURES OF VISCOSITY-TEMPERATURE GRAPHS

Identification	Viscosity	Straight Line Region			Curvature* At:		
	cs. @ 100°F.	From	To	Slope	Low Temp.	End High Temp.	End
SILICONES							
Polymethylsiloxanes							
DC 500 "A"	17.11	-40	100	0.374	None	Convex	(vy)
DC 500 "B"	44.6	-40	210	0.288	None	Convex	(vy)
DC 500 "C"	74.7	-40	300	0.264	None	Convex	(med)
DC 200 "D"	173.7	-40	400	0.225	None	Convex	(med)
DC 200 "E"	279.	-40	400	0.207	None	Convex	(sl)
Polyethylsiloxanes							
DC 400 "A"	13.51	0	130	0.563	Convex (sl)	Convex	(vy)
DC 400 "B"	51.7	-40	300	0.522	None	Convex	(sl)
DC 400 "C"	98.6	0	350	0.449	Concave (sl)	Convex	(sl)
Poly(methyl-phenyl)siloxanes							
DC 702	17.29	100	300	0.702	Concave (sl)	Convex	(vy)
DC 703	34.2	100	350	0.629	Concave (vy)	Convex	(vy)
DC 710	122.0	210	400	0.448	Concave (vy)	Convex	(vy?)
DC 550	95.4	100	400	0.441	Concave (sl)	Convex	(vy)
POLYALKYLENE GLYCOLS AND DERIVATIVES							
Polyethylene Glycols							
Polyethylene Glycol 200	25.5	-40	210	0.787	Concave(med)	Convex	(sl)
Carbowax 1000	Solid	130	250	0.520		Convex	(med)
Polypropylene Glycols							
Polyglycol P-400	33.5	-20	300	0.852	None	Convex	(med)
Polyglycol P-750	51.6	100	350	0.715	Concave(med)	Convex	(med)
Polyglycol P-1200	73.4	130	400	0.609	Concave(med)	Convex	(sl)
Polypropylene Glycol Derivatives							
Ucon LB-100	19.59	32	350	0.744	Concave(sl)	Convex	(sl)
Ucon LB-100-X	20.87	0	400	0.756	Concave(sl)	Convex	(sl)
Ucon LB-250	53.9	100	450	0.570	Concave(med)	Convex	(med)
Ucon LB-250-X	57.5	100	500	0.582	Concave(med)	Convex	(sl)
Ucon LB-550-X	121.7	210	500	0.476	Concave(vy)	Convex	(sl)
Poly(ethylene-propylene)Glycol Derivatives							
Ucon HB-100-X	21.72	100	400	0.687	Concave(sl)	Convex	(sl)
Ucon HB-260	49.7	100	550	0.520	Concave(med)	None	
Ucon HB-260-X	52.3	100	550	0.552	Concave(med)	None	
Ucon HB-600-X	134.2	210	500	0.449	Concave(vy)	None	
PETROLEUM OILS							
Navy Symbol 2135	68.5	0	300	0.830	None	Convex	(med)
Navy Symbol 2250	169.1	32	300	0.865	None	Convex	(med)
Grade 1065	120.6	32	300	0.685	None	Convex	(med)
Grade 1120	369	32	350	0.661	None	Convex	(med)
DIESTER OILS							
Di(n-butyl)sebacate	6.10	32	210	0.713	None	Convex	(vy)
Di(2-ethylhexyl) sebacate	12.61	-40	300	0.713	None	Convex	(med)
Di(undecyl) sebacate	23.03	-40	300	0.719	None	Convex	(med)
HALOCARBONS							
Fluoro Lubricant FCD-331	49.0	32	210(?)	1.184	Convex (sl)	Convex	(vy)
Fluorolube-Standard	86.9	None	--	None	Convex(med)	Convex	(vy)
Fluorolube-Light Grease	Solid	210	400(?)	0.939	Convex(med)	Convex	(vy)

* Vy = Very; Med = Medium; Sl = slight.

fluids show both convexity at elevated and concavity at low temperatures. In any homologous series of fluids, increasing molecular weight or viscosity causes the convexity at high temperatures to decrease. The halocarbons are unique among the fluids studied in that their graphs are convex rather than concave at low temperatures.

From the straight-line portion of each viscosity-temperature graph, the tangent has been calculated and is given as the "slope" in the fifth column of Table 4. The uniquely low slopes of the polymethylsiloxanes are evident here, as is the increase in the slope going from methyl to ethyl to phenyl side chains, this being analogous to the rule for hydrocarbons that the slope increases with the ratio of the cross-sectional diameter of the molecule to its length. Another example of this rule is shown by the increases in the slopes with aromaticity of the series of methyl-phenyl siloxane copolymers. Due to the presence of methyl side chains in the polypropylene glycols, they have larger slopes than the polyethylene glycols of comparable viscosity. As the Ucon fluids are polymers of ethylene and/or propylene glycol, chain-stoppered at one end by a monohydric alcohol (10, 11), comparison with the analogous polyglycols of the same viscosities reveals they have much lower slopes (or higher VI) than would be expected from the increase in chain length caused by the chain stopper. It is obvious to assume that this improvement is caused by the decrease in association or hydrogen bonding resulting from blocking of one OH at the end of the polyglycol.

The addition of the Ucon antioxidant had little effect on the slope of the viscosity-temperature graph at the higher temperatures. At high temperatures the graphs of the LB and HB fluids having the same viscosity at 100 F became practically coincident. Relations between the molecular structures and the slopes (or VI) of diesters and petroleum oils have been discussed previously (15). The aliphatic diesters and the high VI petroleum oils behave similarly over the entire temperature range. It is notable that the halocarbons had the highest slopes encountered.

It is suggestive to treat the linear viscosity-temperature region of each fluid as a region of "more normal" or "more ideal" behavior, and the regions of convexity or concavity as regions of "abnormal" behavior having their origin in the increasing prominence of some chemical or physical transformation peculiar to the particular class of liquids studied. Such an approach makes it necessary to learn more about the other properties of the fluid at extreme temperatures, and it is justifiable only if it is thereby possible to collate a diverse group of phenomena. Because of the uniform gradation possible in all properties of the synthetic fluids like the silicones, Ucons, and diesters, their comparative study is especially valuable in such a connection.

The convexity of the graphs at high temperatures cannot be explained by evaporation, cracking, or oxidation since these would cause much larger irreversible changes in the viscosities than were encountered. It is suggested that this is due to a statistical uncoiling of long-chain molecules as the temperature rises due to the disorganizing effects of thermal agitation. This coiling is pronounced in the polymethylsiloxanes and is decreased by the number and size of the side chains as outlined earlier (5). In linear molecules with freedom of rotation about the chemical bonds in the main chain, coiling should be most pronounced the higher the molecular weight (and the higher the viscosity of the fluid). As high temperatures are approached, a more random distribution results, and the molecule exhausts its capacity to uncoil, with a resulting rapid dropping away from the "normal" straight line on the viscosity-temperature chart. The convexity occurs at lower temperatures for the lower members of any homologous series.

The point of deviation from linearity on the ASTM chart for

all the polymethylsiloxanes occurs at a viscosity of between 18 and 20 centistokes. This unexpected regularity may be another manifestation of the uncoiling mechanism. The similar behavior with respect to curvature on the ASTM chart of high VI petroleum oils, aliphatic diesters, and polyethylsiloxanes may be attributed to their essentially straight-chain and short-branched structure. At high temperatures the graphs of the highly aromatic silicones are very convex, and this is probably due to the great hindrance offered to coiling by large phenyl groups. A similar situation holds with respect to the halocarbons, but here the cause is the large diameter of fluorine or chlorine atoms and the slightly shorter carbon-carbon bond distance because of which hindrance is created to free rotation about carbon to carbon bonds. The exceptional behavior of the Ucon fluids at high temperatures may be caused by an increase in association at high temperatures resulting from the loss of hydrated water. This needs further study.

At low temperatures the lack of concavity of the viscosity-temperature graphs of the polymethylsiloxanes, petroleum oils, and aliphatic diesters studied is to be considered evidence that no new phenomena are occurring such as precipitation, crystallization, or association. The slight curvature for the polyethylsiloxanes can be associated with the precipitation of an insoluble fraction as reported earlier. Similarly, the concavity of the graphs for the poly(methyl-phenyl) siloxanes is also due to the precipitation of an insoluble fraction, and it is more pronounced the higher the aromaticity, due possibly to the van der Waal's forces developing at low temperatures between adjacent phenyl groups of different polymer molecules.

The viscosity data presented here when plotted, using the Walther equation with $c = 0.6$, gave graphs essentially the same as those on the ASTM chart, except that the convexity was greater in the low-viscosity region. When c was chosen as 0.75, the convexity at low viscosities was less than on the ASTM chart. The linear portion of some of the graphs was extended to higher temperatures (notably the fluorocarbons), but in no case was the convexity eliminated at extremely high temperatures. By using the Walther equation as three constant equations it is evident that c can be selected for each liquid so as to increase the range of linearity of the graph. The arbitrary introduction of the constant c , having values greater than the viscosity of the liquids under investigation, would appear as an artificial means of obtaining linear graphs. Lack of theoretical justification also makes this inappropriate here. The attempt to correlate convexity at high temperature on the ASTM chart with the properties of the liquids studied is probably mathematically equivalent to correlating such properties with the values of c .

In Fig. 5 are some typical graphs made by plotting the absolute viscosities in centipoises on the extended ASTM chart. No particularly new features are revealed by these graphs, except that the linear region is shortened and the convexity at high temperatures is increased as the density of the fluid decreases from unity. Conversely, the graphs of the halocarbons whose densities exceed unity are straighter for longer temperature ranges and show less convexity at the high temperatures. These data were also plotted on a $\log \eta$ versus $1/T$ chart, but very curved graphs resulted for all fluids except the polymethyl siloxanes. Further study is being made of the applicability of other equations for predicting the viscosity-temperature characteristics of these fluids.

CONCLUSIONS

It is shown that the deviations of the graphs from linearity on the modified ASTM viscosity-temperature chart can be given reasonable explanation based upon other physical and chemical

FIG. 5 ABSOLUTE VISCOSITY FOR VARIOUS
FLUIDS
(Viscosity-temperature charts modified from
ASTM, D 841-39.)

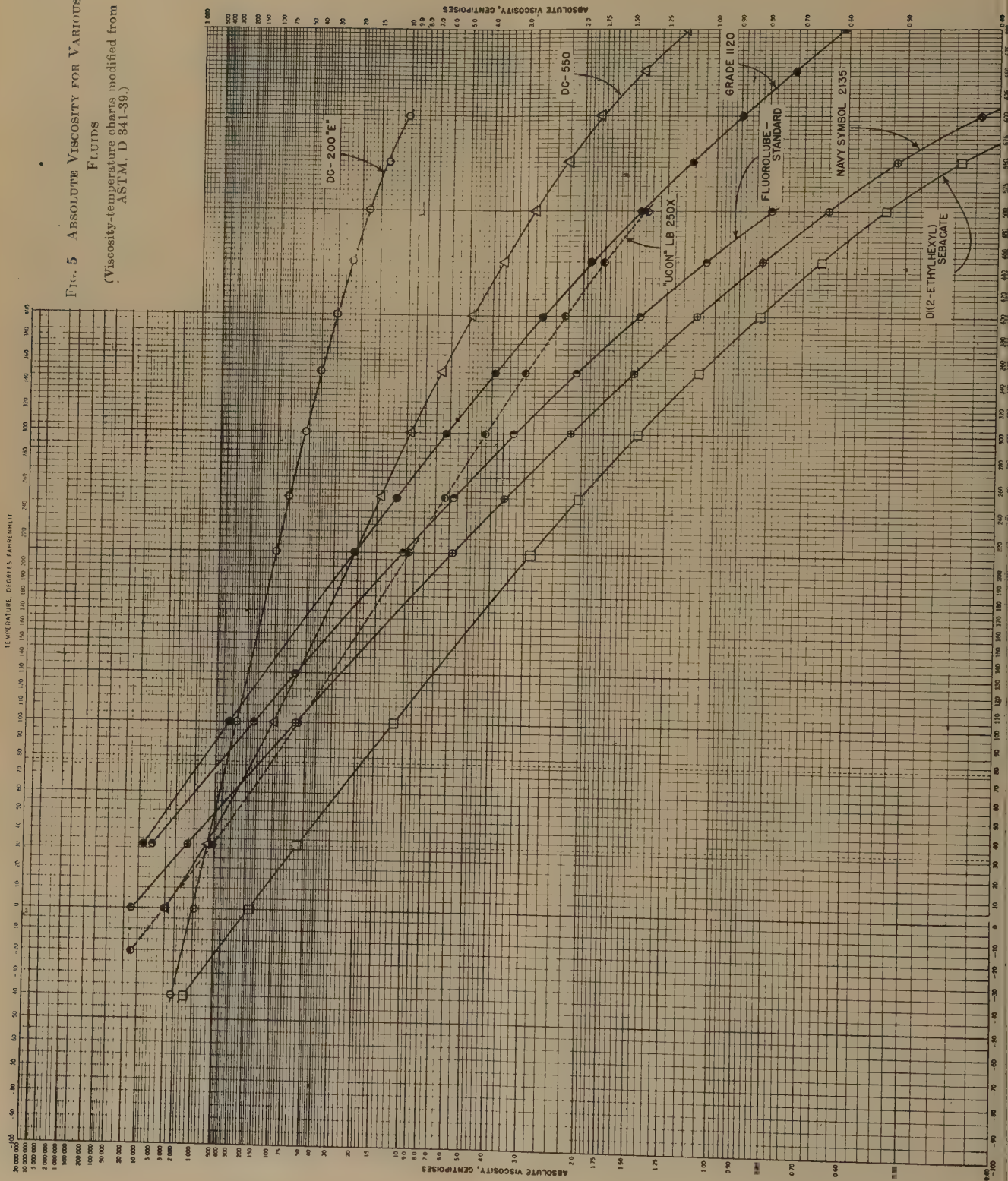


TABLE 5 ABSOLUTE VISCOSITIES OF FLUIDS

Identification	Viscosity, centipoises at °P																	
	-40	-20	0	32	100	130	150	210	250	300	350	400	450	500	550	600	650	700
SILICONES																		
Polyethylsiloxanes																		
DC 500 "A"	121.1		56.3	34.8	16.07	6.34	4.79	3.50	2.52	2.00	1.560	1.223	0.978					
DC 500 "B"	336.		152.8	121.1	62.4	26.35	12.29	8.90	6.99	5.00	3.84	3.01	2.37					
DC 500 "C"	578.		260.	71.2	23.58	10.1	34.79	14.79	10.94	8.25	6.39	4.97	3.91					
DC 200 "D"	1360.		613.	165.4	55.3	21.2	50.1	34.3	25.3	19.19	14.77	11.60	9.21					
DC 200 "E"	2210.		989.	237	101.5	76.1	51.7	35.0	25.3	19.19	14.77	11.60	9.21					
Polypropylsiloxanes																		
DC 400 "A"	361.		98.9	44.5	12.74	8.52	3.66	2.64	1.867	1.372	1.066	0.820						
DC 400 "B"																		
DC 400 "C"	8520.		1519.	500.	97.2	21.50	11.39	9.29	6.24	4.57	3.25							
Poly(methyl-phenyl)siloxanes																		
DC 702																		
DC 703			727.	124.3	18.33	4.12	2.87	1.976	1.434	1.081	0.811	0.666						
DC 710			2904.	340.0	36.6	5.99	4.75	3.19	2.280	1.691	1.305	1.017						
DC 550			20600.	1728.	132.0	71.6	21.83	14.59	9.56	6.70	4.96	3.73	2.89					
			3270.	667.	101.2	59.0	21.01	14.55	9.56	6.70	4.89	3.58	2.84					
POLYALKYLENE GLYCOLS and DERIVATIVES																		
Polyethylene Glycols																		
Carbowax 1000	34800.	5570.	1347.	252.4	28.3	15.0	4.61	3.01	1.964	1.372	1.015	0.621	0.502	0.412				
Polypropylene Glycols																		
Polyglycol P-400																		
Polyglycol P-750	23380.	3940.	477.	477.	33.4	16.02	4.38	2.78	1.792	1.259	0.928	0.718	0.567	0.456				
Polyglycol P-1200	33800.	5730.	697.	51.3	25.0	6.93	4.10	2.85	1.967	1.471	1.122	0.889	0.70					
Polypropylene Glycol Derivatives																		
Ucon LB-100	43300.	7430.	951.	72.8	35.9	10.17	6.49	4.20	2.91	2.151	1.64	1.32	1.02					
Ucon																		
Ucon LB-100-X	12300.	2298.	626.	136.9	18.81	3.79	2.50	1.777	1.312	0.986	0.80	0.63						
Ucon LB-250	17020.	2950.	767.	157.5	20.11	3.88	2.55	1.815	1.350	1.014	0.799	0.64	0.52					
Ucon LB-250-X	10700.	3097.	542.	56.7	35.2	10.17	6.49	4.20	2.98	2.24	1.74	1.36	1.1					
Ucon LB-550-X	11150.	9670.	1378.	119.8	17.75	11.55	7.53	5.15	3.87	2.96	2.35	1.8						
Poly(ethylene-propylene) Glycol Derivatives																		
Ucon HB-100-X	10180.	2095.	582.	145.2	21.87	4.56	3.16	2.188	1.600	1.233	0.970	0.785	0.643					
Ucon HB-250	7000.	2950.	780.	382.	50.9	9.76	6.64	4.45	3.23	2.45	1.92	1.52	1.28					
Ucon HB-250-X	8430.	2108.	429.	429.	53.8	9.89	6.69	4.48	3.25	2.44	1.92	1.55	1.28					
Ucon HB-600-X	33900.	7190.	1313.	1313.	139.7	23.00	15.25	9.95	7.03	5.24	4.05	3.20	2.7					
PETROLEUM OILS																		
Navy Symbol 2135																		
Navy Symbol 2250	11060.	1296.	61.7	61.7	3.55	2.17	1.481	1.076	0.818	0.639	0.514	0.418	0.346	0.29				
Grade 1065		7710.	157.1	157.1	8.70	4.83	2.79	1.819	1.304	0.970	0.748	0.593	0.480	0.394				
Grade 1120		1556.	104.4	104.4	6.10	3.66	2.42	1.721	1.286	0.993	0.789	0.637	0.521	0.443				
		7950.	324.	324.	20.93	11.41	6.26	3.91	2.65	1.91	1.439	1.119	0.894	0.726	0.61			
DIESTER OILS																		
DI(n-butyl)sebacate																		
DI(2-ethylhexyl)sebacate	1360.	176.	5.62	5.62	1.822	1.364	1.012	0.774	0.611	0.492	0.435	0.333						
DI(undecyl)sebacate	6170.	654.	11.40	11.40	2.031	1.435	1.035	0.825	0.657	0.531	0.438	0.359						
			20.52	20.52	3.95	2.73	1.849	1.339	1.012	0.800	0.621							
HALOCARBONS																		
Fluoro Lubricant RCD-331																		
Fluorolube - Standard	10910.	98.9	34.27	34.27	3.56	2.108	1.377	0.949	0.687	0.521	0.442	0.371						
Fluorolube - Light Grease	5360.	157.9	60.6	60.6	19.5	19.16	9.42	5.36	3.45	2.376	1.712	1.282						

properties of the fluids. It is believed that any equation capable of predicting the change of viscosity with temperature of a fluid over such a large temperature range will have to take account of the other physical and chemical changes which take place. The observed convexity at high temperatures appears general, and it has been found possible to present a qualitative explanation based on the idea of coiling of linear molecules and its disappearance under conditions of excessive thermal agitation. Data have been presented permitting better definition of the regions for which linearity can be safely assumed on the extended ASTM viscosity-temperature chart. Comparative data have been presented on the kinematic and absolute viscosities, densities, and cubical coefficients of expansion as functions of temperature over the entire useful range of the fluids investigated. It is pointed out that hindrances to rotation about carbon-carbon bonds may be responsible for the large decrease in viscosity index with halogenation of hydrocarbons.

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Strength and Failure Characteristics of Metal Membranes in Circular Bulging

By W. F. BROWN, JR.,¹ AND F. C. THOMPSON²

Circular hydraulic bulges were formed from a group of materials having widely varying strain-hardening rates. The complete development of the shapes and strain distributions was determined experimentally, and the stress and radius of curvature at the pole were calculated as a function of the maximum strain. An analysis of the data revealed that strain gradients and, therefore, the bulge heights were influenced by the stress-strain characteristics of the metal. It was also found that the bulge contour was closely approximated by a sphere only at strains in the vicinity of the instability strain. Instability was exhibited by all materials having a sufficient ductility at strains varying from $\epsilon_3 = -0.47$ for 75S-O to $\epsilon_3 = -0.64$ for annealed low-carbon steel. The phenomenon of instability was related to both the development of the shape and the strain distribution. The previously reported differential equation for instability in a circular bulge was found to yield strains which agreed with the maximum load strain and the instability strains derived from the analysis of the shape and strain distributions. These instability strains were not found to be simply related to the necking strains in uniaxial tension. The height and maximum strains (forming limits) obtainable in bulging were found to be greatly reduced by the presence of surface flaws or a large grain size.

NOMENCLATURE

The following nomenclature is used in the paper:

- $s_1 = s_2$ = two principal stresses in plane of sheet
- s_3 = principal stress acting perpendicular (normal) to plane of sheet
- $\bar{s} = \frac{1}{\sqrt{2}} \sqrt{(s_1 - s_2)^2 + (s_2 - s_3)^2 + (s_3 - s_1)^2} \dots$ effective stress
- e_1, e_2, e_3 = conventional principal strains in directions of corresponding principal stresses
- $\epsilon = \ln(1 + e)$ = natural strain
- $\bar{\epsilon} = \frac{\sqrt{2}}{3} \sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2} =$ effective strain
- R = radius of curvature at pole
- R' = computed (average) radius of curvature for chord length L
- h = thickness of metal membrane at any point
- h_0 = initial thickness of metal membrane

¹ Formerly Research Metallurgist, Research Laboratory for Mechanical Metallurgy, Case Institute of Technology, presently Research Metallurgist, Flight Propulsion Research Laboratory, NACA, Cleveland, Ohio.

² Metallurgical Engineer, Allegheny Ludlum Steel Corporation, Dunkirk, N. Y.; formerly with Research Laboratory for Mechanical Metallurgy, Case Institute of Technology, Cleveland, Ohio.

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p = hydraulic pressure

L = chord length

n = strain-hardening exponent in generalized stress-strain function equal to necking strain in uniaxial tension

H = height of bulge at pole

INTRODUCTION

The load and geometrical conditions present in many structures subjected to underwater explosion or to the action of high hydrostatic pressures are similar to those encountered when a metal membrane restricted at its periphery is deformed by means of a hydraulic pressure. Problems of a similar nature arise in the design of safety diaphragms used in the protection of pressure vessels. In addition, the study of such membranes offers the possibility of investigating the fundamentals of plastic flow and fracturing in a biaxial stress state.

Circular membranes are, for reasons of symmetry, subjected to a state of balanced biaxial tension ($s_1 = s_2$). In a previous publication by Sachs and Lubahn (1),³ a theory of tensional instability similar to that for the necking of a tensile test specimen (2) was extended to several other load and geometrical conditions, including a circular hydraulic bulge. In this analysis it was shown that instability in a circular bulge should occur at much higher strains than in a tensile test. These conceptions were verified by an experimental analysis of a hard-rolled and an annealed electrolytic-copper bulge in a previous publication by Sachs and Brown (3).⁴ It was shown that the instability was associated with a maximum in the pressure-strain curve, and that the instability strain could be derived from a suitable analysis of the development of the meridional strain distributions.

The deformation characteristics of metal membranes have also been investigated by Gensamer and associates (4) and by Saibel (5). Gensamer investigated both circular and elliptical bulges of the aluminum alloy 24S-T clad and 24S-O clad. Saibel attempted to develop an expression which for the circular bulge would yield the strain distribution in terms of the height at the pole and radius of the die. This analysis was then extended using the condition of instability (1) to yield an expression for the instability strain in terms of the strain-hardening exponent n , of the generalized stress-strain relationship (6, 7) relating the effective stress $\bar{s}$, to the effective strain, $\bar{\epsilon}$, (8)

$$\bar{s} = A(\bar{\epsilon})^n$$

These results, however, do not appear to be in agreement with the experimental data presented in this paper, as will be discussed later.

A considerable amount of experimental and theoretical work on circular bulging of large steel diaphragms has been carried out at the David Taylor Model Basin of the Navy Department. These results are described in recently declassified reports by Gleyzal (9, 10), Greenfield (11), and Mostow (12). However,

³ Numbers in parentheses refer to the Bibliography at the end of the paper.

⁴ These results have been added to many of the graphical representations in this paper and some additional data have been presented for these materials.

this information was not available at the time this paper was prepared.

Related to the problem discussed here are also some bursting tests on hemispherical shells by Marin and Wilson (13). However, the data reported are not sufficient to permit an analysis of the shape, or to determine whether instability may have occurred.

This investigation is an extension of the previous experimental work on circular bulges (3) to further analyze their shape and determine the effects of the metal strain-hardening characteristics. For several materials, the meridional strain distributions, the bulge contours, and the pressures were experimentally determined. From these measurements the radii of curvature and the meridional stresses at the pole were computed. Both the strain distributions and the contours were analyzed to reveal whether or not the bulge exhibited an instability. In addition, the previously developed instability equation was solved graphically using the calculated values of stress and radius of curvature at the pole.

An experimental analysis of the type presented here yields many useful basic relations which will be the subject of later papers. Thus the energy to fracture may be determined from the pressure-strain and height-strain curves. This quantity is an important consideration in the resistance of a structure to the effects of underwater explosion. Also, as was previously pointed out (3), the stress-strain relation derived in circular bulging can be represented in such a manner that it is equivalent to the stress-strain relation in pure tension, but over a much wider range of strains than could be investigated by the conventional tensile or compression test.

The analysis of the data presented here is being extended with the purpose of relating the stress distribution in a circular bulge to the strain-hardening constant of the particular material tested. It now appears that such an attack may yield the correct strain distributions for the materials reported here, and, consequently, permit the instability strains to be calculated from the stress-strain relations in uniaxial tension.

In this investigation the following assumptions were made regarding conditions at the pole:

- 1 The normal principal stress s_3 , is negligible compared with the principal stresses ($s_1 = s_2$) in the plane of the sheet.
- 2 The effects of the edge conditions may be neglected.
- 3 The bending stresses (due to the flexural rigidity of the sheet) are negligible compared with the tensile stresses.
- 4 The material is isotropic, i.e., the ratio between the principal strains, e_2/e_1 , is unity during the entire deformation.

MATERIAL AND PROCEDURE

Five materials (Table 1) covering a wide range of stress-strain characteristics were investigated. All the alloys were tested in the "as-received" condition, with the exception of the annealed low-carbon steel. This steel (0.040 in. thickness) was available as 60 per cent cold-rolled sheet. The annealed condition was produced by holding at 1250 F for 2 hours, furnace-cooling to 700 F, and then cooling to room temperature in still air.⁵

⁵ This heat-treatment should approximate a full commercial process anneal.

In addition, attempts were made to bulge the aluminum alloy 52S-1/2H, 50 per cent cold-rolled low-carbon steel (0.050 in. thickness) and 70 per cent cold-rolled low-carbon steel (0.030 in. thickness). However, for various reasons to be discussed later, these attempts were unsuccessful.

Before testing, the (12 in. $\times$ 12 in.) bulging blanks were photo-gridded with a 20-line-to-the-inch net (14), the grid lines being parallel and perpendicular to the rolling direction of the sheet.

Tensile tests were made for all materials.⁶ For each alloy, specimens were cut in the rolling direction, transverse, and at 45 deg to the rolling direction. The average rate of straining in these tests was comparable to that in bulging. The conventional yield strength⁶ and tensile strength derived from an average of tests in the three directions are reported in Table 1.

The bulging equipment and the preparation of the gridded specimen blank have been described in detail in previous papers (3, 15). However, in this investigation a Riehle testing machine was used to furnish oil to the bulging head at pressures up to 3000 psi. The pendulum-type load-measuring device on this machine was adapted to indicate the bulging pressure to ± 0.5 per cent. When testing the stronger alloys, lead fuse wire (0.030 in. diam) was used as a gasket between the specimen blank and the bulging head to prevent oil leakage past the studs.

A complete experimental analysis consisted of determining the pressures, the distribution of meridional strains, and the contours at appropriate intervals of strain (at the pole) up to fracture. Strain and contour measurements were made in the transverse direction (90 deg to the rolling direction),⁷ using the same techniques described previously (3). All measurements were made after removal of the applied pressure. The strain distribution after fracture was determined for all alloys except 24S-T. The many-branched fracture of 24S-T (15), interfered with strain measurements made after bursting.

In order to check the reproducibility of the results and the magnitude of possible material variations, pressure versus meridional-strain-at-the-pole curves were determined for an additional specimen of each material, Fig. 1. In general, the data obtained on the two specimens, and in the case of low-carbon steel on three specimens, agreed quite well within the limits of experimental error. It should be pointed out in this respect that the agreement of the pressure-strain data does not preclude the possibility of material variations having a significant effect on the distribution of strain and curvature.

RESULTS

In this paper the data have been represented by the use of a "universal stress-strain relationship," i.e., one which yields the same relation between some selected function of a principal stress and some selected function of a principal strain, for any stress state considered.

⁶ Tensile specimens were $3/4$ in. in width and similar to those recommended for sheet materials by the ASTM but possessing a longer (5 in.) gage length. The conventional yield strength is that determined by the 0.20 per cent (plastic) strain-offset method.

⁷ A certain degree of anisotropy of strain and shape was noted, particularly for the annealed low-carbon steel, and these effects of directionality will be considered in a subsequent investigation.

TABLE 1 MATERIALS TESTED

Material	Initial sheet thickness, in. (± 0.001)	Yield strength, psi				Tensile strength, psi			
		Longitudinal	45 deg	Transverse	Average ^a	Longitudinal	45 deg	Transverse	Average ^a
Annealed stainless steel (Type 347)	0.036	35500	35000	34500	35000	95200	93700	93000	94000
Annealed low-carbon steel	0.041	29700	34400	33200	32400	45700	50000	48700	48100
24S-T bare	0.039	53600	45500	45600	48200	72500	69500	70300	70800
24S-O bare	0.041	15200	14700	14700	14900	35900	34800	34300	35000
75S-O bare	0.041	15200	15700	15000	15300	35400	35900	34800	35400

^a Average of one test in each direction, i.e., longitudinal, transverse, and 45 degrees to rolling direction.

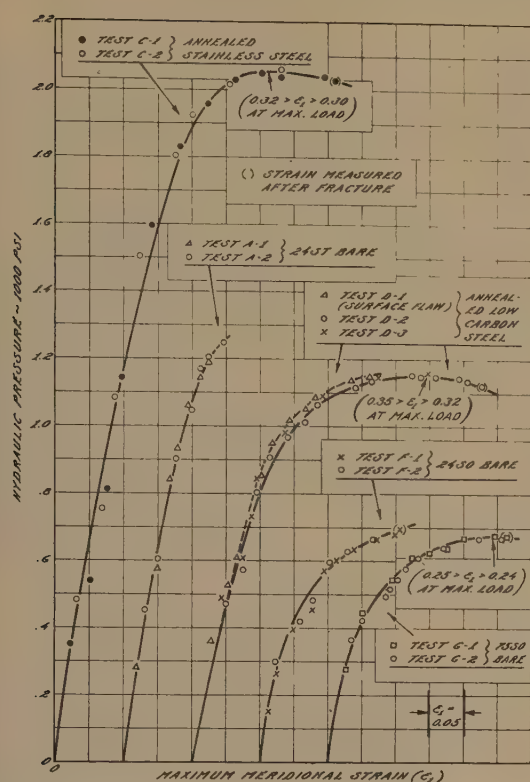


FIG. 1 PRESSURE-STRAIN CURVES FOR CIRCULAR BULGING OF VARIOUS MATERIALS

Of the several relations which have been proposed, the representation of twice the maximum shear stress

$$2 \left[\frac{s_1 - s_3}{2} \right]$$

“decisive” stress, as a function of the greatest absolute natural strain, “decisive” strain, $\epsilon_{\max}$ ⁸ is the most simple. This representation may lead to stress values which are in error by as much as 15 per cent for plane strain ($s_1 = 2s_2$, $s_3 = 0$). However, for the cases of balanced biaxial tension ($s_1 = s_2$, $s_3 = 0$), considered by the authors, this relationship yields the same function between the stresses and strains as would the use of the effective stress and effective strain discussed by Dorn (8). Therefore, the data have been represented in terms of the largest maximum stress, $s_1 = s_2$ (equal to twice the maximum shear stress), and the greatest natural absolute strain, $\epsilon_3 = -2\epsilon_1 = -2\epsilon_2$.

Since the stresses and strains are not uniformly distributed across the contour, the instability equation previously developed (3) is in terms of the maximum values of the decisive stress and decisive strain. These occur at the pole and are referred to as the maximum meridional stress and maximum meridional strain, respectively. In the following discussion, unless otherwise noted, the stress and strain values referred to are those at the pole of the bulge.

Pressure-Strain Relations. Plastic instability of a mechanical system is characterized by an increase in strain at constant or decreasing load. Fig. 1 shows the pressure-strain relations for bulges of various alloys. It can be seen that a pronounced maximum is present in the pressure-strain curves for stainless steel and low-carbon steel; 75S-O bulges exhibited a constant load before fracture; however, no decrease in load could be detected. Both 24S-O and 24S-T bulges exhibited continuously increasing pressure to fracture.

The instability strains (strains at maximum load) are difficult to determine accurately from these plots. However, the values as reported on the graphs varied from approximately $\epsilon_1 = 0.24$ for 75S-O to $\epsilon_1 = 0.33$ for low-carbon steel.

Strain Analysis. Figs. 2 and 3 illustrate the meridional strain distributions at a series of selected pressures. The strains at a given point are plotted against the original position of this point on the undeformed sheet. The strain distribution, measured after fracture, has been added to these figures, with the previously mentioned exception of 24S-T.

In general, the strain increases continuously from the die periphery to a maximum at the pole. However, the strain is not necessarily zero at the die edge (3 in. from the pole), since for the higher bulges an appreciable strain may occur over the radius of the die.

For a particular material, the shape of the distribution curves varies considerably, depending on the magnitude of the strain. In general, the strain distributions become less uniform (steeper) with increasing strain at the pole. However, this change to higher strain gradients may not be continuous at the lower strains. Considering, for example, low-carbon steel, Fig. 2, the strain gradient becomes increasingly steeper until a strain at the pole of $\epsilon_1 = 0.08$ is reached; however, the next distribution shown is nearly uniform. This cycle is repeated once more between $\epsilon_1 = 0.095$ and $\epsilon_1 = 0.18$; with further deformation the strain gradient increases continuously to fracture.

In addition, it appears for most alloys that after a certain relatively high strain, increasing strains at the pole produce only a very small increase in strain at distances greater than approximately 1 in. from the pole.

Comparing the strain distributions for bulges of the various alloys, it is evident, particularly for relatively high strains at the pole, that considerable variation in strain gradients exists.

Shape Analysis. The height of the bulge at the pole is represented as a function of the normal strain in Figs. 4 and 5.

In general, the height increases at a decreasing rate with progressive strain. At a particular strain the rate of increase is most rapid for the very ductile alloys, i.e., annealed copper, low-carbon steel, and stainless steel. The shapes of the curves for this group of materials are very similar, particularly at the higher strains. Height-strain curves for the less ductile aluminum alloys, 24S-O, 24S-T, and 75S-O are practically identical, the only difference being in extent. On the other hand, the height for hard copper at a particular strain is less than the other alloys in spite of its high ductility.

It is obvious that the heights at fracture are not a simple function of the total strain (ductility).

The radius of curvature at the pole may be determined from the contour measurements. Thus as described previously (3), curves relating the radius to the chord length (see Figs. 8 and 9) are extrapolated to zero chord length. This method has the advantage that it yields a very close approximation to the true curvature at the pole for any smoothly contoured shape. In Figs. 6 and 7, the radius is plotted as a function of the normal strain. For a particular material, the radius of curvature decreases at a decreasing rate with increasing strain.

In the past, it has been tacitly assumed by most investigators⁹ (4, 5) that the contour of the circular bulge closely approximates a sphere during the entire deformation. That such an assumption is correct over a large portion of the contour at strains approaching the instability strain has been shown for an annealed-copper bulge (3).

Variations in the curvature (reciprocal of the radius of curvature) are not readily obtained from plots of the contours. How-

⁹ Such an assumption is necessary when the radius of the curvature at the pole is determined by means of a sphereometer (4, 5).

⁸ ϵ is the natural strain = $\ln(1 + e)$.

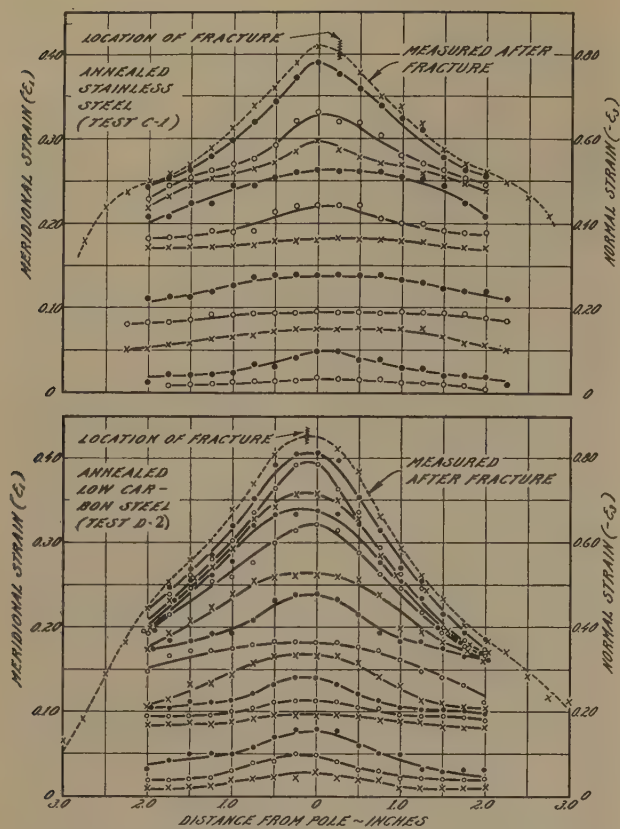


FIG. 2 DISTRIBUTION OF MERIDIONAL STRAINS AT VARIOUS LOADS AND AFTER FRACTURE FOR CIRCULAR BULGING OF TWO STEELS

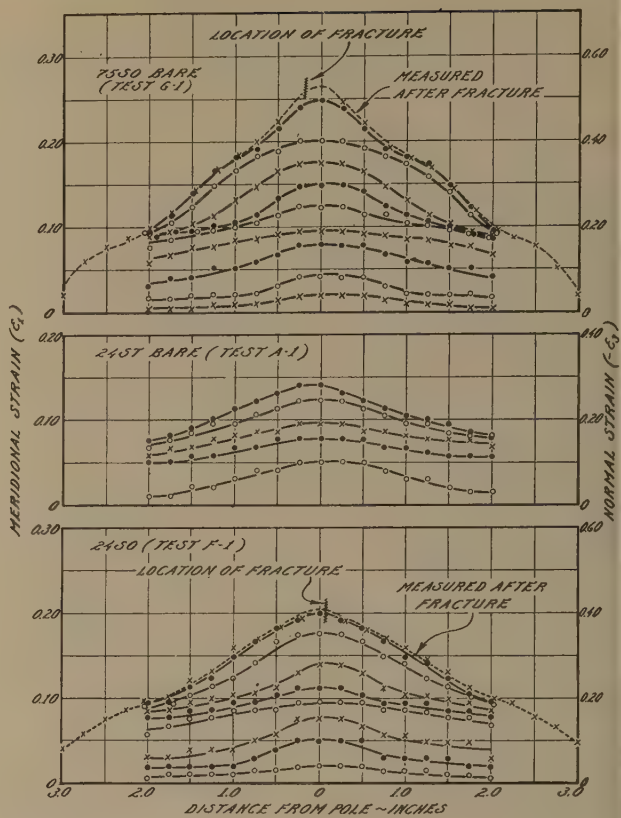


FIG. 3 DISTRIBUTION OF MERIDIONAL STRAINS AT VARIOUS LOADS AND AFTER FRACTURE FOR CIRCULAR BULGING OF SEVERAL ALUMINUM ALLOYS

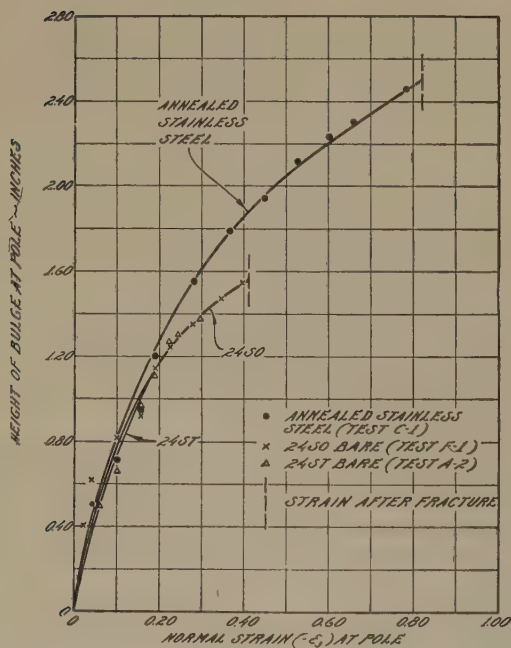


FIG. 4 RELATION BETWEEN NORMAL STRAIN ϵ_2 AND HEIGHT AT POLE FOR CIRCULAR BULGES OF VARIOUS MATERIALS

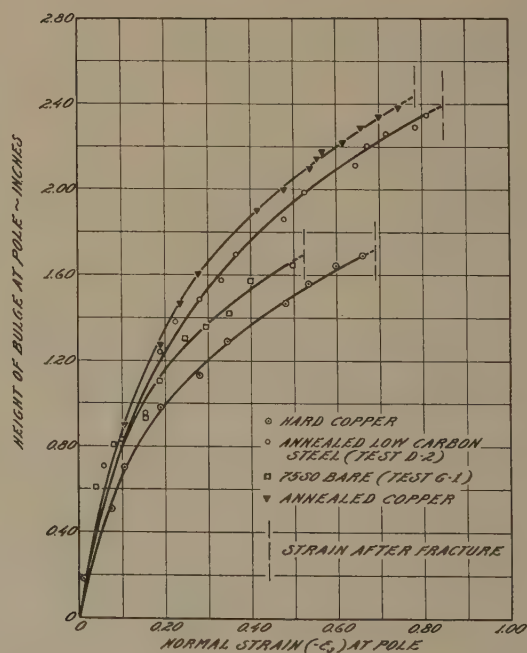


FIG. 5 RELATION BETWEEN NORMAL STRAIN ϵ_2 AND HEIGHT AT POLE FOR CIRCULAR BULGES OF VARIOUS MATERIALS

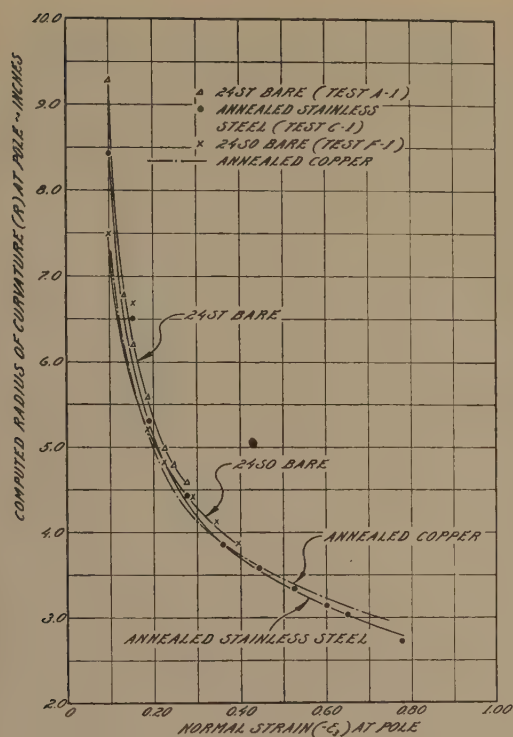


FIG. 6 RELATION BETWEEN NORMAL STRAIN ϵ_3 AND RADIUS OF CURVATURE R AT POLE FOR CIRCULAR BULGES OF VARIOUS MATERIALS

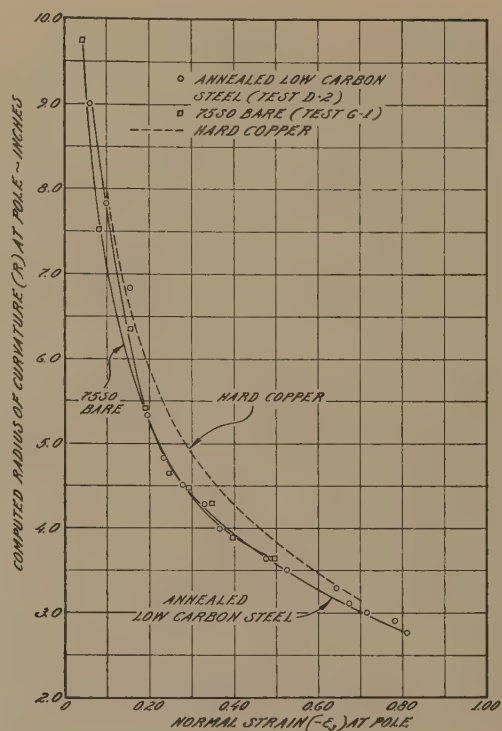


FIG. 7 RELATION BETWEEN NORMAL STRAIN ϵ_3 AND RADIUS OF CURVATURE R AT POLE FOR CIRCULAR BULGES OF VARIOUS MATERIALS

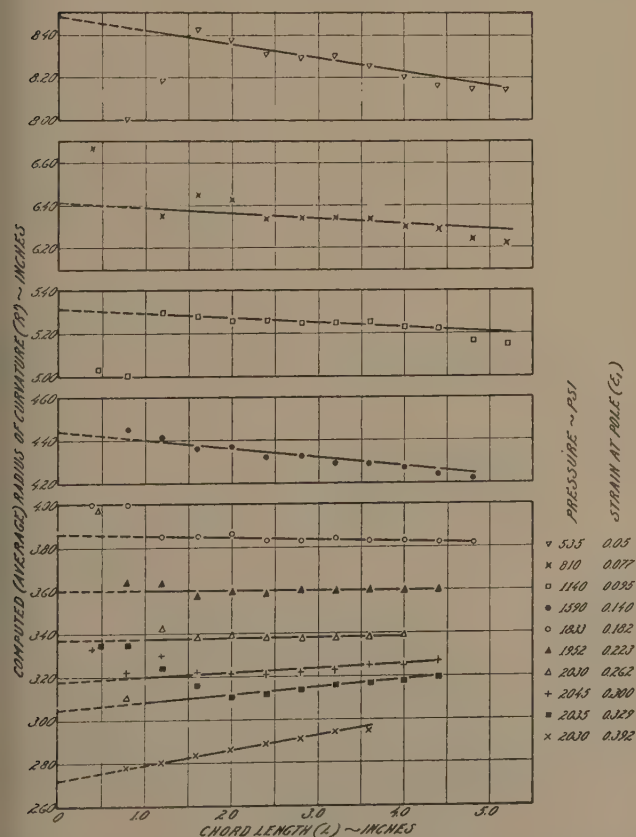


FIG. 8 DETERMINATION OF RADIUS OF CURVATURE R AT POLE FROM RADII OF CURVATURE CALCULATED FOR VARIOUS CHORD LENGTHS FOR AN ANNEALED STAINLESS-STEEL BULGE; TEST C-1

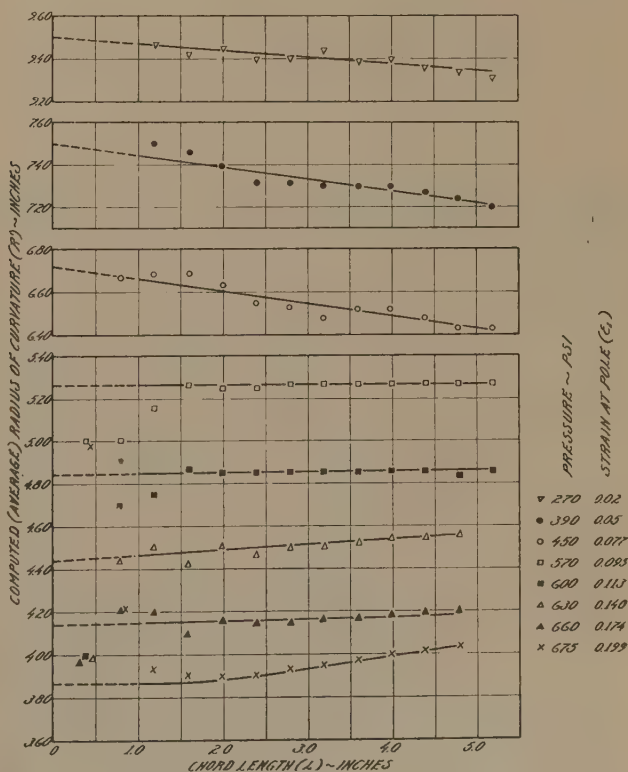


FIG. 9 DETERMINATION OF RADIUS OF CURVATURE R AT POLE FROM RADII OF CURVATURE CALCULATED FOR VARIOUS CHORD LENGTHS FOR A 24S-O BARE BULGE; TEST F-1

ever, the technique for determining the radius of curvature at the pole used in this investigation clearly reveals such variations in the curvature. Figs. 8 and 9 show the computed radius of curvature as a function of the chord length with strain as a parameter, for 24S-O and stainless steel. The trends noted are typical for all materials investigated thus far. At small strains these curves have a negative slope, indicating that the curvature, $1/R$, decreases toward the pole. With progressing strain, the slope gradually approaches zero, i.e., a straight line. This is an indication of uniform curvature over the distances investigated.¹⁰ With further increasing strains in the vicinity of the instability strain, the slope of these curves increases in a positive direction, showing that the curvature is greatest at the pole.

The development of the bulge shape may be further analyzed by a consideration of the height strain (see Figs. 4 and 5), and curvature-strain relations. Thus for a particular maximum meridional strain, using the corresponding values of radius of curvature and height at the pole, a circle may be constructed with its center on the vertical axis of symmetry. This construction has been made in Fig. 10, for stainless steel at several different strains.

At a low strain, $\epsilon_3 = -0.15$, the constructed circle intersects the base plane at the point B , which is at a greater distance from the vertical axis of symmetry than is the point of tangency I , between the die radius and base plane. From the point I , the actual bulge shape must conform to the die radius for a distance which is greater the larger the deformation. To satisfy this requirement and in order for the contour to be smooth, the curvature must decrease as the pole is approached. This has been shown schematically by the dotted line IC .

With increasing strain the intersection between the constructed circle and the base plane moves toward the vertical axis of symmetry. Consequently, the constructed circles conform more and more closely to the requirement that the shape be tangent to the die radius. This results in an increasingly more uniform curvature distribution as the instability strain is approached.

It was observed that, at instability ($\epsilon_3 = -0.60$), the arc of the constructed circle intersected the base plane at point P , a distance of 3 in. from the vertical axis of symmetry;¹¹ i.e., one

half the die diameter. With still further deformation ($\epsilon_3 = -0.78$) the radius at the pole decreases and the point of intersection G , of the arc of the constructed circle with the base plane moves inside the die diameter. Therefore, straining beyond a certain point in the vicinity of instability would result in the arcs of the constructed circles conforming less and less closely to the requirement that the shape be tangent to the die radius. Thus the curvature of the actual shape must increase toward the pole, as is shown by the dotted line IH . At still higher strains, the increase in curvature becomes more pronounced.

It should be noted that following instability only comparatively small changes in curvature of the dotted construction lines (such as IH) in Fig. 10, would be necessary for rather large deformation in the region of the pole. In other words, the strains at sufficient distances from the pole may remain practically constant for a rather large change of strain at the pole. These conclusions in regard to changes in curvature during the development of the bulge shape are in qualitative agreement with those derived from the variation of radius of curvature with chord length (see Figs. 8 and 9).

By another method of analysis (discussed later, Figs. 14 and 15), it has been demonstrated that the shape of all material tested develops in the manner described in the foregoing. Apparently the assumption, that the bulge contour is spherical during the entire deformation, is in error, the amount of error being variable and a minimum at strains in the vicinity of the maximum load strain.

The observation that the curvature decreases toward the pole at comparatively low strains may be explained as follows: Calculations for several cases have revealed that the stress distribution becomes more uniform with increasing strain at the pole. At low strains the stress decreases rather rapidly toward the die periphery. This reduction in stress may be caused by either (a) an increase in thickness, or (b) a decrease in radius. The change in thickness at low strains is, however, practically negligible. Therefore, this reduction in stress must be explained by an increase in curvature as the die periphery is approached.

Stress-Strain Relationships. The true maximum meridional stress s_1 , for a particular value of the measured maximum meridional strain ϵ_1 , may be obtained by the following relation

$$s_1 = \frac{pR(1 + \epsilon_1)^2}{2h_0} = \frac{pR}{2h_0(1 + \epsilon_3)}$$

where p is the hydraulic pressure, h_0 the initial sheet thickness and R the radius of curvature at the pole.

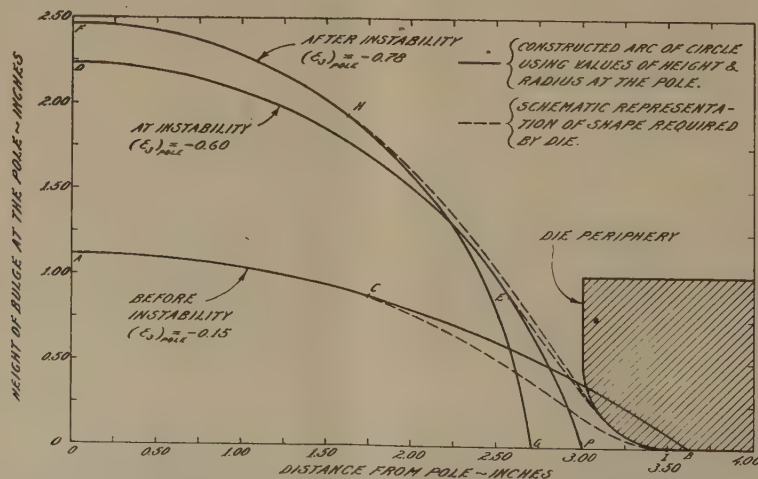


FIG. 10 SCHEMATIC CONSTRUCTION OF CIRCULAR BULGE SHAPES AT VARIOUS STRAINS FOR ANNEALED STAINLESS STEEL FROM HEIGHTS AND RADII OF CURVATURE AT POLE

¹⁰ These measurements were limited by the wall of the die to chord lengths not exceeding approximately five inches. Therefore, changes in curvature beyond approximately $2\frac{1}{2}$ inches from the pole are not revealed.

¹¹ This peculiar relation between the development of the shape and the instability was observed for all materials exhibiting instability (see Figs. 14 and 15) and cannot be explained at present.

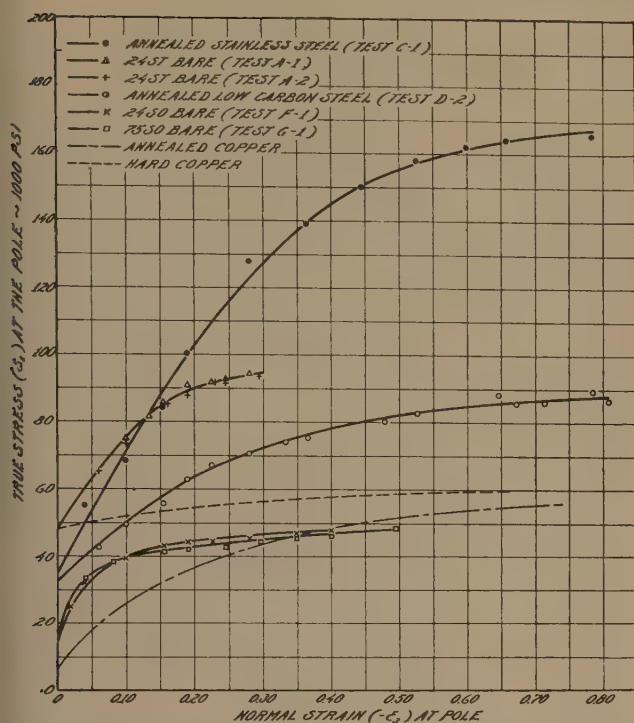


FIG. 11 RELATION BETWEEN TRUE STRESS S_1 AND NORMAL OR PLASTIC STRAIN ϵ_3 AT POLE IN CIRCULAR BULGING OF VARIOUS MATERIALS

Fig. 11 shows the stress-strain relations obtained for the various materials.¹² It is evident that a wide variation in stress-strain characteristics is represented.

It has been shown (3) that the stress-strain relationships in circular bulging conform to the general laws of plasticity and are a function only of the stress state, being independent of changes in curvature. These relations as represented in terms of the decisive stress and decisive strain should be identical to the stress-strain curves for these materials in uniaxial tension. However, the range of strain in uniaxial stress which can be covered in a conventional tensile test is usually considerably restricted, due to necking which alters the stress state.

Fracturing Characteristics. All the materials fractured in the vicinity of the pole, the locus of fracture extending parallel to the rolling direction, approximately equal distances either side of the pole (with the exception of 24S-T, the total length of metal separation was from 1 to 2 in.). The alloy 24S-T, on the other hand, fractured in an "explosive manner" (15), the fracture extending over a considerable meridional distance and exhibiting many branches. In some cases a large section was blown out of the top of the 24S-T bulges.

It has been shown previously by Gensamer (4) and Sachs (15) that for certain aluminum alloys a localized reduction in thickness occurs in the immediate vicinity of the fracture. This would indicate that for those materials the actual metal separation may take place in a state of plane strain, i.e., strain parallel to the fracture being constant.

The existence of such a phenomenon would render impossible the determination of the actual fracture stress and strain in a biaxial stress state by means of a hydraulic bulge. As yet, it is not known whether such effects are associated with fracturing of the materials investigated here.

¹² Since the pressure was removed before strain measurements were made, these curves do not include the elastic strains and, therefore, have been terminated at the conventional yield stress.

The maximum normal strains (ductility) at fracture were obtained by extrapolation of the strain distributions measured after fracture (dotted curves in Figs. 2 and 3), and therefore do not include the local region of high strain mentioned. These values are larger than the maximum load strains, Fig. 1. However, they are less than the fracturing strains which would be expected from the reduction in area (ductility) of these metals in a tensile test. Thus annealed stainless steel exhibits a ductility in a tensile test of approximately 70 per cent reduction in area, or $\epsilon_1 = 1.2$. In bulging, the corresponding maximum value of ϵ_3 obtained from the strain distributions is -0.82 .

Several rather interesting phenomena which determined the forming limits were noted in some preliminary tests made on various materials. Thus it was found that the bulge was very sensitive to imperfections which would constitute a groove or notch in the metal surface. A small defect of this type was observed at a considerable distance from the pole in a test made on low-carbon steel, Fig. 1 (test D-1). This developed into a very small fracture at $\epsilon_3 = -0.52$.

Several attempts were made to form bulges of 52S-1/2H, a cold-rolled aluminum alloy which develops severe stretcher strains in a tensile test. In bulging, these stretcher strains which constituted narrow trough-shaped regions of reduced thickness, appeared after various small amounts of deformation. Fracture would then occur shortly in one of these stretcher strains. The maximum meridional strains at fracture varied from $\epsilon_3 = -0.10$ to $\epsilon_3 = -0.30$ for presumably identical specimens of 52S-1/2H.

In a previous investigation of bulging of aircraft aluminum alloys (15), the experimental high-strength aluminum alloy XA 75S-O bare, was found to develop severe "orange peel" due to a large grain size. This phenomenon resulted in the formation of irregular localized regions of reduced thickness which considerably lowered the forming limits and introduced considerable scattering.

DISCUSSION OF RESULTS

As previously mentioned, when instability occurs, the pressure-strain curve exhibits a maximum. To be correct the strain at instability obtained by any indirect method of analysis must be in agreement with this maximum load strain.

In subsequent discussion, an attempt will be made to determine the instability strains from a consideration of the strain distribution, the shape, and also the differential equation of instability (1), and to compare these values with the maximum load strains. Also, it will be shown that the development of the shape and strain distribution may be related to the strain-hardening characteristics of the metal.

Analysis of Instability. The instability of a tensile test specimen results in a restriction of plastic flow to a small region (neck) and thereby produces a distinct change in the contour of the specimen. Also, in uniaxial tension the nonuniformity in strain distribution, resulting from necking, is readily apparent, since the strain is essentially uniform before instability.

However, in certain shapes which include the bulge, it is difficult to visualize how plastic flow could be confined to a small area and result in a pronounced change in the contour. Also, in bulging, any increase in the nonuniformity of strain as a consequence of instability would not be readily apparent from the strain distributions, because of the steep gradients present before instability.

The variation of strain gradients during the deformation may be analyzed if the meridional strains at several given distances from the pole are plotted against the strains at the pole, Figs. 12 and 13. For all metals except 24S-O and 24S-T, after a certain strain at the pole the strain at a given distance from the pole increases at a considerably reduced rate and may become nearly constant at distances greater than 1.50 in. According to this

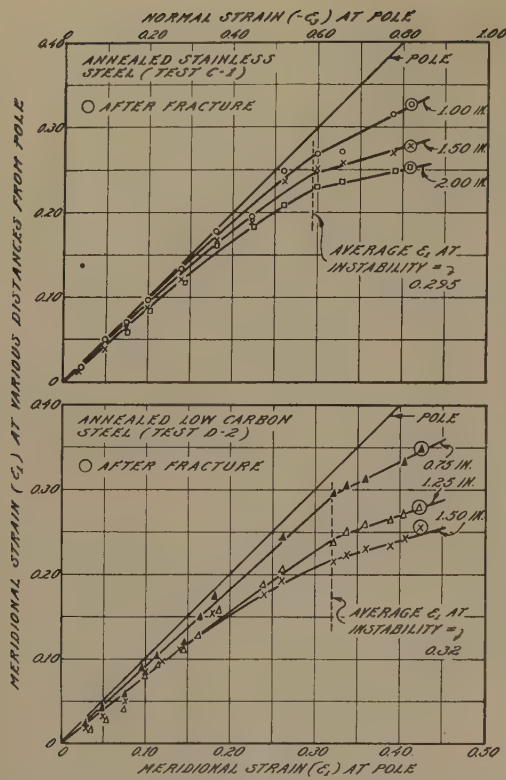


FIG. 12 RELATION BETWEEN MERIDIONAL STRAIN AT POLE AND THAT AT VARIOUS DISTANCES FROM POLE FOR CIRCULAR BULGING OF TWO STEELS

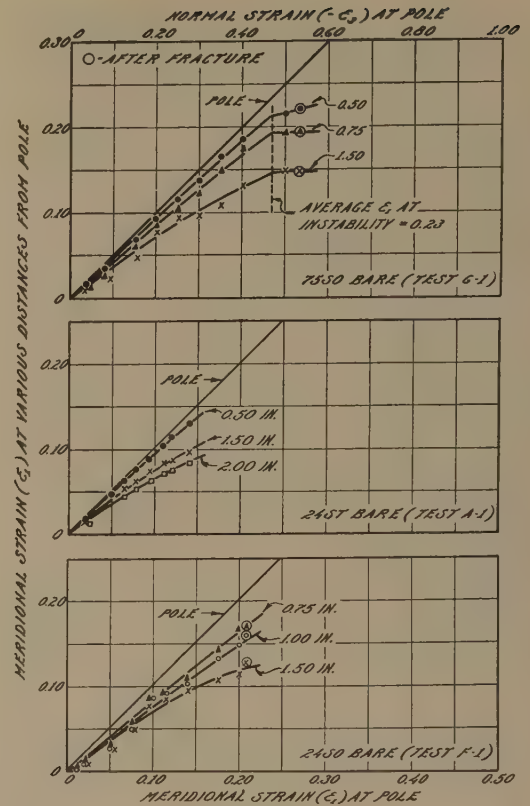


FIG. 13 RELATION BETWEEN MERIDIONAL STRAIN AT POLE AND THAT AT VARIOUS DISTANCES FROM POLE FOR CIRCULAR BULGING OF SEVERAL ALUMINUM ALLOYS

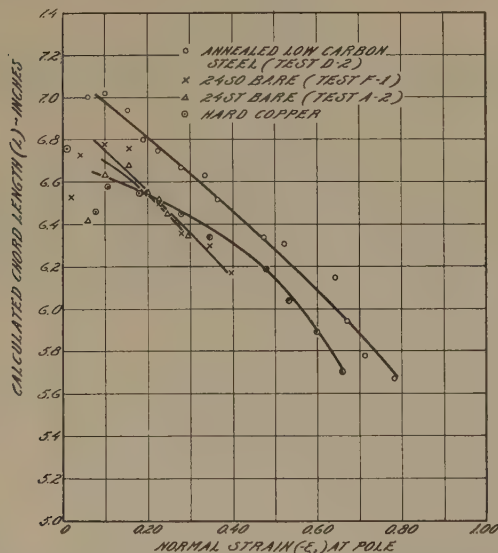


FIG. 14 CHORD LENGTH L AS CALCULATED FROM MEASURED HEIGHT AND RADIUS OF CURVATURE R AT POLE AS A FUNCTION OF NORMAL STRAIN ϵ_3 AT POLE FOR CIRCULAR BULGES OF VARIOUS MATERIALS

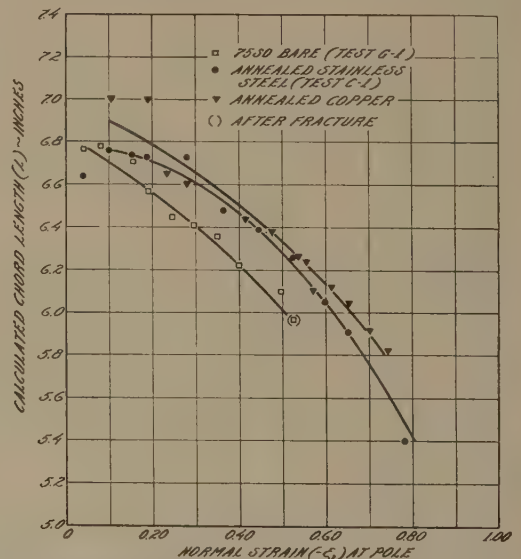


FIG. 15 CHORD LENGTH L AS CALCULATED FROM MEASURED HEIGHT AND RADIUS OF CURVATURE R AT POLE AS A FUNCTION OF NORMAL STRAIN ϵ_3 AT POLE FOR CIRCULAR BULGES OF VARIOUS MATERIALS

concept, the strain at which these curves show a sudden change in slope should be the instability strain. In late discussion, it will be referred to as the "experimental instability strain." This constancy of strain at large distances from the pole might also be concluded from the shape analysis, Fig. 10, which revealed that only small changes in curvature occur in this region after instability.

The phenomenon of instability may also be related to the de-

velopment of the shape. Thus it was shown by a process of graphical construction (see Fig. 10) that at large strains, in the vicinity of the maximum load, the contour of a stainless-steel bulge could be closely approximated by a sphere which passes through the pole and intersected the base plane at the die periphery. This analysis may be made in another manner by calculating for a given strain the chord length L , corresponding to

the experimentally determined radius of curvature R and the height H at the pole according to the following equation

$$L = (8RH - 4H^2)^{1/2}$$

In Figs. 14 and 15, these calculated chord lengths have been plotted as a function of the normal strain at the pole for all materials investigated. It is evident that with increasing strain the calculated chord length decreases at a rapidly increasing rate. In those cases where instability occurred, this calculated chord length becomes smaller than 6 in. (the die diameter). It is apparent that, at a calculated chord length of 6 in., the strain is approximately equal to the instability strain. This peculiar relation between the shape and the instability has not yet been explained satisfactorily. Results of such an estimate of the instability are rather inaccurate, since they are subject to errors in the zero reading on the height gage which are difficult to eliminate. In addition, the effects of die radius are variable, depending upon the metal, and this introduces additional uncertainties.

The strain at which the bulge becomes unstable may also be determined from the differential equation of instability (1). This expression is written in terms of maximum meridional stress s_1 , the natural normal strain ϵ_3 , and the radius of curvature R , at the pole, as follows

$$\frac{d \ln s_1}{d \epsilon_3} = \frac{d \ln R}{d \epsilon_3} - 1$$

or

$$\frac{ds_1}{d \epsilon_3} \frac{1}{s_1} = \frac{dR}{d \epsilon_3} \frac{1}{R} - 1$$

The functions

$$\frac{d \ln s_1}{d \epsilon_3} = \frac{1}{s_1} \frac{ds_1}{d \epsilon_3}$$

and

$$\frac{d \ln R}{d \epsilon_3} = \frac{1}{R} \frac{dR}{d \epsilon_3}$$

occurring in this equation may be obtained graphically from Fig. 11 and Figs. 6 and 7, respectively. If the functions

$$\frac{1}{s_1} \frac{ds_1}{d \epsilon_3}$$

and

$$\frac{1}{R} \frac{dR}{d \epsilon_3} - 1$$

are plotted against the normal strain ϵ_3 , the intersection will represent the solution of the instability equation. Such graphical solutions, yielding the "theoretical instability strain," are presented in Figs. 16 and 17.

The instability strains obtained by these various methods of analysis are assembled in Table 2, together with the maximum load strains. The agreement between these various values and

TABLE 2 INSTABILITY STRAIN VALUES IN BULGING

Material	Maximum load strain $-\epsilon_3$	Theoretical instability strain $-\epsilon_3$	Instability strain from shape analysis $-\epsilon_3$	Experimental instability strain $-\epsilon_3$
annealed stainless steel (Type 347)...	0.62	0.60	0.60	0.59
annealed copper...	0.60	0.60	0.65	0.62
annealed low-carbon steel.....	0.66	0.64	0.64	0.64
58-O bare.....	0.48	0.47	0.60	0.46
hard copper.....	0.56	0.59	0.67	0.58

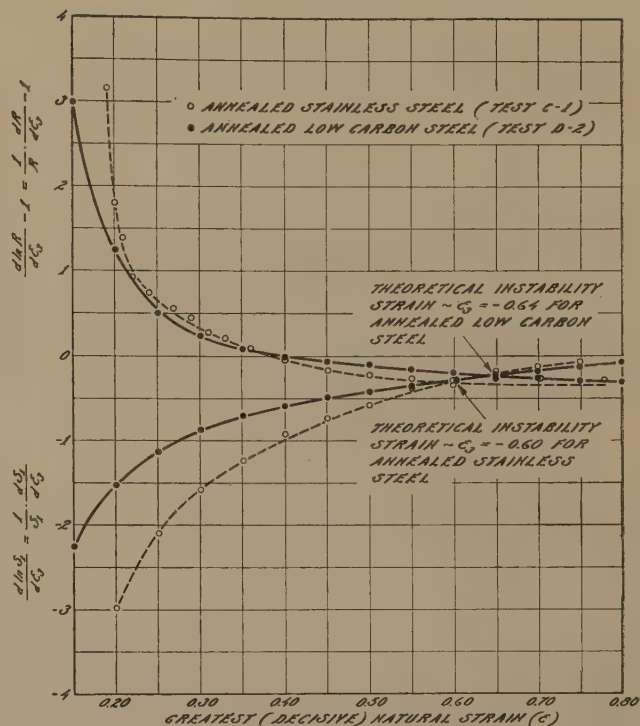


FIG. 16 GRAPHICAL SOLUTION FOR INSTABILITY STRAIN IN CIRCULAR BULGING OF TWO STEELS

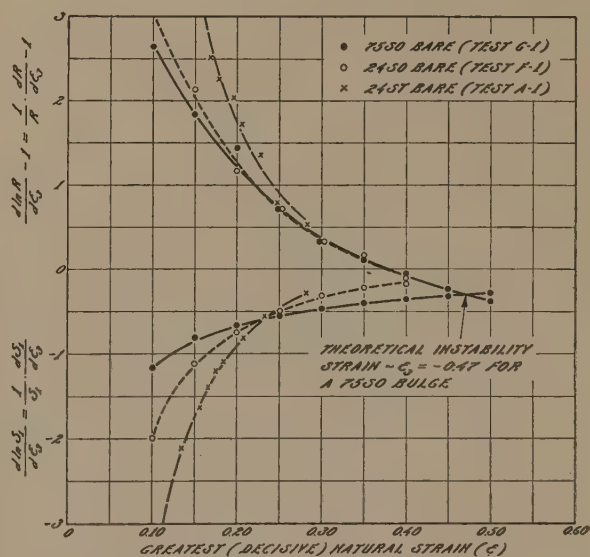


FIG. 17 GRAPHICAL SOLUTION FOR INSTABILITY STRAIN IN CIRCULAR BULGING OF SEVERAL ALUMINUM-ALLOYS

the maximum load strain for a particular material is good (the maximum variation in no case being greater than approximately 8 per cent). These analyses also indicated that instability was not reached for 24S-O and 24S-T. This is confirmed by the absence of a maximum in the pressure strain curves of these metals.

These instability strains in bulging are considerably higher than those in uniaxial tension. This is explained by the fact that in bulging the instability is dependent not only upon the stress-strain characteristics but also on the curvature changes. Instability of the system will occur when the rate of increase in pressure due to strain-hardening of the metal and increasing curvature of the shape is no longer able to compensate for the

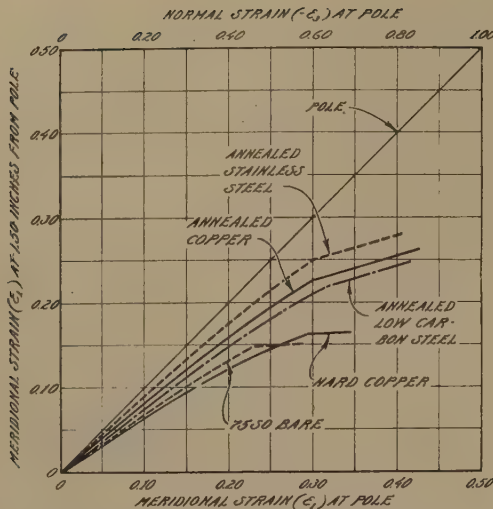


FIG. 18 RELATION BETWEEN MERIDIONAL STRAIN AT POLE AND THAT AT 1.50 IN. FROM POLE IN CIRCULAR BULGING OF VARIOUS MATERIALS

decrease in pressure due to the decreasing thickness of the metal.¹³ Thus the curvature change of the bulge co-operates with the metal strain-hardening to increase the pressure and, consequently, to delay instability. In some cases the metal ductility will not be sufficiently great to reach the instability strain as evidenced in tests on both 24S-T and 24S-O.

Influence of the Strain-Hardening Rate. At instability, all materials exhibit a very nearly spherical contour, while the strain gradients, Figs. 2 and 3, vary considerably. Referring to Figs. 12 and 13, it is apparent that for the same strain at the pole the average strain over the contour may be quite different, depending on the material. This is shown more clearly if the relation between the meridional strain at the pole and that at 1.50 in. from the pole for the various materials is compared, Fig. 18. The slope of these curves for a particular strain at the pole is a function of the average strain. Thus the highest possible average strain for a given strain at the pole (uniform strain distribution) would be represented by the 45-deg line. According to this analysis the material with the highest strain-hardening rate possesses the most uniform strain distribution. Consequently, the curves increase in slope with increasing necking strains in uniaxial tension (Table 3), which are a measure of the strain-hardening ability of a metal. It may be concluded, therefore, that in the case of a metal membrane the strain distribution and, consequently, the functions relating the curvature to the strain at the pole are dependent on the stress-strain characteristics of the metal.

In addition, the height-strain relations should also be dependent on the strain-hardening characteristics. This would be expected since for a given strain at the pole the total arc length would be dependent on the average strain over the contour. Thus, bulges having equal average strains would have equal heights (assuming the shape is a sphere). As pointed out, the average strain is a function of the strain gradient and, for a given strain at the pole, it will be larger the higher the strain-hardening rate. Therefore, considering bulges of various materials having equal strains at the pole, the height will be greatest for the material having the highest strain-hardening rate. This explains the fact that the height of the ductile hard copper is lower at a particular strain than the height of the less ductile aluminum alloys, Figs. 4 and 5.

¹³ An analogous statement was made for instability in simple uniaxial tension by Ludwik (16).

TABLE 3 COMPARISON OF PREDICTED AND THEORETICAL INSTABILITY STRAINS IN BULGING

Material	Necking strain in uniaxial tension $\epsilon_1 = \eta$	Predicted instability strain in bulging (Saibel) $-\epsilon_2$	Theoretic instability strain in bulging $-\epsilon_3$
Annealed stainless steel (Type 347).....	0.40	0.57	0.60
Annealed copper.....	0.35	0.53	0.60
Annealed low-carbon steel.....	0.29	0.47	0.64
75S-O bare.....	0.13	0.28	0.47
Hard copper.....	<0.01	<0.04	0.59

The instability strains in bulging show less variation with the stress-strain characteristics of the metal than do the necking strains in pure tension. This is explained by the fact that, while a decrease in the strain-hardening rate tends to reduce the instability strain by decreasing the slopes of the stress-strain curve it also tends to increase the instability strain by increasing the slopes of the radius-strain curve.

It is evident that while the instability strain in bulging is dependent on the stress-strain function of the metal, it is not simply related to the necking strain in uniaxial tension.

Comparison With Published Results. The stress-strain relations in bulging reported by the authors for 24S-T and 24S-O are compared with those obtained by Gensamer, et al (4) for 12-in. diam bulges of these alloys in the clad condition. The differences in these curves, Fig. 19, are primarily due to the cladding, which should reduce all stress values by an equal amount. However, the curves for 24S-T are not parallel at strains less than approximately 10 per cent, and the maximum strain observed by the authors is somewhat higher than that reported by Gensamer. These differences may possibly be explained by commercial material variations.

An attempt has been made by Saibel (5) to develop an expression yielding the strain distribution for a circular bulge in terms of the radius of the die and height at the pole. This analysis is combined with the use of the instability condition to yield a rela-

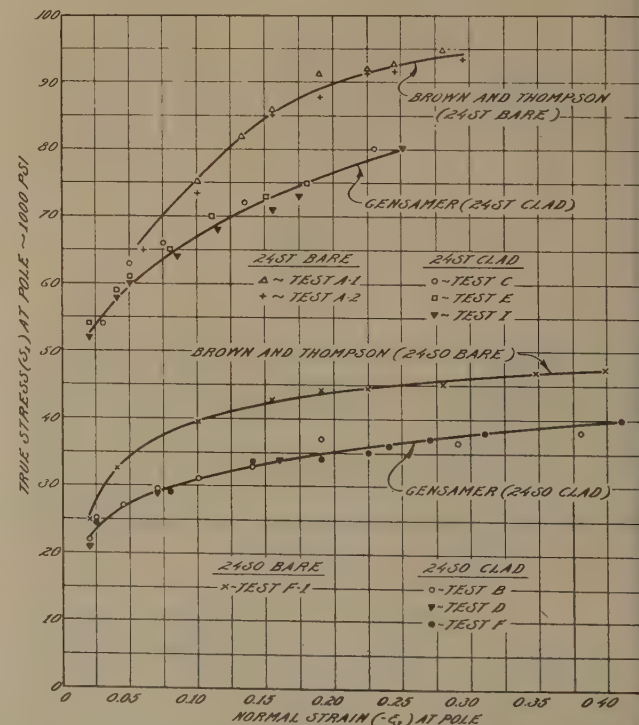


FIG. 19 COMPARISON OF STRESS-STRAIN CURVES OBTAINED BY GENSAMER WITH THOSE OF AUTHORS (Circular bulging of 24S-O and 24S-T, bare and clad.)

tion between the strain-hardening exponent n , in the generalized stress-strain equation

$$\bar{s} = A(\bar{\epsilon})^n$$

and the instability strain in circular bulging. However, this approach appears to neglect the important effects of the stress-strain characteristics on the strain distribution and, consequently, on the strain-curvature relations. Table 3 shows instability strains predicted by means of Saibel's (5) formula are not in agreement with the experimental values reported in this investigation.

CONCLUSIONS

Summarizing the results of this investigation, the following conclusions may be drawn regarding the deformation and failure characteristics of metal membranes subjected to hydraulic pressure:

- 1 An analysis of the development of the shape reveals that the bulge contour is closely approximated by a sphere when the strain at the pole is in the vicinity of the instability strain.
- 2 The characteristics of the strain distribution, and therefore the function relating curvature and strain, are influenced by the strain-hardening characteristics of the metal.
- 3 If sufficiently ductile, a circular metal membrane will exhibit an instability characterized by a large increase in strain at the pole at constant or decreasing pressure.
- 4 For a wide variety of materials, the previously developed differential equation for instability in a circular bulge yields strains which agree with the maximum load strains.
- 5 The strain at instability may also be derived from the development of either the shape or the strain distribution.
- 6 In circular bulging, the instability strain is less influenced by the strain-hardening function than is the necking strain in uniaxial tension.
- 7 Instability strains in bulging cannot be derived from the necking strains in pure tension by any known simple relation.
- 8 If a metal exhibits any discontinuities, such as stretcher strains, large grain size, or surface flaws, the forming limits (maximum strains) in bulging are greatly reduced.

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A Reconsideration of Deformation Theories of Plasticity¹

By D. C. DRUCKER,² PROVIDENCE, R. I.

Deformation theories of plasticity for strain-hardening materials are defined as those which postulate that the components of elastic and permanent strain are completely determined by the existing components of stress. The assumption is made that there is some criterion, in terms of stress, for loading or increase in plastic deformation. A common supposition of this nature is that the permanent strain will increase when the octahedral shearing stress increases.

It is shown that no matter what the loading criterion these deformation theories of plasticity lead to the unacceptable conclusion that large (finite) changes in the components of permanent strain may accompany infinitesimal increases in loading despite strain hardening. Simple illustrations are given and it is demonstrated that an incremental, or so-called flow theory eliminates this undesirable feature.

ORIGIN OF DEFORMATION EXPRESSIONS

A GREAT deal of experimental work has been carried out to find laws which will predict the behavior of materials under combined stress from their action in simple tension. With a few exceptions, the useful tests have been combinations of tension, torsion, and internal pressure applied in various but fixed ratios to thin-walled tubes. For practical purposes, the states of stress can be treated as statically determinate; they are of the form

$$\sigma_{ij} = k\sigma^o_{ij} \dots \dots \dots [1]$$

where σ^o_{ij} is a three-dimensional state of stress (Cartesian components $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}$) and k is a scalar which starts at zero and increases in the course of the experiment. Therefore the directions of the axes of principal stress and also the ratios of the principal stresses remain constant.

These apparently general three-dimensional but actually specialized experiments can be correlated very well with each other and with results in simple tension by (1, 2)³

$$\frac{\sigma_1 - \sigma_2}{\epsilon_1 - \epsilon_2} = \frac{\sigma_2 - \sigma_3}{\epsilon_2 - \epsilon_3} = \frac{\sigma_3 - \sigma_1}{\epsilon_3 - \epsilon_1} \dots \dots \dots [2]$$

a volume-change relation, and the choice of the maximum shearing stress as the criterion of loading. An even better agreement is obtained when the octahedral shearing stress is taken as the

criterion. In this connection, the work of Morrison (5) on size effect is of considerable interest and importance. Tests of Taylor and Quinney (6) show that Equation [2] is an oversimplification of the physical behavior of metals and therefore a more elaborate relation is required to obtain truly excellent agreement with experiment. The much more general relation given by Prager (3) for the components of permanent strain ϵ_{ij}'' ($\epsilon_x'', \epsilon_y'', \epsilon_z'', 1/2\gamma_{xy}'', 1/2\gamma_{yz}'', 1/2\gamma_{zx}''$ in the usual Cartesian form)

$$\epsilon_{ij}'' = P s_{ij} + Q t_{ij} \dots \dots \dots [3]$$

where, with the usual summation convention for repeated subscripts,

P and Q	are functions of J_2 and J_3
$J_2 = \frac{1}{2} s_{ij} s_{ji}$	is proportional to the square of the octahedral shearing stress
$J_3 = \frac{1}{3} s_{ij} s_{jk} s_{ki}$	is the third-order invariant of the stress deviation; J_2 is the second
$s_{ij} = \sigma_{ij} - s \delta_{ij}$	is the stress deviation
$s = \frac{1}{3} \sigma_{ii}$	is the average principal stress
σ_{ij}	is the stress
$t_{ij} = s_{ik} s_{kj} - \frac{2}{3} J_2 \delta_{ij}$	is the deviation of the square of the stress deviation

includes all deformation expressions for incompressible isotropic materials and can be adjusted to fit the data. The difficulty introduced by the change in Poisson's ratio from the elastic to the plastic state has been discussed by Swainger (4).

The analysis of experimental results, or conversely, the prediction of the state of strain from the state of stress, when the directions of the principal stresses and their ratios remain unchanged, is now at a satisfactory stage. Recent experimental work is directed toward improving an already high accuracy or investigating new materials.

Perhaps for this reason a misunderstanding has arisen. The stress-strain relations which correlate the tube tests are often believed to be applicable in a general mathematical theory of plasticity (actually only a few solutions are available for materials with strain hardening). However, such extension is not necessarily permissible. Only under very special conditions (7) will the directions of the axes of principal stress and the ratios of the principal stresses remain constant in actual structures or machine parts in the plastic range.

DEFORMATION THEORY

There are several variations of what has been termed the *deformation* type of mathematical theory of plasticity (1, 2, 3) but their unifying characteristic is that the total strain at a point, and therefore the permanent strain as well as the elastic part, is determined uniquely by the state of stress at the point. The exact form of the dependence and the numeric quantities involved will of course vary with the particular material considered and are not important in this discussion. However, it can be seen that complete isotropy is implied by such a theory if the directions

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² Associate Professor of Engineering, Graduate Division of Applied Mathematics, Brown University. Jun. ASME.

³ Numbers in parentheses refer to the Bibliography at the end of the paper.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society. Paper No. 48-A-81.

of the co-ordinate axes are chosen without regard to the material or the state of stress. This assumption is generally made because of the resulting simplification and the (actually only partly relevant) fact that many important polycrystalline metals are quite isotropic in a statistical sense. Complete isotropy means that the directions of the axes of principal stress and of principal strain coincide.

Some qualifying statements must be made about the dependence of strain on stress even for simple tension, Fig. 1. As long as the stress σ increases, the strain ϵ will increase and will be a

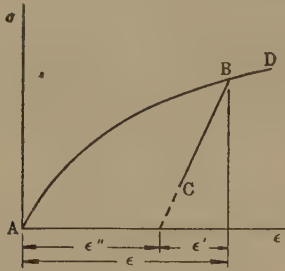


Fig. 1 SIMPLE TENSION

unique function of σ . If, at some point B on the stress-strain curve AB the tension is decreased, the strain will decrease also, but along another line BC which usually has the same slope as the tangent to the original curve at point A . The change in strain on unloading is elastic, as subsequent reloading retraces the path CB and then goes on to D with relatively minor deviations. The process can be followed easily and the final strain calculated if the stress history is given. However, a general mathematical expression for strain in terms of stress becomes too elaborate after a first unloading. Therefore in this discussion only one loading followed by, at most, one unloading will be supposed.

CRITERION OF LOADING

As soon as general variations in the state of stress are considered, the formulation of a loading criterion is no longer a trivial problem. There is no obvious single variable, such as σ in the case of simple tension, or k for the three-dimensional states of Equation [1], whose increase definitely represents loading and decrease unloading.

It is reasonable to define loading in the plastic range as a change in the state of stress which produces a change in the permanent strain. The criterion of loading for each material may be a matter for experimental determination, but the theory of plasticity postulated and the criterion used must be consistent. Therefore, if an originally isotropic material with isotropic strain hardening is assumed, the criterion of loading can involve the magnitudes, including sign, of the principal stresses $\sigma_1, \sigma_2, \sigma_3$, but not their directions. A function $f(\sigma_1, \sigma_2, \sigma_3)$ must exist such that a change in the permanent strain occurs when the value of f increases and no change in permanent strain occurs when f decreases. The maximum shear stress $\frac{\sigma_1 - \sigma_3}{2}$ and the octahedral

shear stress $\frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$ criteria are simple examples of f . Note that the elastic, and therefore the total, strain may change whether f increases or decreases.

No matter what the criterion, it is physically sound to expect, and to require, of a mathematical theory for a strain-hardening material, that a small increase in f produces a small change in the permanent strain. In the limit, the derivative, or the ratio of

the change of (any component of) permanent strain to the change of f , must be finite.

INCONSISTENCY FOR ROTATION OF PRINCIPAL AXES

It can now be demonstrated that the previous conditions and premises of deformation theory are not always consistent. Suppose a state of stress σ_{ij} and plastic strain ϵ_{ij} to exist at any point of any strain-hardening body. Let an infinitesimal change in stress take place of a type constituting loading, e.g., k increase in Equation [1]. An infinitesimal increase in the permanent principal strains will necessarily take place also. For the purpose of description, call the principal stresses corresponding to σ_{ij} $\sigma_1, \sigma_2, \sigma_3$ and the principal strains $\epsilon_1, \epsilon_2, \epsilon_3$. If the material is assumed to be completely isotropic, the principal directions of stress and strain coincide with each other and with the axes of the principal permanent strains $\epsilon_1'', \epsilon_2'', \epsilon_3''$ also. The increase in f , or the amount of loading, depends upon the change in the magnitudes of $\sigma_1, \sigma_2, \sigma_3$ only and is independent of the directions of the principal stresses. Therefore the same infinitesimal increase in f will take place if the directions of $\sigma_1, \sigma_2, \sigma_3$ are rotated through finite angles at the same time their magnitudes are being changed slightly. As previously shown, an isotropic deformation law requires that the axes of permanent strain coincide with the axes of stress as long as loading takes place. Therefore the axes of permanent strain and necessarily the components of strain will change by finite amounts although the loading is infinitesimal and the magnitudes of the principal strains and stresses change only infinitesimally. Passing to the limit, this means that although the magnitudes of the principal stresses are still $\sigma_1, \sigma_2, \sigma_3$ and the principal strains still $\epsilon_1, \epsilon_2, \epsilon_3$, the derivatives of some or all of the permanent strain components ϵ_{ij}'' with respect to f (the loading) become infinite. An infinitesimal increase in f can produce a finite physical change in permanent strain.

This type of inconsistency was first pointed out by Handelman, Lin, and Prager (8) in a different manner for a special state of stress with the equivalent of the octahedral shearing stress as the criterion of loading. The preceding generalization shows the impossibility of overcoming the difficulty by choosing another loading criterion or avoiding a special state of stress. Anisotropic strain-hardening theories of the deformation type would lead to a similar contradiction.

The objection might be raised that the finite rotation with infinitesimal loading is an unusual condition and probably would not be encountered. Although this may be true in most cases, it is, nevertheless, evident that if rotation occurs, and it almost always will, the prediction of a deformation theory will be unreliable. The higher the ratio of rotation to increase in loading, the greater the error.

INCONSISTENCY FOR CONSTANT PRINCIPAL DIRECTIONS

Another and equally, or more, important form of the inconsistency is somewhat difficult to describe in general terms. Suppose the axes of principal stress to remain fixed in direction and consider all possible changes in the magnitudes of the principal stresses. Some changes will constitute loading, some unloading, and for some f will remain constant. It is therefore possible to increase the value of f only infinitesimally (or not at all) while changing $\sigma_1, \sigma_2, \sigma_3$ by finite amounts in many ways. As before, if infinitesimal loading occurs, a deformation theory will predict finite changes in permanent strain for some, not necessarily all, of these finite changes in stress, and in the limit the derivative of the components of permanent strain with respect to the loading f will be infinite.

The proof of this assertion follows if f is a sufficiently continuous function of the principal stresses so that $f = \text{const}$ forms a surface in $\sigma_1, \sigma_2, \sigma_3$ space and $f + df = \text{const}$ is a neighboring

surface. Take a point on f at which⁴ $\sigma_1 = A$, $\sigma_2 = B$, $\sigma_3 = C$ and consider, as an example, the point B , A , C which must lie on the same f surface because f is determined by stress magnitude alone. Any path which lies on the surface $f = \text{const}$ and joins these two points will represent progressive changes in stress which are neither loading nor unloading although the magnitudes of the principal stresses change by finite amounts. Strictly speaking, deformation theories describe only what happens when loading occurs. Thus it is necessary to imagine a path along which f always increases infinitesimally so that the end point lies in the neighborhood of B , A , C , say, at $B + dB$, A , C , if B is algebraically the largest of the three values. As deformation theories are not concerned with which stress is labeled σ_1 , if the permanent principal strains predicted by the particular theory were $\epsilon_1'' = a$, $\epsilon_2'' = b$, $\epsilon_3'' = c$ at A , B , C the prediction for $B + dB$, A , C would differ only infinitesimally from b , a , c so that finite changes of permanent strain of magnitude $|b - a|$ would accompany the infinitesimal loading.

ILLUSTRATIVE EXAMPLES

Simple examples can be used to illustrate the foregoing general discussion. Fig. 2 shows the two-dimensional state of stress produced in a thin-walled tube by internal pressure and axial tension, neglecting the radial stress. A method of rotating the state of stress without changing the magnitudes of the principal stresses has been described previously (9). The addition of torque and the proper variation in internal pressure and axial tension required can be calculated easily. Fig. 3 shows the rotation of a uniaxial tension which has permanently stretched the tube in the axial direction. If this tension is increased in-

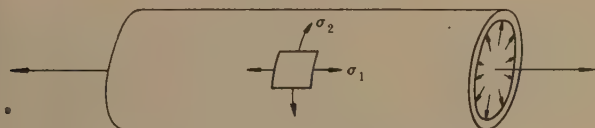


FIG. 2 COMBINED STRESS

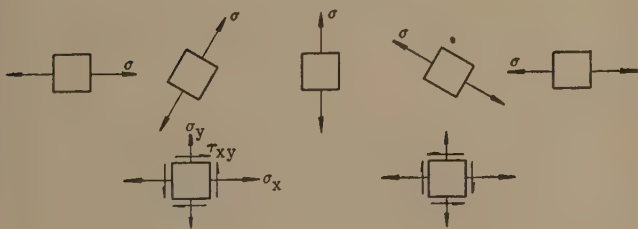


FIG. 3 ROTATION OF A UNIAXIAL TENSION

finitesimally as it is rotated, all isotropic deformation theories predict that the permanent strain will rotate completely with the stress so that when the stress is a pure circumferential tension, the diameter will be permanently stretched and there will be a permanent axial contraction due to the Poisson effect. They therefore lead to the result that for infinitesimal loading a permanent axial elongation has been converted to a contraction. In fact, in the limit, this is in direct contradiction to the assumption of isotropic strain hardening.

In discussing the rotation inconsistency it is not necessary to be explicit about the particular deformation theory or criterion of loading, as the answer for all is the same. However, the incon-

sistency that still arises when the directions remain constant requires a specific illustration for clarity. One commonly accepted system is represented by Equation [2] and the use of the octahedral shearing stress as the loading criterion.

In the case of plane stress, with the 3 direction normal to the plane, the equations reduce to

$$\frac{\sigma_1 - \sigma_2}{\epsilon_1'' - \epsilon_2''} = \frac{\sigma_2}{\epsilon_2'' - \epsilon_3''} = \frac{-\sigma_1}{\epsilon_3'' - \epsilon_1''} \dots \dots \dots [4]$$

and

$$f = \sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 \dots \dots \dots [5]$$

where the coefficient and the square root have been dropped.

Suppose a state of simple tension to exist in the tube; principal stresses A , O , O , and principal permanent strains $\epsilon_1'' = a$, $\epsilon_2'' = -a/2$, $\epsilon_3'' = -a/2$. Substitution in Equation [5] gives $f = A^2$. If the magnitudes of σ_1 and σ_2 are changed in such a manner as to keep f constant (no change in the octahedral shearing stress), there should be no change in the permanent strain. If f is increased infinitesimally then the change in permanent strain should also be infinitesimal. Equation [5] gives the required value of σ_2 for an arbitrary choice of σ_1 . By proper increase of the internal pressure and change in the axial force on the tube, the state of stress $\sigma_1 = 2A/\sqrt{3}$, $\sigma_2 = A/\sqrt{3}$ can be reached for which f is still A^2 , and always was during the process. Taking the permanent change in volume to be zero, Equation [4] gives $\epsilon_1'' = -\epsilon_3''$ and $\epsilon_2'' = 0$. Despite the unchanged octahedral shearing stress, the theory predicts an increase in the permanent circumferential strain of $a/2$.

It might be thought that the inconsistencies could be resolved by simply saying that loading should not be defined as a change in stress producing a change in *any* component of permanent strain but rather a change in some strain invariant, as, for example, the octahedral shear strain. Unfortunately, it can be shown that such an assumption leads, in general, to conclusions which strongly violate known physical facts such as (approximate) isotropic strain hardening.

A comparison with experiment of the problems involved in both the rotation and the constant direction inconsistencies would be of great value.

AN ALTERNATIVE POINT OF VIEW

Another point of view, which is more direct, can be used to analyze explicit stress-strain equations. Consider the simple relation analogous to Hooke's law but containing a plastic instead of elastic modulus

$$\epsilon_1 = \frac{1}{C}[\sigma_1 - \nu(\sigma_2 + \sigma_3)] \dots \dots \dots [6]$$

or the more elaborate one of Swainger (4) involving an elastic modulus E , plastic modulus P , elastic Poisson's ratio q , and plastic p , and stresses at which yield occurred σ_{o1} , σ_{o2} , σ_{o3}

$$\epsilon_1 = \frac{1}{P}[\sigma_1 - p(\sigma_2 + \sigma_3) - \sigma_{o1} + p(\sigma_{o2} + \sigma_{o3})] + \frac{1}{E}[(p - q)(\sigma_2 + \sigma_3) + \sigma_{o1} - p(\sigma_{o2} + \sigma_{o3})] \dots \dots \dots [7]$$

each of which is to be used with the octahedral shear stress or equivalent criterion. C or P is determined by the octahedral shear stress.

As shown in the preceding example, the state of stress σ_1 , σ_2 , σ_3 can be changed completely while increasing the octahedral shear stress infinitesimally or not at all. Substitution in the foregoing

⁴ In this treatment, the subscripts 1, 2, 3 do not represent either ascending or descending algebraic values. Any principal direction may be called 1 or 2 or 3.

equations leads to inconsistent large changes in a component of the permanent strain which the theories predict under this negligible or zero increase in loading. Note that the elastic changes in strain calculated from Hooke's law must be subtracted from the total to get the permanent strain.

REDUCTION TO INCREMENTAL (OR FLOW) THEORY

Differentiation of the most general deformation expression, Equation [3] with respect to the loading criterion f gives^a

$$\frac{d\epsilon_{ij}''}{df} = \frac{dP}{df} s_{ij} + \frac{dQ}{df} t_{ij} + P \frac{ds_{ij}}{df} + Q \frac{dt_{ij}}{df} \dots\dots [8]$$

The limited applicability of deformation theory appears sharply in this form, for as has been shown, the derivatives of the deviations may, according to such theories, become infinite. Therefore a blunt approach is to throw the last two terms out, and suitably restrict P and Q , to leave

$$\frac{d\epsilon_{ij}''}{df} = P_1 s_{ij} + Q_1 t_{ij}$$

or

$$d\epsilon_{ij}'' = (P_1 s_{ij} + Q_1 t_{ij}) df \dots\dots\dots [9]^b$$

an incremental, or flow, theory (8). This approach has some validity because the experimental evidence and the thinking involved in the derivation of deformation theories was not concerned with terms of this nature. Conversely, Equation [9] can be integrated directly when $\sigma_{ij} = k\sigma_{ij}^n$ to give Equation [3].

Consideration of the integration process shows how deformation laws lack required generality. Taking $P_1 = P_1(J_2, J_3^2)$ and $Q_1 = J_3 Q_2(J_2, J_3^2)$, the incremental theory (8), Equation [9], assumes isotropic strain hardening and a criterion of loading just as all isotropic deformation theories do. However, only for very special paths of loading does Equation [9] integrate to give a deformation law. In general, the final strain will depend upon the stress history despite the isotropy, and step-by-step integration will be required. However, none of the preceding statements rules out the possibility that another relationship than Equation [9] may actually apply. Some experimental evidence does exist for a flow-modified deformation theory (10).

CONCLUSION

It has been shown that for general stress histories the assumptions of a deformation type of mathematical theory of plasticity for strain-hardening materials are not completely self-consistent for any criterion of loading. In particular, it was demonstrated that the usual stress-strain relations coupled with the choice of the octahedral shearing stress as the criterion are inconsistent with the also accepted basic notion of isotropic strain hardening. On the one hand the theory postulates that plastic deformation will not take place unless the octahedral shear stress increases a finite amount; on the other hand the equations can and do yield large changes in the permanent strain for infinitesimal or zero increase in the octahedral shearing stress.

Solutions of plasticity problems based on these or any other equations giving total or permanent strain as a function of the existing state of stress may therefore be seriously in error if the

^a The terms $d\epsilon''/df$, dP/df are not quite derivatives of the usual type. They mean the limit of the ratio of the increment in the quantity (due to a change in the state of stress) to the increment in f (due to stress) as the change in f approaches zero.

^b The total strain increment is obtained by adding the change in the elastic components of strain (Hooke's law terms) to the plastic changes.

It is of interest to note that if f is taken as J_2 or the octahedral shearing stress, Equation [9] leads to a form similar to Equation [2] with the total strains replaced by strain increments.

state of stress at each point does not just increase in the scalar manner of Equation [1].

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Discussion

WENDELL P. ROOP.⁷ The argument makes use of a phase space in which the co-ordinates are values of the three principal stresses at a single point in the geometrical space. The history of a given loading sequence is depicted by a line in the phase space which passes from each to the next of a series of loading conditions. As these conditions succeed each other in time, each point on the curve is associated with an instant of time. Or this scheme may be generalized by substituting another variable for time which designates the level of loading, but for which the symbol " t " is also used. It is naturally monotonic with respect to time.

A function " f " of the principal stresses is now imagined, such as the octahedral stress, or any of the functions which are invariant with respect to orientation in the geometrical space, or any other function which may happen to be chosen for a special physical significance. Contoured surfaces of constant f are established in the phase space.

The slope of f on t varies with orientation in the phase space; thus in passing normally from one of the contour surfaces to an adjoining one with a higher value of f , the load-sequence curve would show a maximum of $\frac{df}{dt}$. But if this curve lies within such

a surface, or lies tangentially to it, then $\frac{df}{dt} = 0$. The writer would like to exclude the case where $\frac{df}{dt} < 0$ and to refer to the remaining cases as those of monotonic loading.

The argument, as the writer understands it, relates to the cases where $\frac{df}{dt} = 0$.

Among a group of theories of plasticity which he calls "deforma-

⁷ Swarthmore College, Swarthmore, Pa.

tion theories" such a case, the author says, leads to contradictions. The following characteristic of the theory which the writer would like to use is an assumption relating to the strains:

Start from a point at which the principal stresses are $\sigma_1 = A$, $\sigma_2 = B$, $\sigma_3 = C$; the principal strains which exist at this stage will be called $\epsilon_1 = a$, $\epsilon_2 = b$, $\epsilon_3 = c$. Now increase each of the principal stresses by a factor $1 + \Delta$ where Δ is a small fraction, say, one per cent. Then it is common experience in a great variety of cases of plastic flow that each of the principal strains will also increase by a small though different fraction. If the axes of principal strain remain parallel to those of stress then this fraction of increase of strain will be the same for all three principal strains, in the ratio $1 + \delta$ to 1. The ratio Δ/δ may vary; and as the fracture point is approached the whole analysis may break down. But in a large number of practical cases the conditions described will continue to hold up to fairly large strains, say, 10 or 20 per cent or even higher values.

Cases like this are certainly included among those for which a deformation theory may be proposed. But the author uses the expression deformation theory in a broader sense. He asks us to consider especially cases in which f does not change, as it does not, for example, when σ_1 is changed from A to B and at the same time σ_2 from B to A . It is quite clear that under these circumstances the principal strains may find themselves quite undecided about what to do.

But might we not now restate the problem? We have asked whether deformation theories are valid and given the answer that they are not since in general no unique function of the strains exists which can be linked with f .

But under certain circumstances such a strain function does exist. The writer has seen several sets of experimental data, notably in tube tests under various ratios of internal pressure to axial tension, in which the same curve of octahedral stress on octahedral strain is obtained under widely varying values of that ratio. May we not inquire rather about the limits within which this is true rather than those within which it is false?

The author cites cases in which a contrary result is found, notably that in which the tube is subjected also to torsion; this is a method of "rotating the state of stress without changing the magnitudes of the principal stresses." If the author is willing to say that under these conditions no function of the principal strains exists which stands in a unique relation to the octahedral stress, the writer is ready to take his word for it. However, he goes further and refers to an "inconsistency that still exists when the directions remain constant."

For example, under simplified conditions the author considered the case where $\sigma_3 = 0$ as in the familiar tube tests. The whole range of cases which the writer wishes to consider is included when n , the ratio of hoop stress to axial stress, is varied from zero to unity. When hoop stress exceeds axial stress, the subscript 1, as ordinarily used, is applied to the hoop stress, and as it continues to increase $n = \sigma_2/\sigma_1$ diminishes toward zero.

In the simplified phase-diagram the co-ordinates are σ_1 and σ_2 and one octant is all that is needed to cover the whole range of cases now considered. Such a diagram is shown in Fig. 4. Half of the diagram has been used to show contours of equal octahedral stress and the other half for octahedral strain, calculated by the simple deformation theory.

On this diagram a path of progressive change has been marked out, beginning at A and ending at D . If the change is made by passing from A directly to D , the simple theory gives the correct result. By this it is meant that when the principal stresses are varied as shown in the lower octant the octahedral strain will have the progressive values shown in the upper octant. The principal strains will have the values

$$\epsilon_1 = \frac{\gamma}{\tau} \cdot \frac{2}{3} (2 - n) \sigma_1$$

$$\epsilon_2 = -\frac{1 - 2n}{2 - n} \epsilon_1$$

$$\epsilon_3 = -\frac{1 + n}{2 - n} \epsilon_1$$

It is easy to check the agreement of these results with the basic formula

$$\frac{\sigma_1 - \sigma_2}{\epsilon_1 - \epsilon_2} = \frac{\sigma_2 - \sigma_3}{\epsilon_2 - \epsilon_3} = \frac{\sigma_3 - \sigma_1}{\epsilon_3 - \epsilon_1} = \frac{\sigma_1}{\epsilon_1} \frac{2 - n}{3} = \frac{1}{2} \frac{\tau}{\gamma}$$

May the writer ask the author if he is willing to agree that if the alternative path $A B C D$ does not deviate too widely from $A D$ the results obtained by following $A D$ may offer a usable approximation? This seems to be a common case in the processes which the writer is studying. If the writer knew how to calculate the strain increments on path $A B C D$, the writer would like to do it.

Another feature of the work which the writer has in hand is the application of measured residual strains to estimate of the energy used in producing them. It is quite possible that in approaching

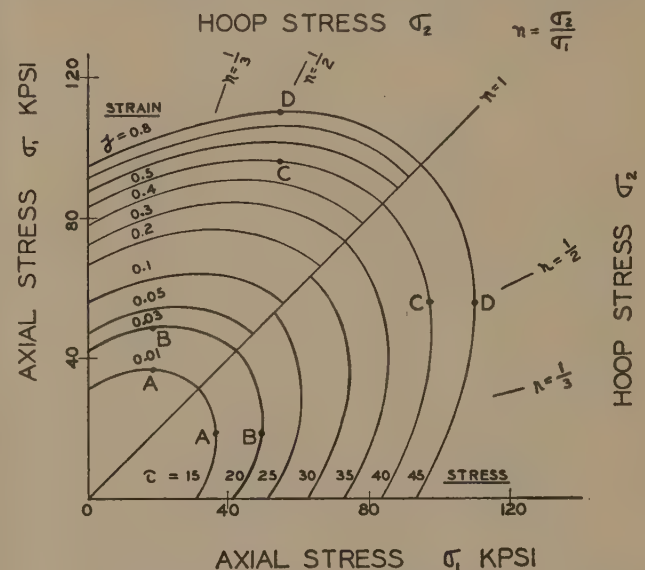


FIG. 4 CONTOURS OF CONSTANT OCTAHEDRAL STRESS τ AND OF CONSTANT OCTAHEDRAL STRAIN $\gamma = (\tau/47.6)^4$ AS OBSERVED FOR SHIP STEEL

the final pattern of residual strains the metal at a given point may have passed through stages in which n did not maintain a strictly constant value. But is, or is not, the energy value the writer got (by neglecting those changes in the value of n) more acceptable than the values of the separate principal stresses would be?

AUTHOR'S CLOSURE

Captain Roop brings up the very important question of how much deviation from "radial" loading, $\sigma_{ij} = k\sigma_{ij}^0$, will put deformation theory seriously in error. The answer is quite easily obtained by integration if a particular incremental form is assumed, e.g., $d\epsilon_{ij}'' = P_1(J_2)s_{ij}dJ_2$ may be compared with the octahedral form $\epsilon_{ij}'' = P(J_2)s_{ij}$.

This, however, is not quite the problem of greatest interest. When the state of stress is nonhomogeneous, as in the case of a notched bar, the stress path followed by each point depends not

only upon the history of the boundary loading but also upon the stress-strain curve of the material under simple stress. If strains are very large, the curve usually is very closely a simple power term for most of the strain range. It is probable therefore that an energy expression based on elementary deformation theory will not differ enormously from one based on elementary incremental or flow theory. On the other hand, large differences show up in the stress-analysis problems of the machine designer where strains are still small.

A word of caution seems in order. One should not allow either past experimental habits or apparently rigorous mathematical formulations to obscure the true physical picture. The behavior of material is most complex. It is clear that any of the very simple stress-strain relations, even of the incremental type, will fit only relatively simple loading paths adequately. Far more experimental information is required to enable a decision to be made as to the least complicated theory which will be useful in particular application.

Power-Plant Cycle Evaluation

By J. K. SALISBURY,¹ SCHENECTADY, N. Y.

Thermodynamic losses in the heater system of a steam-power-plant cycle are analyzed. All heater systems are rated with respect to an "ideal" heater system. Simple algebraic formulations permit determination of arrangement loss, distribution loss, and loss due to terminal difference and pressure drop in any heater cycle. Miscellaneous corrections to cycle heat rate are described and two typical corrections are analyzed. The paper constitutes a new fundamental approach to analysis of feedwater heating cycles, based largely upon a simplifying assumption, valid for the purpose. It may be regarded as a sequel to a previous paper covering major parameters of steam-plant regenerative cycles.

INTRODUCTION

THIS work is a sequel to a paper published by the author² outlining a method of analysis for steam-power-plant cycles. The previous paper considered major parameters of steam-cycle performance, using several simplifying assumptions. All heaters were considered to be contact-type heaters having identical condensate enthalpy rises. No pressure drop in extraction lines or terminal difference in heaters was taken into account. Deviations of actual cycles from this ideal cycle will be investigated in this paper and methods illustrated for determining the loss which results.

The previous paper presented data for determination of the theoretical gain in heat rate resulting from use of an infinite number of heaters heating feedwater to the saturated liquid enthalpy corresponding to throttle pressure. The fraction of this gain realizable in an actual cycle was presented as a function of the fraction of the "possible" rise, for various numbers of heaters. In determination of the theoretical gain with an infinite number of heaters, a "quadratic approximation" was used to represent analytically the state line of a steam turbine. In this empirical equation, written for a known state line, the independent variable was the fraction of the possible rise in feedwater enthalpy. The quadratic approximation is of great usefulness in analyzing actual heater cycles. Its form is $(H - h) = ax^2 + bx + t$.

In any heater cycle there are three major types of loss, as well as miscellaneous heat-rate corrections for subcooling in the condenser hot well, use of condensate-cooled generator coolers, and boiler-feed-pump power. The major losses may be classed as follows:

- 1 Arrangement loss.
- 2 Distribution loss.
- 3 Pressure drop and terminal difference loss.

These losses will be considered in sequence, and a method out-

lined for their analysis. Brief discussion of some of the miscellaneous losses is also given.

The need for such analysis arises in at least three ways: (a) It is of value to the designer of a feedwater-heating cycle, in selection of its characteristics, to know the effect of various minor changes. (b) It is useful to the test engineer in correcting test results for uncontrollable deviations of test conditions from guarantee cycle conditions. (c) It is of value to the operator in determining readily whether the feedwater-heating system requires maintenance which will justify temporarily taking a heater out of service.

METHOD OF ANALYSIS

Evaluation of two heater arrangements usually is accomplished by actual calculation of the two cycles and comparison of the results. This method has the disadvantage that a small difference of two large numbers is sought, and extreme care in calculation is necessary. Presentation of a strictly accurate method of analysis for all types of cycle seems to be a prodigious task. However, by use of a simple device the difference in actual cycles easily may be calculated with accuracy. All cycles are rated by their loss with respect to the ideal cycle. The error in this simple method has been shown by the author to be of completely negligible magnitude.

In the previous paper,² it was demonstrated that the quantity of heat given up in a feedwater heater by 1 lb of steam ($H - h$), varies within a very narrow range throughout the turbine state line. If, for purposes of cycle comparison, we assume the state line to have a shape such that the variation in this quantity is zero, many of the difficulties disappear. This situation is entirely conceivable, and requires only that the various portions of the turbine have efficiencies which cause the state line to have a slightly different shape, that is, by assuming a turbine with certain internal characteristics closely approximating the actual ones, we are provided with a state line which greatly simplifies analysis, and is valid for the purpose. The effect of this assumption is made clear in Fig. 1, illustrating the true and assumed state lines, and characteristic variation of $(H - h)$. Since both the actual cycle and the ideal with which it is compared are assumed to operate on the assumed state line, the difference found is strictly correct for the state line used, and very close indeed to the difference for the true state line.

Another device, also of great usefulness in such analysis, consists in use of a dimensionless variable y , the meaning of which is made clear in Fig. 2. The heat given up by 1 lb of steam in the condenser ($H_c - h_c$), is used to represent the value of this quantity throughout the state line to a first approximation.³ Using the nomenclature of the previous paper, $(H - h) \cong t$. Thus in Fig. 2 we may set up a heat balance

$$(1 + w) R/n = w_1 (H - h) = w_1 t \dots \dots \dots [1]$$

Solving this equation

$$w_1 = (1 + w) R/nt = (1 + w) y \dots \dots \dots [2]$$

$$y = \frac{w_1}{1 + w} = \frac{R}{nt} \dots \dots \dots [3]$$

³ A further refinement is of course to use an average value of $(H - h)$, easily found by integration of the quadratic approximation

$$(H - h)_{\text{avg}} = 1/3 ax^2 + 1/2 bx + t$$

¹ Division Engineer, Thermal Power Systems Division, General Engineering and Consulting Laboratory, General Electric Company. Mem. ASME.

² "The Steam-Turbine Regenerative Cycle—An Analytical Approach," by J. K. Salisbury, Trans. ASME, vol. 64, 1942, pp. 231-245.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society. Paper No. 48-A-57.

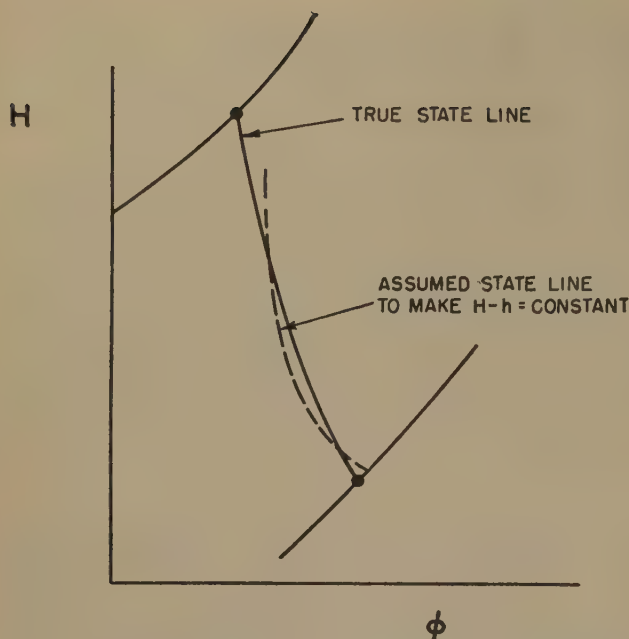
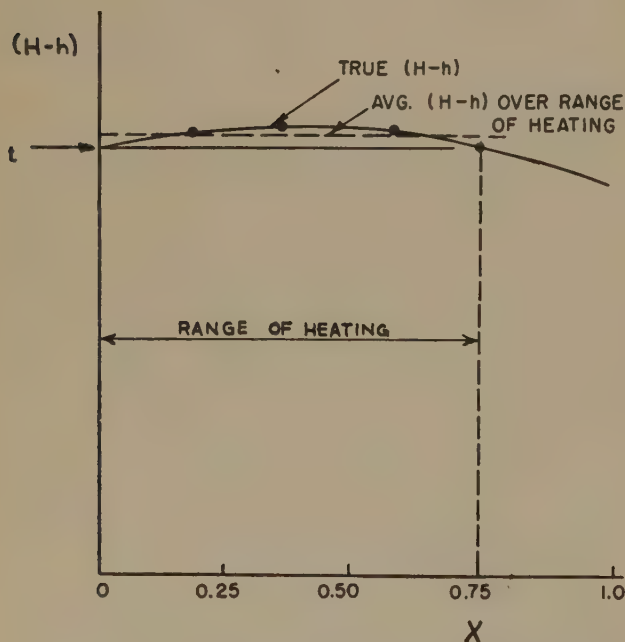


FIG. 1 (a) COMPARISON OF ACTUAL AND APPROXIMATE STATE LINE

FIG. 1 (b) VARIATION OF $(H-h)$ WITH FEEDWATER ENTHALPY RISE

The dimensionless variable y thus is defined as the ratio of the average rise per heater to the average heat given up per pound in each heater. In the illustration used, the latter is represented by the quantity t , for simplicity. It is seen also that in this simple case the quantity y represents the ratio of the extraction quantity to the quantity of condensate flowing into the heater. This quantity can be used to build equations for various heater cycles from which the loss easily can be determined.

BASIC RELATIONS

There are four types of heaters, namely, the contact heater, the flashed heater, the drain-cooler heater, and the pumped heater.

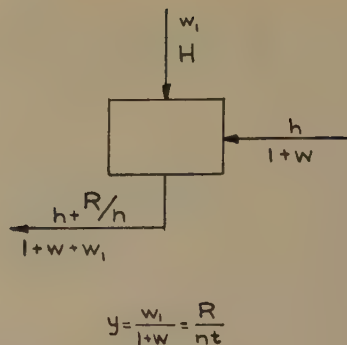


FIG. 2 NOMENCLATURE FOR CONTACT-TYPE HEATER

The first three are distinct types. The last type becomes thermodynamically identical with the first, if the terminal difference is assumed to be zero. In analysis of arrangement loss, the latter assumption is made; the loss due to use of a pumped heater arises entirely as a result of the existence of a terminal difference. Thus for arrangement loss we are concerned with only three distinct types, namely, the contact, the flashed, and the drain-cooler heater.

Nomenclature. In general, the nomenclature of the previous paper² is used. However, it is necessary to introduce a few additional symbols, as follows:

- a = subscript, actual cycle; coefficient in the quadratic approximation
- b = coefficient in the quadratic approximation
- c = number of contact heaters; subscript, contact-heater cycle
- f = number of flashed heaters (includes also drain-cooler heaters, since they are a special type of flashed heater)
- d = number of drain-cooler heaters
- w = extraction flow per pound of condenser (turbine exhaust) flow in a regenerative cycle
- e = extraction flow per pound of throttle flow in a regenerative cycle. Note that $(1-e) = \frac{1}{(1+w)}$
- C = $(1+y)$ in equations; alternatively used to designate a series of consecutive contact heaters
- F = $(1-y)$ in equations; alternatively used to designate a series of consecutive flashed heaters
- D = $\left(1 - \frac{y}{1+ky}\right)$ in equations; alternatively used to designate a series of consecutive drain-cooler heaters
- h = subscript, hot well (used in e_h , low-pressure heater drains flashed to the hot well)
- k = effectiveness of a drain cooler; decrease in temperature of drains passing through drain-cooler, divided by temperature rise of heater with which drain cooler is associated, and from which the drains come
- P = series of consecutive pumped heaters
- q = heat rejected in regenerative cycle
- Q = heat supplied in regenerative cycle
- r = heat rate, in Btu supplied per Btu useful energy output, based upon shaft output of turbine, assuming zero mechanical losses (bearings, governor, etc.)
- L = approximate⁴ fractional loss in heat rate due to deviations of the cycle from the ideal. The use of this variable greatly simplifies many of the equations.

⁴ The fractional error in this approximation is equal to the loss itself, i.e., for 1 per cent loss the error in heat rate is only 0.01 per cent. Even this error may be eliminated, however, as shown.

$$\text{The true loss, } \frac{\Delta r}{r} = \frac{L}{1-L}$$

1,2... = subscripts on C, F, D, and P indicating number of heaters in series

Change in Cycle Heat Rate. It readily may be shown that for any change in heat rejected or heat supplied the true change in cycle heat rate is given by

$$\frac{\Delta r}{r} = \frac{(r-1) \left(\frac{1 + \frac{\Delta q}{q}}{1 + \frac{\Delta Q}{Q}} - 1 \right)}{1 - (r-1) \left(\frac{1 + \frac{\Delta q}{q}}{1 + \frac{\Delta Q}{Q}} - 1 \right)} \dots \dots \dots [4]$$

Based on the foregoing definition of L it is seen that

$$\frac{L}{r-1} = \frac{1 + \frac{\Delta q}{q}}{1 + \frac{\Delta Q}{Q}} - 1 \dots \dots \dots [5]$$

If use of an actual cycle does not cause extra heat to be rejected in the condenser, Equation [5] may be simplified. In this instance the loss in heat rate due to terminal difference, pressure drop, and other deviations from the ideal cycle is readily seen to be given by

$$L = -(r-1) \left[\frac{\frac{\Delta Q}{Q}}{1 + \frac{\Delta Q}{Q}} \right] \dots \dots \dots [6]$$

If the final feedwater enthalpies in the actual and ideal cycles are identical (as is frequently the case), the fractional change in heat supplied is identical with the fractional change in throttle flow per pound of condenser flow, that is

$$\frac{\Delta Q}{Q} = \frac{\Delta w}{1+w} \dots \dots \dots [7]$$

Thus the loss in heat rate is entirely a function of the change in throttle flow per pound of condenser flow.

In some instances a low-pressure heater is used from which drains are flashed to the condenser hot well. Introduction of this extra heat rejection to the condenser necessitates use of Equation [5]. It may be shown that the loss is given by

$$\frac{L}{r-1} = (1+w)_e [(1-e)_a + e_h y (1-k)] - 1 \dots [8]$$

In Equation [8] both methods of expressing the extraction quantity are used. Although the original method of expression of this quantity in terms of pounds per pound of condenser flow is most convenient for the ideal cycle, for actual cycles this quantity can be expressed more conveniently as pounds extraction per pound of throttle flow, e . The quantity $(1+w)_e$ applies to the ideal cycle.

ARRANGEMENT LOSS

Arrangement loss is that loss which results from the use of surface-type heaters (either with or without drain coolers) instead of contact heaters. It is a function also of the sequence in

which various types of heaters are used in a given cycle. In analysis of arrangement loss it is assumed that all heaters are spaced equally, and that none has terminal difference or pressure drop. Since terminal difference is eliminated from consideration, the pumped heater becomes identical with the contact heater. Distribution loss (due to unequal heater rises), and terminal-difference and pressure-drop loss are taken into account by separate analysis.

It is required that we determine the fractional change in throttle flow in an actual cycle with respect to the ideal cycle. By using the variable y , under the assumptions heretofore given, we are able to write equations of the throttle flow for any type of cycle. Equations are illustrated for simple cases.

In the previous paper,² it was demonstrated (although the variable y was not explicitly used therein) that the throttle flow per pound of condenser flow for an all-contact-heater cycle could be expressed by the equation

$$(1+w) = (1+y)^n \dots \dots \dots [9]$$

For a cycle in which all heaters are spaced equally, and all are flashed heaters, each flashing its drains to the next lower heater, the lowest heater flashing its drains to the condenser hot well, as shown in Fig. 3, it may be proved that

$$(1-e) = (1-y)^n \dots \dots \dots [10]$$

From the definition of w and e

$$1+w = \frac{1}{1-e} \dots \dots \dots [11]$$

Rewriting Equations [9] and [10] in the binominal expansion, we see that

$$(1+w)_e = 1 + ny + \frac{1}{2} n(n-1) y^2 + \dots \dots [12]$$

$$(1+w)_f = \frac{1}{1 - ny + \frac{1}{2} n(n-1) y^2 - \dots \dots [13]}$$

for the contact and flashed-heater cycles, respectively. It is apparent that the two differ only in second approximation.

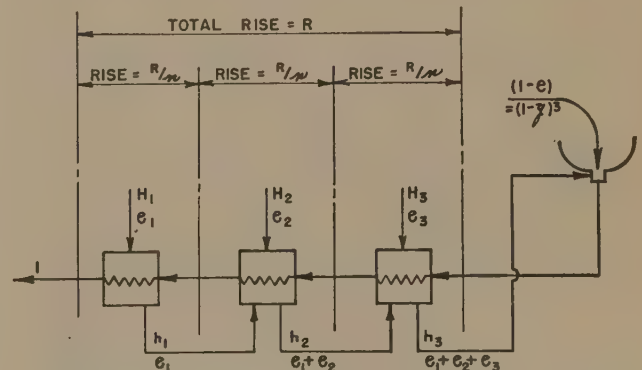


FIG. 3 NOMENCLATURE FOR FLASHED HEATER CYCLE

It also may be shown that in a cycle consisting entirely of drain-cooler heaters, as shown in Fig. 4, the condenser flow per pound of throttle flow is given by

$$(1-e) = \left(1 - \frac{y}{1+ky} \right)^d \dots \dots \dots [14]$$

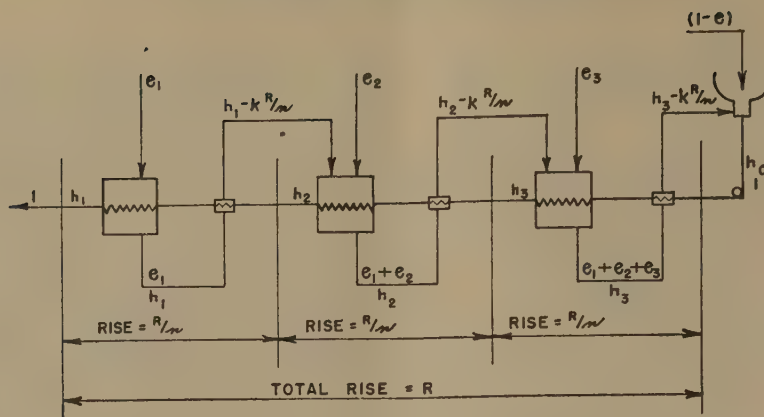


FIG. 4 NOMENCLATURE FOR DRAIN-COOLER HEATER CYCLE

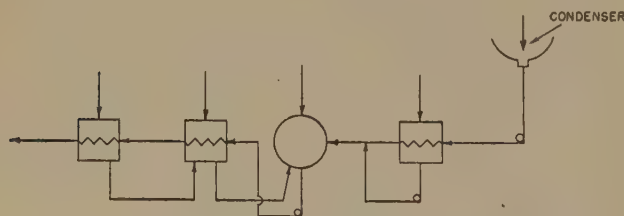
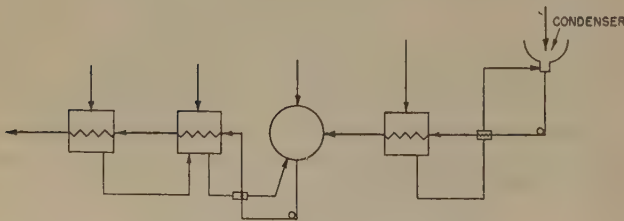
It is interesting to note that if the effectiveness, k , of the drain coolers is 1.0, the drain-cooler-heater cycle becomes identical with the contact-heater cycle; if the effectiveness of the drain coolers is zero, the drain-cooler-heater cycle becomes identical with the flashed-heater cycle. Thus Equation [14] is a generalized form for the homogeneous cycle, a cycle in which all heaters are alike. Furthermore, drain-cooler heaters are thermal hybrids of the contact- and flashed-heater types, the similarity to either being measured entirely by the value of effectiveness of the drain cooler, k .

An extension of this approach provides a method whereby equations may be written, more complex, to be sure, for all other arrangements of heaters, in nonhomogeneous cycles. While it is the author's intention to publish details of the derivation of equations for all conceivable cycles, in the interest of space-saving only two illustrative results are given here, together with curves of the loss for certain commonly used four-heater cycles.

The cycle shown in Fig. 5 is commonly used in modern power plants. It is known as the $F_2C_1P_1$ cycle. The arrangement loss of this cycle may be derived from an equation expressing condenser flow per pound of throttle flow

$$(1 - e) = \frac{(1 - y)^3 + y}{(1 + y)^2} \dots \dots \dots [15]$$

Combining this equation with Equation [8], it is found that

FIG. 5 DIAGRAM OF $F_2C_1P_1$ CYCLEFIG. 6 DIAGRAM OF $F_1D_1C_1D_1$ CYCLE

$$\frac{L}{r - 1} = (1 + y)^2 [(1 - y)^3 + y] - 1 \dots \dots \dots [16]$$

In this arrangement if the total enthalpy rise through the feedwater-heating system is 300 Btu, so that the average rise per heater is 75 Btu, the average value of y is approximately 0.070 for commonly used steam conditions. Substituting this value in Equation [16]

$$\frac{L}{r - 1} = 1.07^2 [0.93^3 + 0.07] - 1 = 0.0010513$$

If the ideal cycle heat rate in Btu per Btu is 3.0, the loss is 0.21026 per cent, or more accurately

$$\frac{\Delta r}{r} = \frac{0.0021026}{0.9978974} = 0.2107 \text{ per cent}$$

The arrangement shown in Fig. 6 is designated as the $F_1D_1C_1D_1$ cycle. The equation for the loss in this cycle is

$$\frac{L}{r - 1} = (1 + y)^3 \left[\frac{(1 - y)^2 + ky}{1 + ky} (1 - y + ky) + y (1 - k) \right] \times \left[1 - \frac{y}{1 + ky} + \frac{y^2 (1 - k)}{1 + ky} \right] - 1 \dots \dots [17]$$

Alternatively written as

$$\frac{L}{r - 1} = C^3 \left[(F^2 + ky) D + y (1 - k) \right] \left[1 + \frac{y^2 (1 - k)}{1 - y + ky} \right] D - 1 \dots \dots [17a]$$

For the conditions of the previous example, and $k = 0.75$, the loss is found to be 0.1169 per cent. Thus it is clear that under these conditions by the use of an extra drain cooler on the second heater from the top and a drain cooler on the lowest pressure heater, there is an improvement in the cycle of 0.0938 per cent. By an extension of this method the contribution of each drain cooler to this gain may be determined.

The foregoing cases illustrate a simple and a complex loss equation. By consideration of the combinations and permutations of the various types of heater one may set up equations for all possible combinations. These equations may be written in generalized form, so that by substitution of the number of heaters of various types at various locations the determination of loss becomes a simple matter of numerical evaluation of the equation. Fig. 7 illustrates the loss for twelve typical four-heater cycles, derived from such generalized equations. The ordinate is to be

multiplied by the quantity $(r - 1)$ to arrive at the true loss. It may be demonstrated from the loss equations that the approximately cubic character of the curves in Fig. 7 is correct.

DISTRIBUTION LOSS

In choosing extraction openings for a feedwater-heating cycle, the turbine designer or consulting engineer frequently is confronted with a choice of two or more extraction openings on the turbine. Much controversy has resulted from the inability of designers to determine the intrinsic loss which results from maldistribution of heaters. Usually the issue is confused by extraneous factors not inherent in heater location. In general, the loss which results from unequal rises across the respective heaters is of extremely small magnitude. The author has established to his own satisfaction that if the enthalpy rises across the respective heaters are identical, the loss which results (with respect to

$$(1 + w) = (1 + y_1)(1 + y_2)(1 + y_3) \dots [18]$$

For convenience let

$$y_1 = (1 + \alpha_1)y; y_2 = (1 + \alpha_2)y; y_3 = (1 + \alpha_3)y \dots [19]$$

It may be shown by algebraic manipulation that to a very close approximation

$$\frac{\Delta w}{(1 + w)} = -1/2 \left(\frac{y}{1 + y} \right)^2 \sum_{n=1}^n \alpha_n^2 + \dots [20]$$

in which the prime designates the new or actual contact-heater cycle with distorted heater distribution. Because the change in throttle flow per pound of condenser flow is expressed as a fraction of the flow in the cycle with distorted distribution, Equation [6] applies directly

$$\frac{L}{r - 1} = \left[1/2 \left(\frac{y}{1 + y} \right)^2 \sum \alpha_n^2 - \dots \right] \dots [21]$$

The value of y in Equation [21] is that of the ideal heater cycle with equal enthalpy rise per heater. The quantity α obviously designates the fractional deviation of the actual value of y from this ideal cycle value. Since the second approximation of Equation [20] involves the quantity $y/(1 + y)$ to the third power, and since the latter normally has a value of approximately 0.05, the error in the calculated loss is considerably less than 5 per cent. Thus Equation [21] is completely acceptable in all cases.

To exemplify the small magnitude of the loss caused by maldistribution it may be shown that in a six-heater cycle, where two heaters have 60 per cent more than the average rise and four heaters have 30 per cent less than the average rise, the net loss in cycle heat rate is approximately 0.24 per cent.

To illustrate graphically the order of magnitude of distribution loss, Fig. 8 is presented. Two types of cycle are considered. The upper set of curves applies to a cycle having all flashed heaters, and the lower set of curves to one having all contact heaters. It is assumed that for each of the four heaters in turn the rise is varied from 0 to 200 per cent of the average rise, the remaining heaters in each instance dividing equally the remaining enthalpy rise of the cycle. In all cases performance is referred to the ideal cycle with equal enthalpy rise per heater. The dash curve in the lower set, calculated on the simplifying assumption that all heaters have equal values of $(H - h)$, applies to any heater. The other curves in this group accurately take into account actual variations of $(H - h)$ throughout the state line for the steam conditions given on the curve. The spread in the curves obviously is due entirely, therefore, to the variation in $(H - h)$, and of only secondary significance.

No simple analysis has been devised for the flashed-heater cycles illustrated by the upper set of curves. The spread in the curves is much greater than for the contact-heater cycle. The phenomenon of distribution loss is much more complex when flashing of drains from a high-rise heater to a lower heater is involved; the spread would occur even if $(H - h)$ were constant throughout the state line. In this type of cycle there appears to be a small gain (insignificant in practice) if the lower pressure heaters have slightly less than the average rise, and the higher pressure heaters have slightly more than the average rise per heater. The reference level for the flashed-heater arrangement is higher than that for the contact-heater cycle because of the inherent (arrangement) loss due to the use of flashed heaters. The magnitude of this loss may be calculated by the methods previously described.

Many other interesting results may be deduced by this method. For example, in a given heater cycle, if an extra heater is inter-

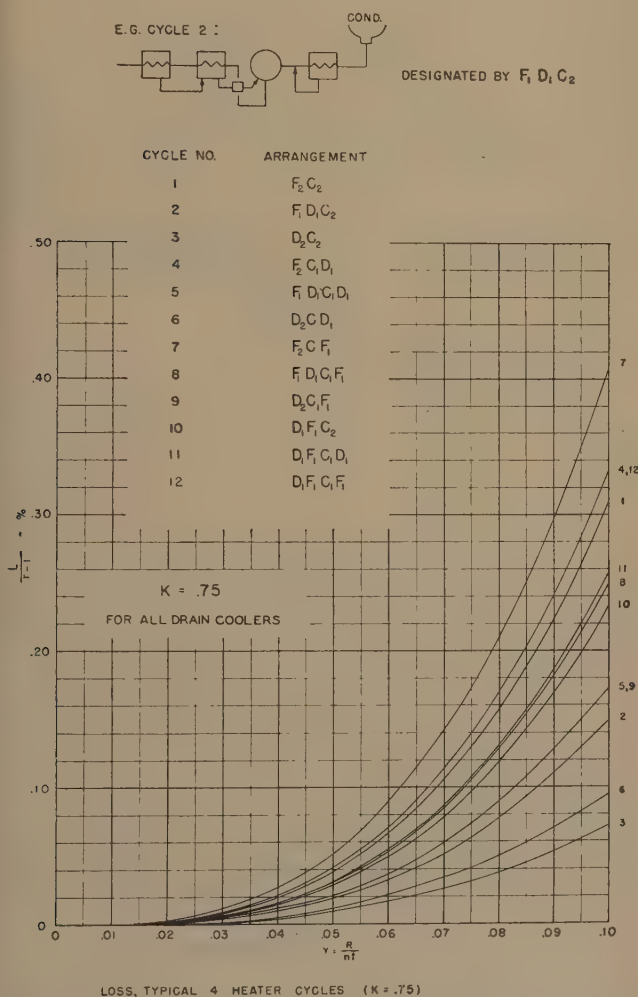


FIG. 7 ARRANGEMENT LOSS FOR TYPICAL 4-HEATER CYCLES AS A FUNCTION OF THE DIMENSIONLESS VARIABLE, y

the best possible distribution) is less than 2 Btu per kw-hr in all normal cases; in most it is less than 1 Btu per kw-hr.

Since we are interested only in the intrinsic loss due to maldistribution, it behooves us to use the simplest cycle for analysis, namely, the ideal or contact heater cycle. If the rises are unequal, then the dimensionless variable y no longer is constant, and must be identified by a subscript corresponding to the order of the heater in the cycle. The throttle flow is clearly given by

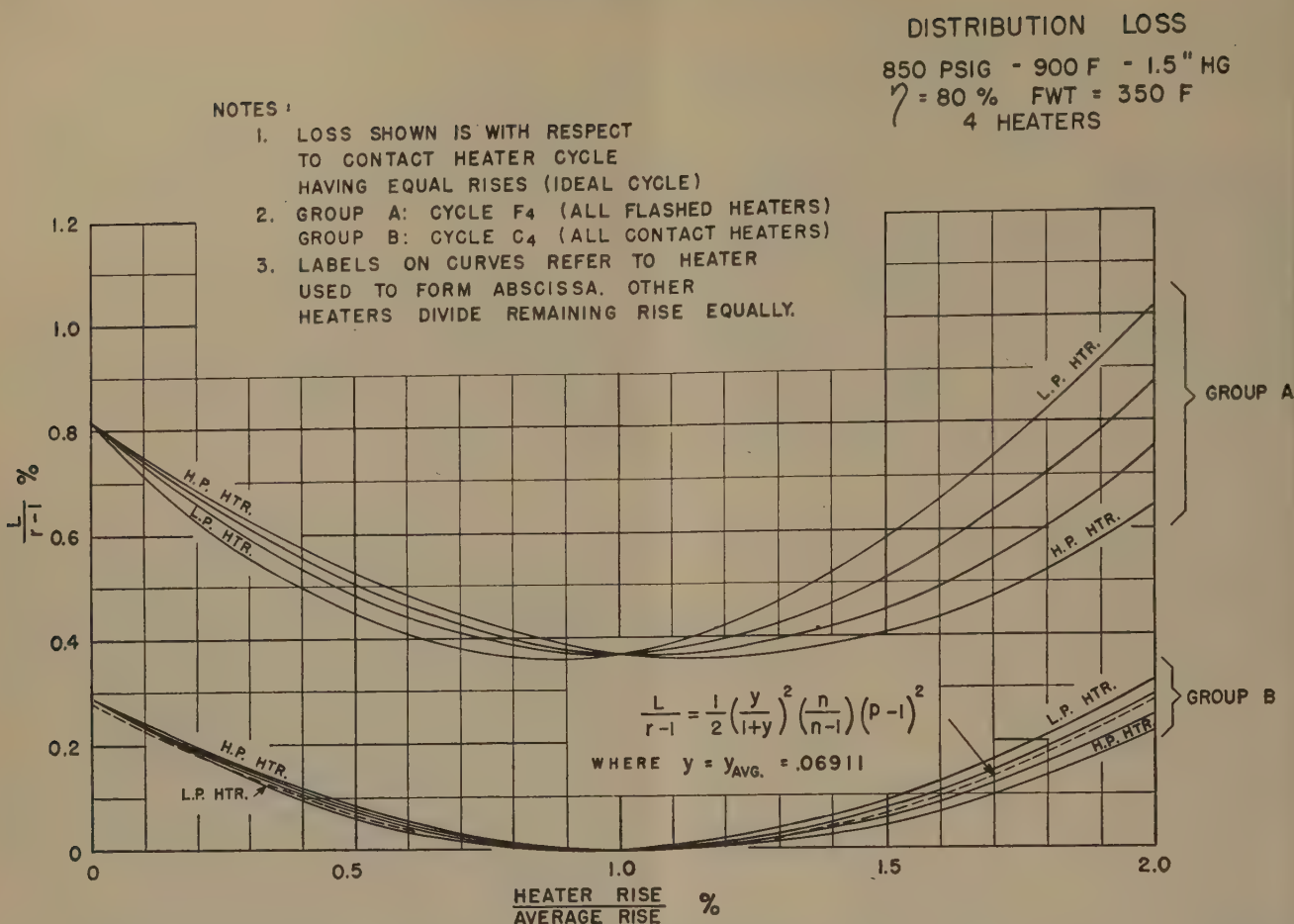


FIG. 8 DISTRIBUTION LOSS FOR 850 PSIG—900 F—1.5 IN. HG CYCLE. GROUP A, ALL FLASHED HEATERS; GROUP B, ALL CONTACT HEATERS

posed midway between any two heaters, it may be shown that the loss (negative, hence a gain) is given by

$$\frac{L}{r-1} = -\frac{1}{4} \frac{y^2}{1+y} \text{ (approx.)} \dots \dots \dots [22]$$

In a typical four-heater cycle this gain was shown to be approximately $\frac{1}{4}$ per cent.

Frequently it is necessary to remove a given heater from service for maintenance or other reasons. Since the source of steam (turbine) is essentially infinite and pressure drops in extraction piping are normally small, the pressures in the next lower and next higher pressure heater will remain unchanged, to a first approximation. For this reason, the heater above the heater removed from service will assume the duty of the heater removed. Thus its duty is essentially doubled; its rise is twice that of a normal heater. The heater which has been "cut out" has zero rise. Thus the values of α for these two heaters are, respectively, 1.0 and -1.0. Assuming the other heaters retain their original rises, the quantity

$$\sum_{n=1}^{n=n} \alpha_n^2$$

in Equation [21] has a value of 2.0. From Equation [21] the loss is

$$\frac{L}{r-1} = \left(\frac{y}{1+y} \right)^2 \dots \dots \dots [23]$$

In the average four-heater cycle the value of y is usually about 0.075. The loss due to cutting out a single heater of a four-heater cycle is therefore approximately 1 per cent. Similar values may be derived for other conditions. It must be remembered that this loss applies only when other conditions of the cycle remain constant. Undoubtedly, the heater which assumes twice its normal duty will have an increase in terminal difference and pressure drop, thus further increasing the total loss. Increased pressure drop in the extraction line also will tend to increase this loss. These effects may be analyzed by other means.

Similar analysis may be used to evaluate the change in heat rate due to adding or removing a heater in design of a cycle, distortion of the rise of a single heater, interposing more than one heater between two existing heaters, etc.

TERMINAL DIFFERENCE AND PRESSURE-DROP LOSS

All losses in a heater cycle with respect to the ideal cycle basically result from some mechanism which causes a decrease in heat supplied (throttle flow per pound of condenser flow), or an increase in heat rejected. Usually these result from a decrease in extraction quantity per unit of condenser flow. This decrease is caused by an increase in heat given up per pound in the heater.

When pressure drop is introduced between turbine and heater, an increase in the quantity $(H - h)$ occurs because there is a normal change (positive or negative) in the quantity $(H - h)$ with pressure throughout the state line; also, since the enthalpy h of liquid in the heater corresponds to a lower pressure than that at which steam is extracted, there is an increase in the quan-

tity $(H - h)$. The total change in $(H - h)$ due to interposition of 10 per cent pressure drop in the extraction line may be expressed algebraically. If we let Δ_P be the change in liquid enthalpy for 10 per cent pressure drop and Δ_{t_P} be the change in heat given up per pound of extracted steam when this pressure drop is introduced

$$\Delta_{t_P} = \Delta(H - h)_P + \Delta_P \dots \dots \dots [24]$$

Using the nomenclature of the previous paper² and differentiating the quadratic approximation to evaluate Equation [24]

$$\Delta_{t_P} = \left(1 + \frac{2ax + b}{R_s}\right) \Delta_P \dots \dots \dots [25]$$

When terminal difference is introduced at a heater, similar effects occur. The normal change in $(H - h)$ is induced by the rise in heater pressure which accompanies the imposition of terminal difference. Its value is

$$\Delta(H - h)_{TD} = \frac{2ax + b}{R_s} \Delta_{TD} \dots \dots \dots [26]$$

where Δ_{TD} is the change in liquid enthalpy of drains corresponding to 5 F terminal difference. In addition, mixing of hot drains with cooler condensate increases the heat given up per pound by the extracted steam. This is the larger of the two effects.

The total increase in heat given up in the heater due to pressure drop and terminal difference is the sum of the two increases, that is

$$\Delta t = \Delta_{t_P} + \Delta_{t_{TD}} \dots \dots \dots [27]$$

Combining Equations [25] and [26], and taking account of cooling of the drains when they are mixed with condensate leaving the heater (as in a pumped heater), we obtain the net change in heat given up in the heater due to the two effects. It is convenient to designate the parenthetical quantity in Equation [25] by K_3 . Thus

$$\Delta t = K_3 \Delta_P + \left(K_3 - \frac{y}{1 + y}\right) \Delta_{TD} \dots \dots \dots [28]$$

The quantities Δ_P and Δ_{TD} of Equation [28] are given in Fig. 9, for typical values; terminal difference = 5 F, and pressure drop = 10 per cent of turbine shell pressure. For other values, the quantities must be prorated.

METHOD OF CALCULATION

Fig. 9 and Equation [28] provide a method for calculation of the increase in heat given up in each heater. It may be shown that to a close approximation the fractional decrease in throttle flow, hence the quantity $L/(r - 1)$, may be found from the following equation

$$\frac{-\Delta w}{1 + w} = \frac{L}{r - 1} = \frac{\sum \Delta t}{t + \frac{\sum \Delta t}{n}} \left(\frac{y}{1 + y}\right) \dots \dots \dots [29]$$

The quantity K_3 may be determined for each heater if the constants a and b of the quadratic approximation for the steam conditions under consideration and the feedwater enthalpy at each heater are known. The constants may be determined from the previous paper,² and the feedwater enthalpies are known for any given cycle from the heat-balance diagram.

To determine the pressure drop or terminal-difference loss for any cycle, determine for each heater the quantities Δ_P and Δ_{TD} from Fig. 9, and evaluate the quantity K_3 for each heater (the

only variable with respect to heaters is the quantity x , a function of the enthalpy of the condensate leaving the heater). Evaluate both components of Equation [28] for each heater. These components are proportional, respectively, to the pressure drop and terminal-difference loss for that heater, and frequently are useful in calculating "change" in loss as the latter are changed. Substitute the results in Equation [29] to determine the total cycle loss due to pressure drop and terminal difference, using an average value of y for the cycle.

This method has been checked by extremely accurate calculations of 16 different cycles from 400 psig to 2400 psig, and 750 F to 1000 F. The results obtained by the usual (long) method check the results found by this analytical method almost precisely.

It may be shown that to a rough approximation terminal difference causes a loss in heat rate given by

$$\text{per cent loss due to 1 deg TD} = 0.01 \times \frac{\text{heater rise, Btu}}{50} \times \frac{r - 1}{2.0} \dots \dots [30]$$

Although not strictly accurate, this form frequently is useful.

The loss due to pressure drop depends upon the range of temperature of the drains of the heater where such loss occurs. Its value may be expressed by

$$\text{per cent loss due to 1 per cent pressure drop} = Z \times \frac{\text{heater rise, Btu}}{50} \times \frac{r - 1}{2.0} \dots \dots [31]$$

where Z , a function of heater-drain temperature, varies from 0.004 at 100 F to 0.011 at 400 F. From this relationship larger pressure drops can be tolerated for the lower pressure heaters, and the loss canceled by lower pressure drops on the high-pressure extractions. Such a shift makes the larger pipes smaller and the smaller pipes only slightly larger. This makes expansion and support of the usually large low-pressure piping easier, and reduces cost without affecting adversely the heat rate.

MISCELLANEOUS CYCLE CORRECTIONS

It is of value to the heat-balance designer to be able to estimate the effect of addition or subtraction of various quantities of heat in the cycle. These quantities arise as the result of generator coolers used in the condensate circuit, external steam or water supplied to a heater, subcooling of condensate, boiler-feed-pump power, etc. In the previous analysis of the cycle, the last item has been disregarded because of the ease with which it may be taken into account by such methods.

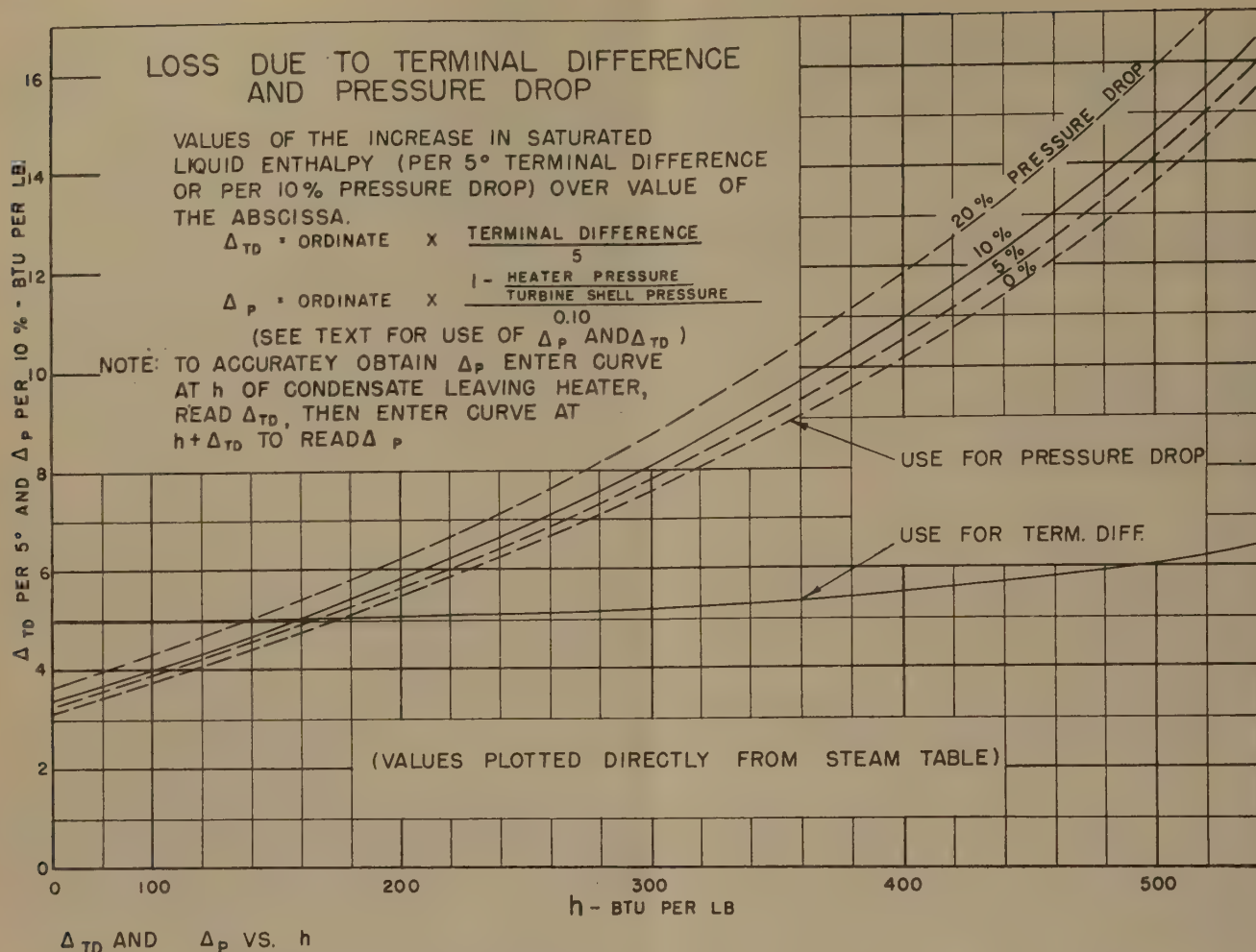
For all deviations of this kind the resulting change in heat rate occurs by virtue of two separate and distinct effects, namely: (1) the major (and obvious) effect of addition or rejection of extra heat from the cycle; and (2) the resulting (but less obvious) change in throttle flow per pound of condenser flow.

Remembering that we have used the symbols Q and q to designate, respectively, heat supplied and heat rejected in a cycle, we may express these separate effects by an equation; in so doing, we use a simplified (approximate⁵) form of Equation [5], separating the two distinct effects

$$\frac{L}{r - 1} = \left(\frac{\Delta q}{q} - \frac{\Delta Q}{Q}\right) - \left(\frac{\Delta w}{1 + w}\right) \dots \dots [32]$$

⁵ For accuracy the right-hand member of Equation [32] should be divided by

$$\left(1 + \frac{\Delta Q}{Q} + \frac{\Delta w}{1 + w}\right)$$

FIG. 9 VALUES OF Δ_{TD} AND Δ_P FOR USE IN CALCULATING TERMINAL DIFFERENCE AND PRESSURE DROP LOSS

For illustrative purposes consider use of this equation to analyze subcooling in the condenser hot well. In this illustration suppose the condensate leaves the hot well at less than saturation enthalpy corresponding to the pressure, by s Btu per lb. Additional heat is rejected in the hot well in an amount s , for a condenser flow of 1 lb per hr. Since the original heat rejection (before subcooling occurred) was t , we may write

$$\frac{\Delta q}{q} = \frac{s}{t} \quad [33]$$

In this instance there is no change in heat supplied.

For simplicity consider the low-pressure heater to be a contact-type heater. The increase in extraction quantity for the low-pressure heater is given by

$$\Delta w_1 = \frac{s}{t} \quad [34]$$

The flow leaving the low-pressure heater is given by

$$(1 + w_1) = (1 + y_1) + \frac{s}{t} \quad [35]$$

The flow leaving the top heater is seen to be

$$(1 + w)_s = \left[(1 + y_1) + \frac{s}{t} \right] (1 + y)^{n-1} \quad [36]$$

from which it is deduced that

$$\frac{\Delta w}{1 + w} = + \frac{s}{t(1 + y_1)} \quad [37]$$

The loss may be found by substitution in Equation [32]

$$\frac{L}{r - 1} = \frac{s}{t} \left(\frac{y_1}{1 + y_1} \right) \quad [38]$$

As a further example, consider the inclusion of generator coolers in the condensate circuit. The generator loss, ΔP , is normally charged against the heat rate, this heat having been rejected to the atmosphere. If it is used to heat condensate, we must regard the change as a "negative" change in heat rejected. Since the heat rejected in a cycle is $P(r - 1)$, where P is the power (energy) output, we may write

$$\frac{\Delta q}{q} = - \frac{\Delta P}{q} = - \frac{\Delta P}{P(r - 1)} \quad [39]$$

In a manner similar to the analysis used for subcooling, it may be shown that

$$\frac{\Delta w}{1 + w} = - \frac{\Delta P}{t(1 + y_1)} = - \frac{\Delta P}{P(r - 1)(1 + y_1)} \quad [40]$$

Using Equation [30] the loss (negative, hence a gain) is given by

$$\left. \begin{aligned} \frac{L}{r-1} &= -\frac{\Delta P}{P(r-1)} - \left[-\frac{\Delta P}{P(r-1)(1+y_1)} \right] \\ &= -\frac{\Delta P}{P} \left(\frac{1}{r-1} \right) \left(\frac{y_1}{1+y_1} \right) \end{aligned} \right\} \dots [41]$$

It is clear from Equation [41] that the greater the rise in the low-pressure heater, the greater is the gain due to recovering generator losses by use of condensate coolers. Since the rise in low-pressure heaters rapidly falls off at lighter loads, the effectiveness of recovery decreases. Partially compensating is the fact that fractional generator losses increase at lighter loads, tending toward a constant net gain.

The foregoing examples are simple illustrations of the power of the dimensionless variable y , used in the analysis of the regenerative cycle. Many other more complex evaluations are possible. For example, in the evaluation of the effect of evaporators on a feedwater-heating cycle several definitions of heat rate frequently are encountered. One may question whether the turbine-heater cycle should be credited for the entire quantity of heat supplied to the evaporated make-up. Application of methods of thought such as have been illustrated readily clarify this situation. A simple method has been developed whereby only a strictly fair credit may be given. Furthermore, one is permitted to discover the effect of locating the evaporator at any given point in the heater cycle, the thermal advantage of evaporator condensers, and other enlightening facts.

ARRANGEMENT LOSS IN TYPICAL FIVE-HEATER CYCLE

Several five-heater cycles have been calculated to illustrate the loss with respect to the ideal cycle, using the following assumptions:

Steam conditions: 1200 psig, 900 F, 1 in. Hg abs
Turbine efficiency = 80 per cent (internal efficiency)
Feedwater heated to 90 per cent of optimum rise
Drain cooler effectiveness, $k = 0.75$

The heat rate for an ideal five-heater cycle with these steam conditions is 8896 Btu per kw-hr. Calculations using the method herein outlined reveal the losses given in Table 1.

TABLE 1 ARRANGEMENT LOSS IN FIVE-HEATER CYCLES

Cycle	Cycle designation ^a	Loss with respect to ideal cycle, per cent
1	F ₅	1.541
2	F ₄ D ₁	1.09
3	F ₃ C ₁ F ₁	0.667
4	F ₃ C ₁ D ₁	0.586
5	F ₂ C ₂	0.565
6	F ₂ D ₁ C ₁ F ₁	0.433
7	F ₂ D ₁ C ₁ D ₁	0.364
8	F ₂ D ₁ C ₂	0.340
9	F ₁ D ₂ C ₂	0.202
10	D ₂ C ₁ D ₁	0.157
11	D ₂ C ₂	0.135

^a Beginning at high-pressure heater.

Table 1 indicates that the poorest cycle is the one using five flashed heaters. However, by addition of a drain cooler to the lowest pressure heater, the loss is reduced approximately 0.45 per cent. Alternatively, by inserting a contact-type heater at the second position, as is frequently done in power-plant practice, the loss is reduced by almost 0.9 per cent. If both substitutions are made, however, the additional gain is less than 0.1 per cent. Comparing cycle 6 with cycle 3, if a drain cooler is inserted on the flashed heater above the contact heater, the gain is almost 0.25 per cent. By modifying further the latter cycle to include a drain cooler on the lowest flashed heater, a gain of less than 0.1 per cent is made. Note that addition of this same drain cooler to the original cycle (cycle 1) resulted in 0.45 per cent gain. The largest gain in the remainder of the list is made between cycles

8 and 9 when two drain coolers are used above the contact heater.

It is important to note that these cycle losses do not take into account the additional loss due to terminal difference in a drain-cooler heater, or in a flashed heater. This must be borne in mind if one of two contact heaters is, in reality, a pumped heater, and the loss is being compared with a similar cycle in which both contact heaters are truly of this type, that is, one must make certain in any given instance that extra terminal difference loss, if it exists, is taken into account.

Example. To illustrate the method of calculating loss in a cycle, consider the turbine-heater cycle in Fig. 10 with the following conditions:

Steam conditions: 1250 psig, 950 F, 1.5 in. Hg
Initial enthalpy = 1468.0 Btu per lb
State line efficiency = 0.85
Theoretical steam rate = 5.804 lb per kw-hr
Used energy = 499.8 Btu per lb
Turbine exhaust enthalpy = 968.2 Btu per lb
Feedwater temperature = 410 F
 $t = 908.5$ Btu per lb
 $a = -154.0$; $b = 132.7$
Average rise per heater = 65.22 Btu per lb
Average $(H - h) = 929.9$
 $y_{avg} = 0.07014$; $\frac{y_{avg}}{1 + y_{avg}} = 0.06554$

Accurate calculation of an ideal cycle with these conditions, but having all contact heaters and no pressure drop or terminal difference, yields a heat rate of 8501.2 Btu per kw-hr. Calculation of the actual cycle in Fig. 10 by extremely accurate methods gives a heat rate of 8572.8 Btu per kw-hr. The total true loss with respect to the ideal is therefore 0.842 per cent.

Calculations similar to those used for Fig. 7 give an arrangement loss for this cycle, described as the F₂D₁C₂ cycle, of 0.197 per cent. Calculations, made by the methods described give a distribution loss of 0.028 per cent. The terminal-difference and pressure-drop losses are calculated in Tables 2 and 3, respectively.

It is seen from the definition of K_3 that

$$K_3 = 1.2545 - 0.591x$$

The "possible rise" for these steam conditions is 520.9 Btu per lb.

Using Equation [29]

$$\frac{L}{r-1} = \frac{58.529}{908.5 + 11.71} (0.06554) = 0.4169 \text{ per cent}$$

The heat rate is approximately 2.51 Btu per Btu. From this the loss due to terminal difference and pressure drop is

$$L = 0.4169 \times 1.51 = 0.63 \text{ per cent}$$

Accurate calculation using conventional means gives a true loss of 0.6351 per cent, an excellent check of the entire method.

Note that the loss due to either pressure drop or terminal difference at any heater can be obtained by proration, using appropriate values in columns (A) and (B) of Tables 2 and 3. Thus the loss due to 10 per cent pressure drop at the top heater is

$$\frac{L}{r-1} = \frac{9.518}{58.529} \times 0.4169 = 0.0678$$

or

$$L = 0.0678 \times 1.51 = 0.1025 \text{ per cent}$$

It is easily determined, too, that more of the 0.63 per cent loss is due to pressure drop than to terminal difference

$$\frac{37.137}{58.529} \times 0.63 = 0.40 \text{ per cent is due to pressure drop}$$

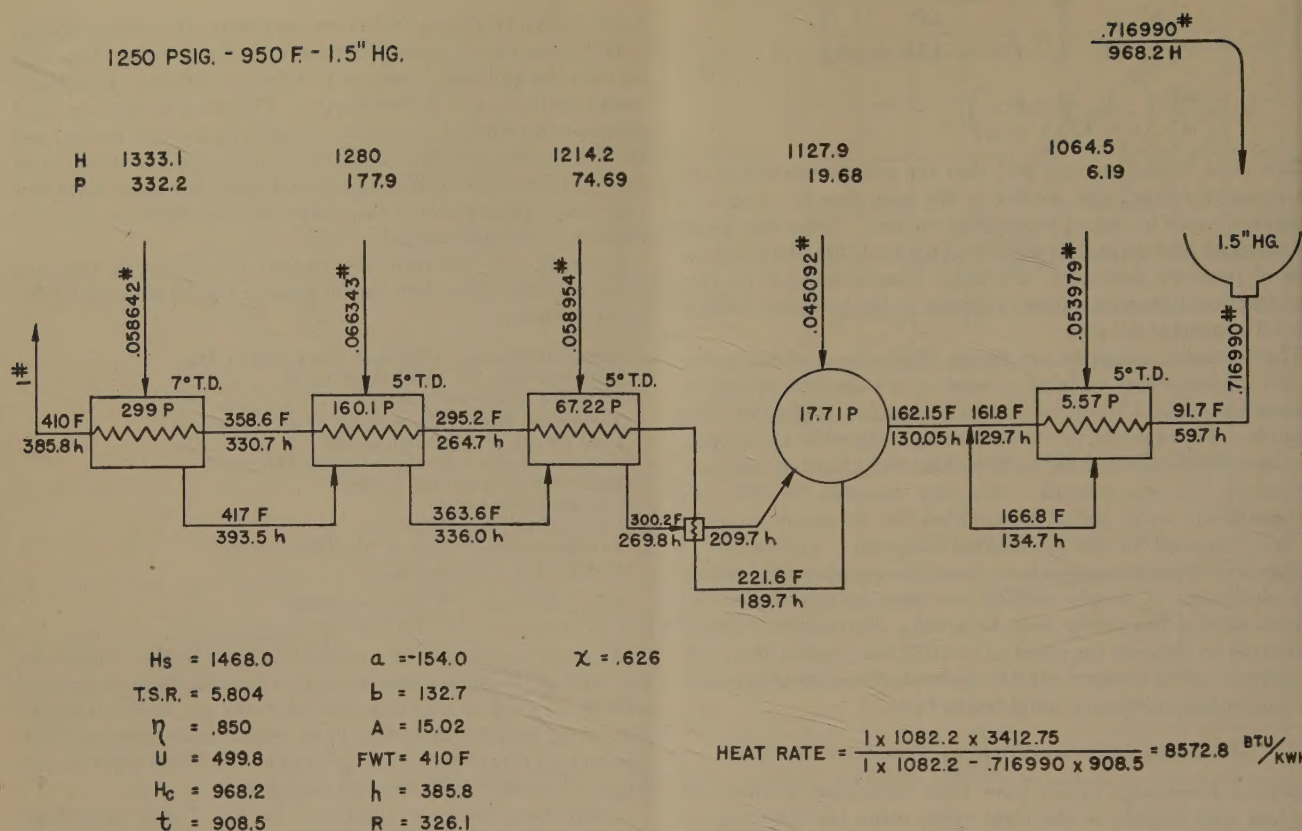
FIG. 10 ACTUAL $F_2D_1C_1P_1$ CYCLE USED IN ILLUSTRATIVE EXAMPLE

TABLE 2 TERMINAL DIFFERENCE LOSS CALCULATION

Heater	Rise, Btu	Σ = accumulated rise	K_2	Terminal difference, deg F	h , Btu per lb	$\Delta TD \times \frac{TD}{5}$, Btu per lb	$K_2 - 0.06554$	Col. (A) $\times \Delta TD = \Delta t_{TD}$
LP	70	0.1344	1.1753	5	129.7	5	1.1098	5.549
4	60	0.2496	1.1071	0	189.7	0	1.0416	0
3	75	0.3936	1.0221	5	264.7	5.13	0.9566	4.907
2	66	0.5203	0.9470	5	330.7	5.28	0.8815	4.654
HP	55.1	0.6260	0.8846	7	385.8	7.67	0.8191	6.282
Total								21.392

NOTE: $\frac{326.5}{5} = 65.22 = \text{average rise.}$

TABLE 3 PRESSURE-DROP LOSS CALCULATION

Heater	Pressure drop, per cent	$h + \left(\Delta TD \times \frac{TD}{5} \right)$	$\Delta P \times \frac{(B)}{10}$ (per cent pressure drop)	Col. (B) $\times K_1 = \Delta t_p$
LP	10	134.7	4.59	5.395
4	10	189.7	5.60	6.200
3	10	269.83	7.32	7.482
2	10	335.98	9.02	8.542
HP	10	393.47	10.76	9.518
From TD calc.				37.137
				21.392
				$\Delta t = 58.529$

NOTE: Average $\Delta t = \frac{58.529}{5} = 11.71.$

and

$$\frac{21.392}{58.529} \times 0.63 = 0.23 \text{ per cent is due to terminal difference.}$$

The total cycle loss is found by adding the various components, Table 4. The results indicate that the analytical method checks the true cycle loss, 0.842 per cent. The difference is only about 0.01 per cent, or 1 Btu per kw-hr.

CONCLUSION

The objective of this paper has been to extend the previous work to cover assumptions made therein, and to present a somewhat unorthodox approach to analysis of feedwater heating. It would be impossible to present all of the ramifications, which unfold as one becomes familiar with the method, within the limited space here available. It is the author's conviction that all of the problems of the regenerative cycle are amenable to

TABLE 4 SUMMARY OF LOSSES
F₂D₁C₂ Cycle, Fig. 10

Loss item	Calculated per cent loss
Terminal-difference loss.....	0.23
Pressure-drop loss.....	0.40
Arrangement loss.....	0.197
Distribution loss.....	0.028
Total loss	0.855

analysis by these general methods. Undoubtedly, those who become familiar with them will make valuable contributions. The author will be obliged if such original contributions are transmitted to him, especially since it is his intention to publish an extensive collection of the results of his own analysis in the near future.

Discussion

C. S. BARKELEW, JR.⁶ Of the various losses analyzed in the present paper the determination of heater terminal-difference losses has perhaps the most general application. However, the author's equation for this loss apparently does not give valuations consistent with figures obtained by heat-balance calculations, based upon the assumption that the turbine-extraction pressures are fixed. It is pertinent to inquire what "conventional" methods of calculation were used to check the author's method.

The principal effect of introducing terminal difference is to shift the feedwater-heating duty to the next higher pressure heater; so that the loss depends upon the heat drop in the turbine between these extraction pressures, rather than upon the temperature rise in the heater. The author's formula apparently is not in accord with this conception of the effect of terminal difference. From certain statements in the text it might be inferred that the increase in heat rate charged to terminal difference is considered to be that which results when the extraction pressures are raised sufficiently to offset the heater terminal differences and maintain specified temperatures of the feedwater leaving the heaters.

A comparison of the terminal-difference-loss evaluation, obtained by applying the author's equation, and by heat-balance calculation, assuming constant extraction pressures, is given in Table 5 herewith, for the turbine in Fig. 10 of the paper, with all flashed heaters cascading drains to the condenser.

TABLE 5 COMPARISON OF TERMINAL-DIFFERENCE-LOSS EVALUATIONS

Heater no.	Extraction pressure, psig	Rise, Btu per lb	Per cent increase in heat rate per Btu (terminal difference)		Difference, per cent
			By Equation	By heat balance	
1	6.2	79.6	0.0122	0.0143	+17
2	19.7	56.0	0.0115	0.0182	+59
3	74.7	81.8	0.0105	0.0125	+19
4	178	67.9	0.0091	0.0094	+4
5	332	59.3	0.0090	0.0146	+62

Inasmuch as the heat-balance calculations are simple and straightforward, the only possibility of error in the loss evaluation appears to be in the assumption that the extraction pressures do not change with variation in feedwater temperatures due to terminal differences. In an actual turbine-heater combination, there is a small variation in the extraction pressures with change in heater performance; and a small change in the "distribution loss" may be associated with the terminal-difference loss. Since the distribution losses generally are of extremely small magnitude, the effect on the terminal-difference-loss valuation pre-

sumably is negligible. However, as a check, a heat balance was calculated with extraction pressures changed to correspond to the changes in turbine internal flows resulting from a terminal difference in the highest pressure heater, where changes in final feed temperature have the most noticeable effect on pressures. The calculated heat rate showed less than 0.001 per cent change for 10 deg F terminal difference over the heat rate calculated on the basis of fixed pressures.

A. O. WHITE.⁷ The writer's company has been using the author's methods for various purposes in connection with the calculation of turbine performance since their publication, and the material and methods he has presented provide a valuable supplement to the previous work, greatly extending its field of usefulness.

The writer considers this whole treatment of the regenerative cycle a marvelous piece of analytical work and it deserves more general use than it has at present, judging from the contacts he has had with the industry. It may be that the lack of an accurate method of evaluating the "cycle correction" has hindered its general acceptance, so that the present paper, by providing an accurate evaluation of the cycle correction for most actual cycle arrangements should promote its use with a resulting saving in calculation time and effort.

The writer knows of no better way of evaluating the relative economy of different cycles than the author's, and upon publication of the results of further analysis, which are promised for the near future, it should eliminate all need for detailed heat-balance calculations except a final set, based upon the actual arrangement to be used, for the purpose of selecting heaters and related equipment and perhaps defining contractual obligations.

There is only one criticism the writer has and that is that the use of the term "arrangement loss" does not seem quite natural as used in the paper. Arrangement loss seems to connote the loss due to all the variables in the heater arrangement, and some other term such as "surface heater loss" would give a better idea of what is involved, leaving arrangement loss and cycle correction as essentially equivalent.

One contribution which would be very valuable would be the extension of the method to the reheat cycle with heaters above the reheat point. The writer has used it for the case with the top heater out of the high-pressure section exhaust, by considering the cycle from the reheater outlet to the condenser as a separate cycle by the author's method, and then adding the high-pressure section to it as a topping unit whose output was additive in terms of flow to the reheater. From this the over-all heat rate could be easily obtained.

AUTHOR'S CLOSURE

Mr. Barkelew's comment that the terminal-difference-loss determination has the most general application probably included the pressure-drop-loss method as well, since they go hand in hand, and the latter loss frequently is the larger. Fortunately, the method is exceptionally accurate for these two losses, usually the chief components of cycle loss.

In last analysis, any loss in the heater cycle is found to depend fundamentally not upon heat drop in the turbine, or upon rise in the heater; minimum loss is obtained by securing the lowest average level of $(H - h)$, hence the maximum quantity of extraction. If one considers feedwater enthalpy at each heater constant (to eliminate the confusing effect of distribution loss), terminal difference increases $(H - h)$, by raising the extraction pressure

⁶ Senior Engineer, Electric Engineering Department, Public Service Electric and Gas Company, Newark, N. J.

⁷ Steam Design Section, Steam Turbine Engineering Division, General Electric Company, Schenectady, N. Y.

(hence H) to a value higher than hypothetically it would have to be to perform the heating task in a contact heater. The method presented in the paper quantitatively evaluates this effect and translates it into cycle loss. By keeping turbine pressures constant Mr. Barkelew takes into account the change in h , but not the change in $(H - h)$, which always occurs along any state line as the pressure changes.

His statement that the principal effect of terminal difference lies in the shift of duty to the next higher heater is inadmissible, by definition. This shift of duty is defined as "distribution loss," and must not be included in the terminal-difference loss. Since distribution loss is from 2 to 3 times as great in an all-flashed-heater cycle as in contact or pumped-heater cycles, under the same conditions, these inadmissible effects are accentuated in the cycle used for checking.

Perhaps the method used by Mr. Barkelew in his attempt to check the accuracy of the method required reading of state-line enthalpy values from a small Mollier chart. This introduces inconsistencies of sufficient magnitude to render a check of these extremely small losses impossible. For checking the method the author used data from a hypothetical state line, analytically defined to any desired degree of accuracy by the quadratic approximation. All data used in making the heat balance were carried to 6 or 7 significant figures, and all chart reading eliminated. The results of a comparison of the idealized basic cycle (having no terminal differences and no pressure drops) with actual cycles checked within 0.01 per cent for the 24 cycles to which the check was applied. It does not seem likely that this is coincidence.

In addition, it would appear that when Mr. Barkelew assumed the turbine pressures constant, there would be a change in final feedwater temperature itself, as the terminal difference on the top heater changes. This assumption introduces an extraneous effect not chargeable, in the author's concept, to terminal difference. This conclusion seems to be borne out by the fact that the largest difference in Table 5 occurs at the top heater.

Despite the fact that the calculation unavoidably included extraneous effects, it may be deduced from Table 5 that Mr. Barkelew's method results in a calculated loss of 0.28 per cent whereas the author's value is 0.23 per cent, a difference of only 0.05 per cent in heat rate. Most of this probably can be attributed to the extraneous effects introduced by the method used.

The author is indebted to Mr. White, his former close associate for his kind words. Some time was required for the original method to receive the acceptance that it now enjoys in the author's company. Once facility in its use is gained, however, the user is well repaid for the time spent.

Mr. White's comment on the use of the term "arrangement loss" is well taken. The word was chosen originally because of the effect of permutation of the heater location on the cycle loss. While all surface heaters cause loss, the magnitude of the loss is a function of their location in the cycle, thus to some small extent justifying the word "arrangement" in this connection. Further addition of a drain cooler to a surface heater—another "arrangement" of the heater—reduces the loss.

In the author's opinion, application of the method to reheat cycles is best accomplished by consideration of the low-pressure portion as one cycle, and the high-pressure portion as an entirely separate cycle performing essentially at 100 per cent thermal efficiency, as Mr. White suggests. Data to be published later will indicate a method whereby the general principles of the terminal-difference-loss calculation probably can be applied to the reheat cycle. Basically, terminal-difference loss results from the equivalent of a "shift" of the state line to the right, giving higher enthalpies for the extracted steam. A heater at the exhaust of the high-pressure turbine in a reheat cycle might be considered as one with a very large "negative" terminal difference, acting to reduce the enthalpy of the steam supplied to it. Whether the method would be of enough advantage for this rather complex cycle to warrant its use is questionable. Probably, consideration of the cycle in two separate parts will be preferable, in the long run.